A refined Eigenstate Thermalization Hypothesis that evades known counterexamples

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Simple many-body setup

$$H = \sum_{j=1}^{N} \left(J_{\alpha\beta}\, \sigma_j^{\alpha} \sigma_{j+1}^{\beta} + h_{\alpha}\, \sigma_j^{\alpha} \right)$$

translation-invariant: $N+1$ "=" 1

$O = \sigma_1^z$ (in general, $O = \prod_{l=1}^{k} \sigma_{j_l}^{\alpha_l}$ - k-local operator, k independent on N)

E_j - eingenenergies, $|E_j\rangle$ - eigenstates, $j = 1, 2, \dots, 2^N$

$\varepsilon_j = \dfrac{E_j}{N}$ - finite in thermodynamic limit $N \to \infty$

Diagonal Eigenstate Thermalization Hypothesis - colloquially

$$H = \sum_{j=1}^{N} \left(J_{\alpha\beta}\, \sigma_j^{\alpha} \sigma_{j+1}^{\beta} + h_{\alpha} \sigma_j^{\alpha} \right)$$

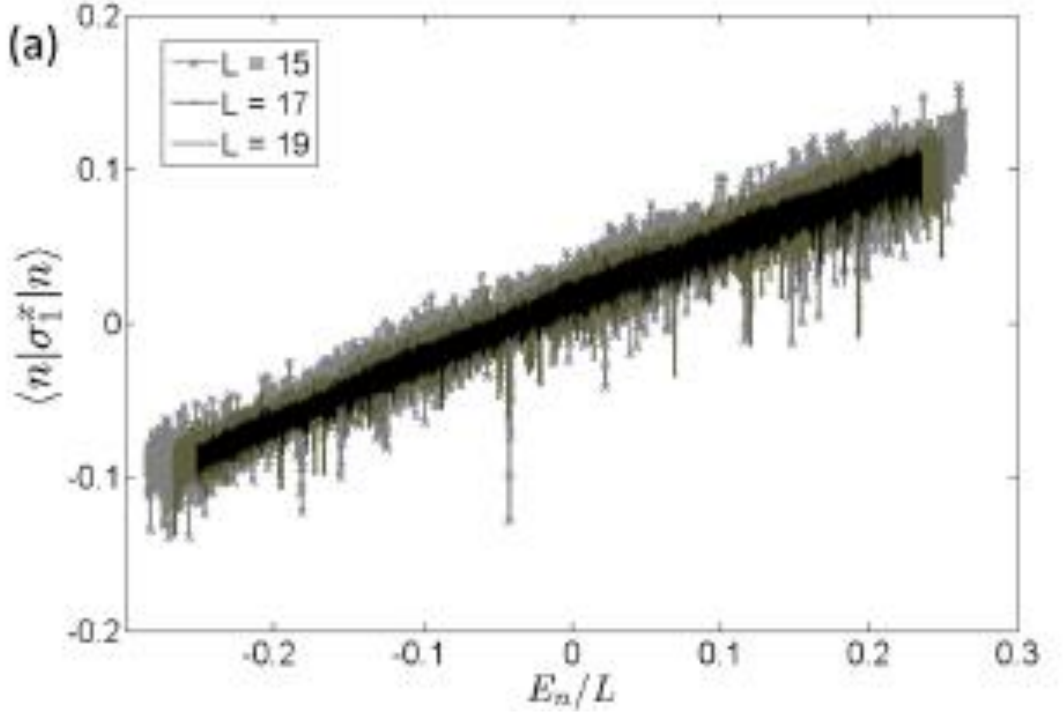
ETH colloquially:

$$\langle E_\nu | O | E_\nu \rangle \approx \langle E_\mu | O | E_\mu \rangle$$

whenever

$\varepsilon_\mu \approx \varepsilon_\nu$ and N is large

Deutsch J M 1991 *Phys. Rev.* A **43** 2046
Srednicki M 1994 *Phys. Rev.* E **50** 888



