Complete evaporation of black holes near extremality

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New Trends in Mathematical Physics

November 7-12, 2022
Moscow

Based on joint works with I. Volovich

"Complete evaporation of black holes and Page curves ", **2202.00548**

"Quantum explosions of black holes and thermal coordinates", **2104.12724**

Overview

1. Black hole explosion
2. Proposal of this talk - constraints on EOS
   - Reissner-Nordstrom
   - Kerr
   - Schwarzschild de Sitter
   - Kerr-Newman,...
   - Does not work for Schwarzschild BH
3. Conclusions

Black hole explosion

- **Hawking temperature of Schwarzschild BH**

$$\mathbf{T = \frac{1}{8\pi M},}$$

M is the mass of BH *(calculations in the stationary background)*

- **If $\mathbf{M \to 0}$ (evaporation) then $\mathbf{T \to \infty}$**

- **Energy density of radiation**

$$\mathbf{E \sim \frac{1}{M^4}}$$

S. W. Hawking, "Black hole explosions?"Nature, v.248 (1974), 30.

Complete evaporation of BH

- **Don't go beyond the Planck scale**

  Beyond Planck scale: quantum gravity, modified gravity,...

- **Complete evaporation:**

  (at least in Semiclassical effective theory)

  Hawking, Page, Susskind, Maldacena, ...

  This talk based on 2202.00548 & 2104.12724

Proposal of this talk - constraints = special equations of state

- **Reissner-Nordstrom, Schwarzschild de Sitter, ...**
  - **Special form of the state equations:**

    Special charge dependence on BH mass

- **Schwarzschild BH**
  - **open problem**
  - **new thermal coordinates**

Hawking temperature of Black Hole

$$\mathbf{ds^2 = -f(r)dt^2 + f(r)^{-1}dr^2 + r^2 d\Omega^2}$$

$$\mathbf{f(r_h) = 0, \qquad f'(r_h) \neq 0}$$

Hawking temperature

$$\mathbf{T = \frac{1}{4\pi} f'(r_h)}$$

$$\mathbf{f = 1 - \frac{2M}{r}, \qquad r_h = 2M, \qquad T = \frac{1}{8\pi M}}$$

- **There is the Big Explosion problem with the completely evaporated Schwarzschild black holes.**

Reissner-Nordstrom BH

- **Reissner-Nordstrom black hole**

$$\mathbf{ds^2 = -f(r)dt^2 + f(r)^{-1}dr^2 + r^2 d\Omega^2}$$

$$\mathbf{f(r) = (1 - \frac{2M}{r} + \frac{Q^2}{r^2}) = (r - r_+)(r - r_-)/r^2}$$

  - **horizons** $\mathbf{r_\pm = M \pm \sqrt{M^2 - Q^2}}, \quad M \geq |Q|$

  - **temperature**

$$\mathbf{T = \frac{1}{4\pi} f'(r_+) = \frac{1}{2\pi} \frac{\sqrt{M^2 - Q^2}}{(M + \sqrt{M^2 - Q^2})^2}}$$

  - **Kretschmann scalar**

$$K_{RN} = \frac{48M^2}{r^6}(1 + \frac{2Q^2}{rM} + \frac{7Q^4}{48r^2M^2})$$

Main result

- **A scenario to avoid the BH explosion is proposed.**
- **We propose to use special EOS (Equations Of State),**

  i.e. special relations of M with other parameters.

RN black hole with special EOS

- We take the charge in the form

$$\mathbf{Q^2 = M^2 - \lambda(M)^2}$$

where $\lambda(M)$ is some given function, $0 < \lambda(M) < M$ *(it regulates proximity to extremeness)*

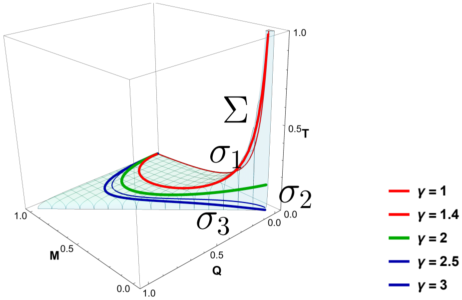
- The Hawking temperature T becomes

$$\mathbf{T = \frac{\lambda(M)}{2\pi(M + \lambda(M))^2}}$$

- If we take $\lambda(M)$ such that for small M it obeys $\mathbf{\lambda(M) = o(M^2)}$, then T tends to 0 as $\mathbf{M \to 0}$ and we get the complete evaporation of BH without $T \to \infty$ at the end of evaporation.
- Hence, for small M the evaporating black hole should be in the near extremal state.

RN with special EOS: curves σ_γ on surface Σ

- $\mathbf{T = \frac{1}{2\pi}\frac{\sqrt{M^2-Q^2}}{(M+\sqrt{M^2-Q^2})^2}}$ **defines surface Σ in (M, Q, T)-space.**
- **EOS:** $\mathbf{Q^2 = M^2 - \lambda(M)^2}$**. We take** $\mathbf{\lambda(M) = CM^\gamma \quad (*)}$



- $T \to \infty$ **as** $M \to 0$ **for** $\gamma < 2$**,**
- $T \to const \neq 0$ **as** $M \to 0$ **for** $\gamma = 2$
- $T \to 0$ **as** $M \to 0$ **for** $\gamma > 2$

Thermodynamics of RN BHs with EOS (*)

- **Temperature**

$$T = \frac{\lambda(M)}{2\pi(M+\lambda(M))^2}.$$

- **Entropy**

$$S_{RN} = \pi r_+^2 = \pi(M+\lambda(M))^2$$

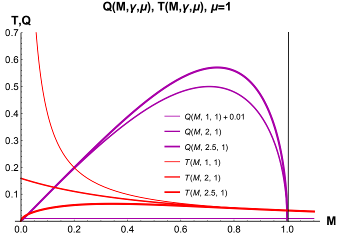
- **Free energy**

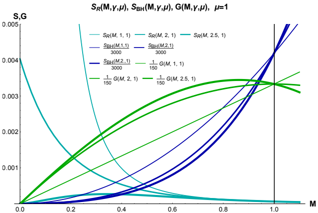
$$G_{RN} = M - TS = M - \frac{1}{2}\lambda(M)$$

- **Entropy S_{RN} and free energy G_{RN} go to 0 as $M \to 0$ for λ satisfying**

$$\mathbf{0 < \lambda(M) \leq CM^{\gamma}, \quad C > 0,\ \gamma > 2}$$

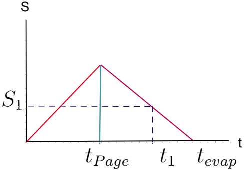
Mass dependence of Q, T, S_R, S_{BH} and F at $\lambda(M) = (M/\mu)^\gamma$ with different γ





- **for all $\gamma > 1$ there is a restriction on M, $M \leq 1$**
- **T and S_R at $\gamma > 2$ starting from $M = 1$, increase to a certain maximum value, then decrease to zero, i.e. the mass dependencies – deformed bell shapes**
- **for $\gamma = 2$ $T \to T_0 \neq 0$ as $M \to 0$**
- **for $\gamma < 2$ $T, S_R \to \infty$**
- **S_{BH} and F tend to zero at $M \to 0$ for all values of γ**
- **F first increases as the black hole mass decreases, but after some M_0 starts to decrease**

- **- Page form of the time evolution of S**



Explosion and information loss problem

- **The information loss problem can be formulated as follows:**
  - **If the Schwarzschild black hole is formed from a pure quantum state and then evaporates completely, this means**

    $|$**pure state** $>\rightarrow|$**mixed temperature state of radiation**$>$

    Information paradox appears when BH complete evaporates

- **The entropy of the initial pure quantum state is zero while entropy of the final radiation state is infinity.**

Black Hole Explosions Problem and Information Paradox

- **The information loss problem (Hawking, 1976) is closely related to the Black Hole Explosions Problem, since the radiation entropy S_R diverges for small M as M^{-3}.**

- **Alternative: the evaporation is incomplete and it stops when the Schwarzschild radius is close to the Planck length.**

  However, Hawking and Page considered complete evaporation process.