H. Kim, T. N. Ikeda, D. A. Huse
Phys. Rev. E **90**, 052105 (2014)

Diagonal ETH – rigorous but wrong formulation

$\forall\ \mathcal{E} > \mathcal{E}_{min}\ ,\ \Delta\mathcal{E}\ \ \ \mathcal{E}+\Delta\mathcal{E} < \mathcal{E}_{max}$

$$\lim_{\Delta\mathcal{E}\to 0}\ \lim_{N\to\infty}\ \max_{\mathcal{E}_\nu,\mathcal{E}_\mu\in[\mathcal{E},\mathcal{E}+\Delta\mathcal{E}]}\ \left|\langle E_\mu|O|E_\mu\rangle-\langle E_\nu|O|E_\nu\rangle\right| = 0$$

Dymarsky 2018-2022

rigorous formulation

however, is falsified by counterexamples:

- integrable systems
- scarred systems
- ...

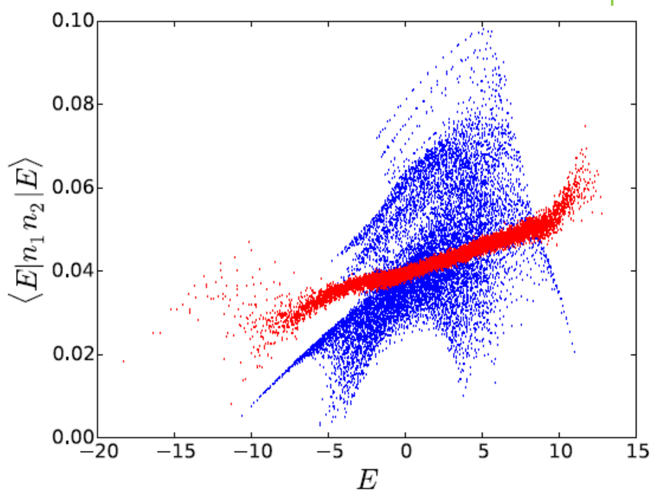
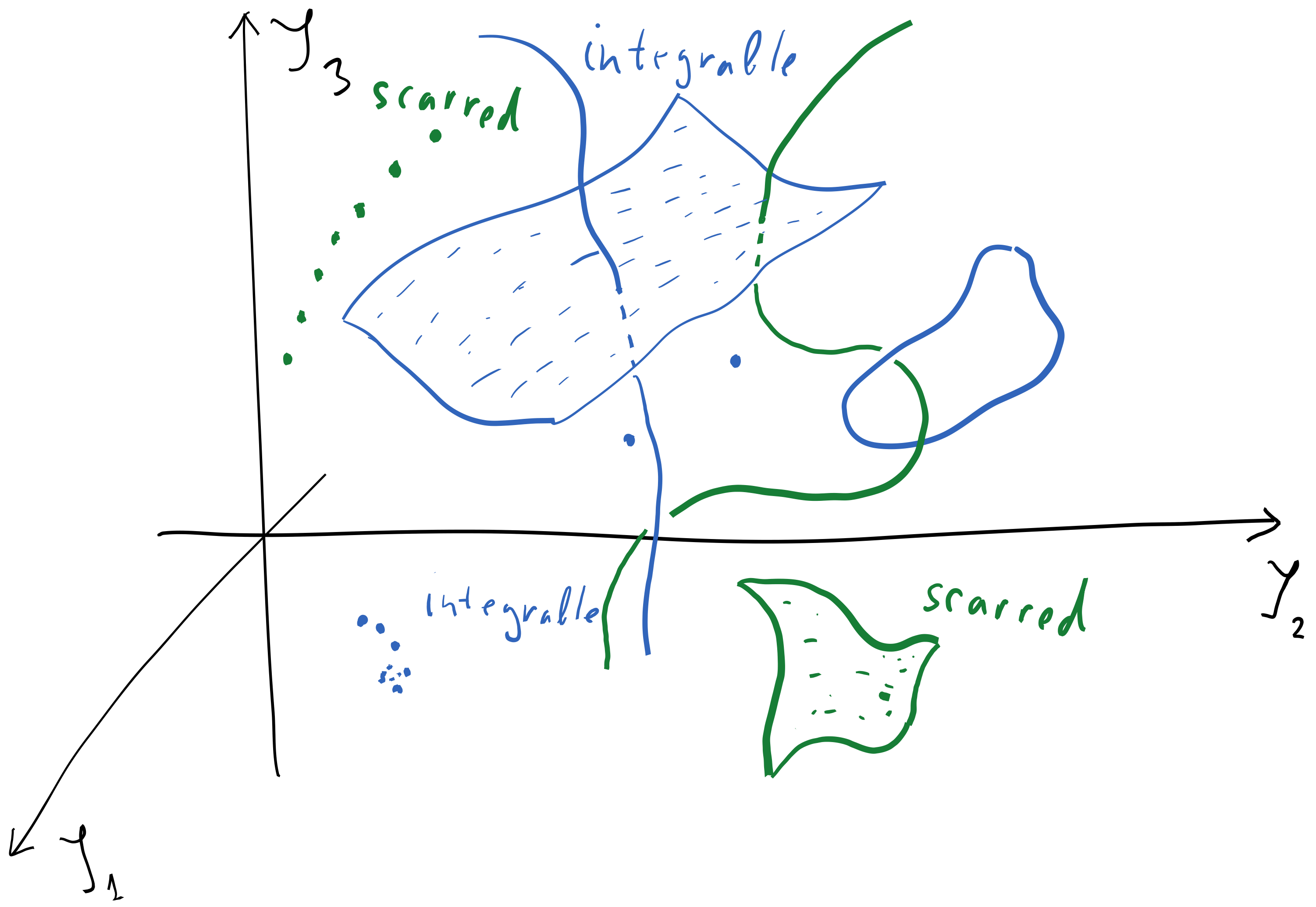


Figure 6. Numerical results for the hard core boson model of equation (18) for 6 particles on 17 lattice sites. The expectation value for the observable $n_1 n_2$, corresponding to the probability that two particles will be next to each other, plotted for different energy eigenstates. The blue points represent the integrable case $V' = t' = 0$, whereas the red points correspond to the non-integrable case $V' = t' = 0.96$

Deutsch 2018 *Rep. Prog. Phys.* 81 082001

Landscape of quantum Hamiltonians



Defining quantum integrability ?

For any H, there are 2^N „integrals of motion":

$$[|E_n\rangle\langle E_n|, H] = 0$$

obviously, some additional requirement of locality should be added

a reasonable definition is (at least) complicated and (at the moment) elusive

- J.-S. Caux and J. Mossel, Remarks on the notion of quantum integrability, *J. Stat. Mech.* P02023 (2011)
- O.Lychkovskiy, A Remark on the Notion of Independence of Quantum Integrals of Motion in the Thermodynamic Limit. *J Stat Phys* **178**, 1028–1038 (2020).

not to mention many-body scarring, Hilbert fragmentation, ...

Refined ETH

$$H = \sum_{j=1}^{N} \left(J_{\alpha\beta}\, \sigma_j^{\alpha} \sigma_{j+1}^{\beta} + h_\alpha \sigma_j^{\alpha} \right)$$

$$V = \sum_{j=1}^{N} v_\alpha \sigma_j^{\alpha}$$

$$\tilde{H}^{\varkappa} = H + \varkappa V$$

$$\forall\ \varepsilon > \varepsilon_{min}\ ,\ \Delta\varepsilon \quad \varepsilon + \Delta\varepsilon < \varepsilon_{max}$$

$$\left\langle \lim_{\varkappa \to 0} \lim_{\Delta\varepsilon \to 0} \lim_{N \to \infty} \max_{\varepsilon_\nu, \varepsilon_\mu \in [\varepsilon, \varepsilon + \Delta\varepsilon]} \left| \langle E_\mu^{\varkappa} | O | E_\mu^{\varkappa} \rangle - \langle E_\nu^{\varkappa} | O | E_\nu^{\varkappa} \rangle \right| \right\rangle_{\vec{v} \in \text{unit sphere}} = 0$$

Thank you for your attention!