Time evolution of RN BH with EOS (*)

- **The loss of the mass and charge during evaporation of RN black hole is a subject of numerous consideration**
  Gibbons, Zaumen, Carter, Damour, Page'76, Hiscock'90, Gabriel'00, Sorkin'01,...

$$\frac{dM}{dt} = -A\sigma T^4 + \frac{Q}{r_+}\frac{dQ}{dt},$$

- **A is a positive constant and the cross-section σ is proportional to M^2 for small M**
- **in the second term**

$$\frac{dQ}{dt} = \frac{d}{dt}\sqrt{M^2 - \lambda^2} = \frac{M - \lambda\lambda'}{\sqrt{M^2 - \lambda^2}}\frac{dM}{dt}$$

Time evolution of RN BH with EOS (*)

- **For** $\lambda(M) = CM^{\gamma}$ **and small** M **one gets** $\frac{dM}{dt} = -C_1 M^{3\gamma-5} (*)$

- **(*) for** $\gamma > 2$ **and** $M(0) = M_0$ **has a solution**

$$\mathbf{M(t)} = \frac{\mathbf{M_0}}{(\mathbf{1} + \mathbf{B\,t})^{\frac{1}{3(\gamma-2)}}}, \quad \mathbf{B} = \frac{\mathbf{3(\gamma-2)C_1}}{\mathbf{M_0^{6-3\gamma}}}, \quad \gamma > \mathbf{2},$$

M_0 **and** B **are positive constants**

- **(*) for** $\gamma = 2$ **has solution** $\mathbf{M(t) = M_0 e^{-C_1 t}}$
  - **i.e. one gets an infinite large time of the complete evaporation of charged black hole under our constraint.**

Kerr black hole

- **Kerr metric in Boyer-Linquist coordinates**

$$
\begin{aligned}
ds^2 &= -\frac{\Delta - a^2\sin^2\theta}{\Sigma}dt^2 - 2a\sin^2\theta\frac{r^2+a^2-\Delta}{\Sigma}dtd\phi \\
&+ \frac{(r^2+a^2)^2 - \Delta a^2\sin^2\theta}{\Sigma}\sin^2\theta d\phi^2 + \frac{\Sigma}{\Delta}dr^2 + \Sigma d\theta^2,
\end{aligned}
$$

$\Sigma = r^2 + a^2\cos^2\theta, \qquad \Delta = (r-r_+)(r-r_-),$
$r_\pm = M \pm \sqrt{M^2-a^2}.$

- **Temperature**

$$T_{Kerr} = \frac{1}{4\pi}\frac{\sqrt{M^2-a^2}}{M(M+\sqrt{M^2-a^2})} \quad \textbf{compare} \quad T_{RN} = \frac{1}{2\pi}\frac{\sqrt{M^2-Q^2}}{(M+\sqrt{M^2-Q^2})^2}$$

- $M^2 \geq a^2.$

The Kerr black hole with EOS (**)

- **If we take** a **to be a function of** M **of the form**

$$\mathbf{a^2 = M^2 - \lambda(M)^2}, \qquad (**)$$

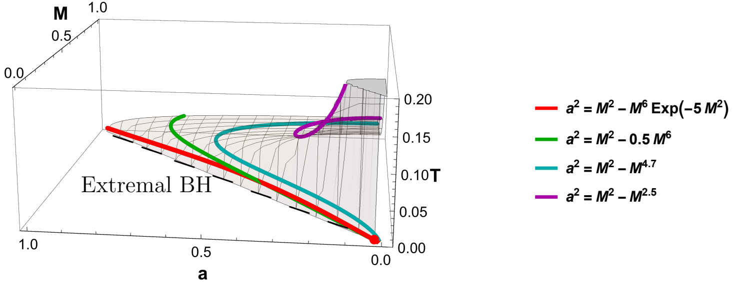
where the function $\lambda(M) > 0$ **then the temperature** T **becomes equal to**

$$T_{Kerr} = \frac{\lambda(M)}{4\pi M(M + \lambda(M))} \quad \textbf{compare} \quad T_{RN} = \frac{\lambda(M)}{2\pi(M + \lambda(M))^2}$$

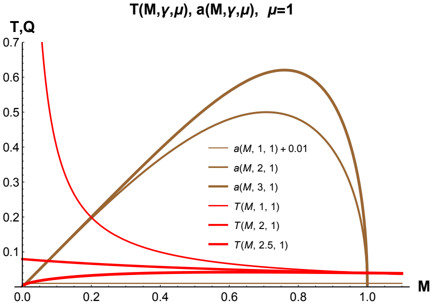
- If the function $\lambda(M)$ satisfies the bounds

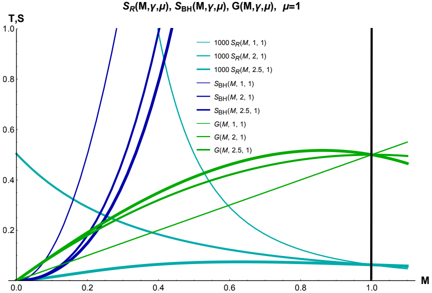
$$\mathbf{0} < \lambda(\mathbf{M}) \leq \mathbf{C}\frac{\mathbf{M}^{\gamma}}{\mathbf{A}+\mathbf{M}^{\gamma+1}}, \quad \mathbf{C} > \mathbf{0},\ \mathbf{A} > \mathbf{0},\ \gamma > \mathbf{2}$$

then $T \to 0$ as $M \to 0$ or $M \to \infty$.



The Kerr black hole with EOS (**)





The plots show the dependence of the temperature T (red), the angular momentum a(brown), the free energy G(green), the black hole entropy S (blue) and the radiation entropy (cyan) for

$$a^2 = M^2 - \lambda(M)^2, \qquad (**)$$

with function

$$\lambda(M) = (M/\mu)^\gamma$$

with different scaling parameter $\gamma = 1, 2, 2.5, 3$ and $\mu = 1$.

Schwarzschild-de Sitter, $\Lambda > 0$

- $ds^2 = -f(r)dt^2 + f(r)^{-1}dr^2 + r^2 d\Omega^2,\ f(r) = 1 - \frac{2M}{r} - \frac{\Lambda}{3}r^2,$
- **For** $0 < 3M\sqrt{\Lambda} < 1\,(\#)$ **3 roots of** $f(r) = 0$**:** $r_+, r_c, r_n < 0$

$$r_+ = \frac{2}{\sqrt{\Lambda}} \sin\left(\frac{1}{3} \arcsin 3M\sqrt{\Lambda}\right),$$

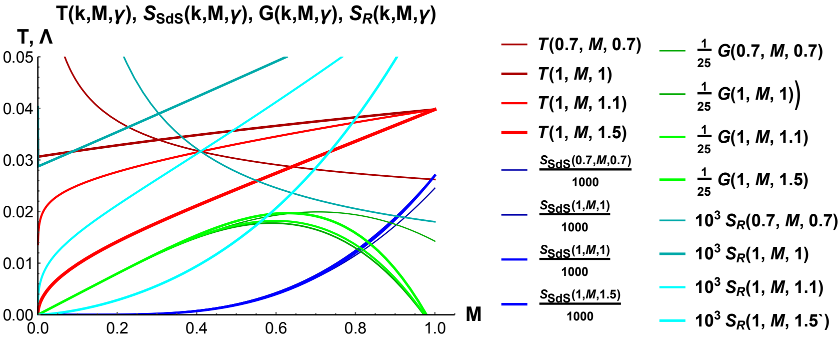
- **The Hawking temperature of BH**

$$T(M,\Lambda) = \frac{1 - 4\sin^2\left(\frac{1}{3}\arcsin\left(3\sqrt{\Lambda}M\right)\right)}{8\pi \sin\left(\frac{1}{3}\arcsin\left(3\sqrt{\Lambda}M\right)\right)}\sqrt{\Lambda}$$

- **For fixed** Λ **the temperature becomes infinite when** $M \to 0$**.**
- **The numerator is equal to zero at** $\Lambda = 1/9M^2$ **and this value of** Λ **realizes the bounded value of** Λ **admissible by inequality (#).**
- **Take** $\Lambda = \frac{1-\lambda(M)^2}{9M^2}$**,** $0 < \lambda \leq 1$

  One can check that if $\lambda(M) = o(M)$**, then** $T \to 0$ **as** $M \to 0$**.**

Schwarzschild-de Sitter, $\Lambda > 0$



The dependence of the temperature T (red), the entropy S_{SdS} (blue), the free energy G(green) and the radiation entropy S (cyan) on M for

$$\lambda(M) = kM^{\gamma} \quad \text{with different scaling parameter} \quad k \quad \text{and} \quad \gamma$$

Plots in light tones correspond dependencies with $T \to 0$ at $M \to 0$
Darker tones correspond to increasing temperature at $M \to 0$, or $T(0) \neq 0$.

Conclusions

- **The problems of the BH explosion of completely evaporating black holes is considered.**
- **It is shown that the special EOS permits to avoid the RN BH explosion.**
- **Similar results are obtained for Kerr and Schwarzschild-de Sitter black holes.**

Thank you !