Non-Markovian quantum dynamics identification and control

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Plan

- Data-driven non-Markovian quantum dynamics identification
- Reduced-order modeling based control of non-Markovian quantum dynamics
- Data-driven control of non-Markovian quantum dynamics
- Outlook

Data-driven non-Markovian quantum dynamics identification

“Quantum trajectories” as alias of system-environment dynamics

Discrete in time system and environment dynamics

$$\varrho_{\mathrm{SE}}(t+1) = \Phi[\varrho_{\mathrm{SE}}(t)]$$

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A “Quantum trajectory”

$$T_K(t) = (\varrho_{\mathrm{S}}(t), \ldots, \varrho_{\mathrm{S}}(t+K))$$

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Connection between a “quantum trajectory” and a state

$$\phi_K : \varrho_{\mathrm{SE}} \mapsto \Big(\mathrm{Tr}_{\mathrm{E}}(\varrho_{\mathrm{SE}}), \mathrm{Tr}_{\mathrm{E}}(\Phi[\varrho_{\mathrm{SE}}], \ldots, \Phi^K[\varrho_{\mathrm{SE}}])\Big)$$

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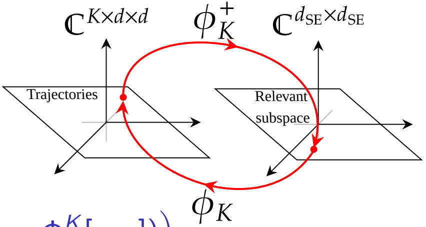
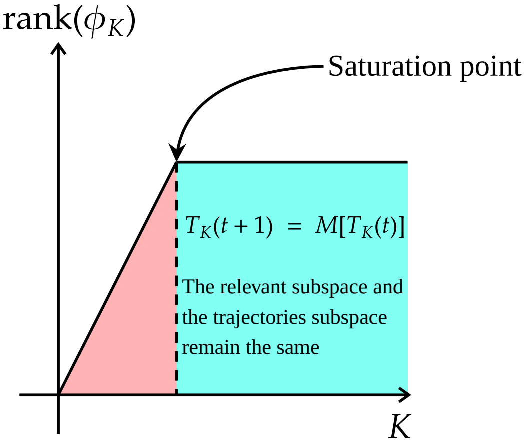
Connection between a “quantum trajectory” and a state

Relevant subspace projector $\pi_K = \phi_K^+ \phi_K$;
Relevant “part” of the density matrix $\varrho_{\mathrm{SE}}^{(r)} = \pi_K[\varrho_{\mathrm{SE}}]$.



$$\phi_K : \varrho_{\mathrm{SE}} \mapsto \left(\mathrm{Tr}_{\mathrm{E}}(\varrho_{\mathrm{SE}}), \mathrm{Tr}_{\mathrm{E}}(\Phi[\varrho_{\mathrm{SE}}], \ldots, \Phi^K[\varrho_{\mathrm{SE}}])\right)$$

What happens if we change K?



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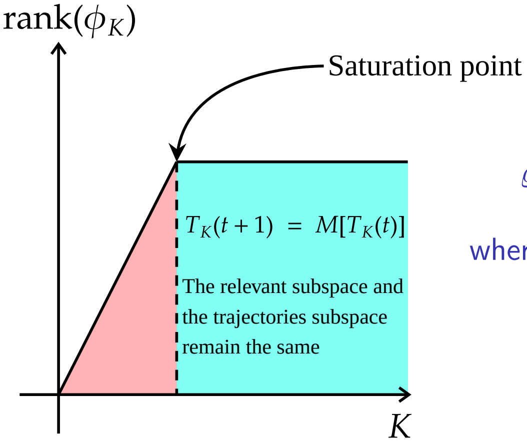

$$\varrho_{\mathrm{SE}}^{(r)}(t+1) = \Phi^{(rr)}\left[\varrho_{\mathrm{SE}}^{(r)}(t)\right]$$

where $\Phi^{(rr)} = \pi_K \Phi \pi_K$;

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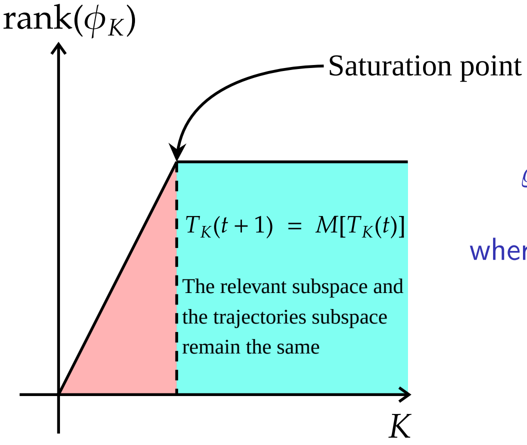

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$$M = \phi_K \Phi \phi_K^{+}$$

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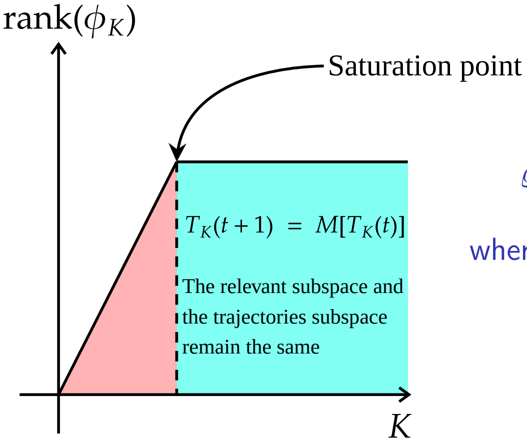

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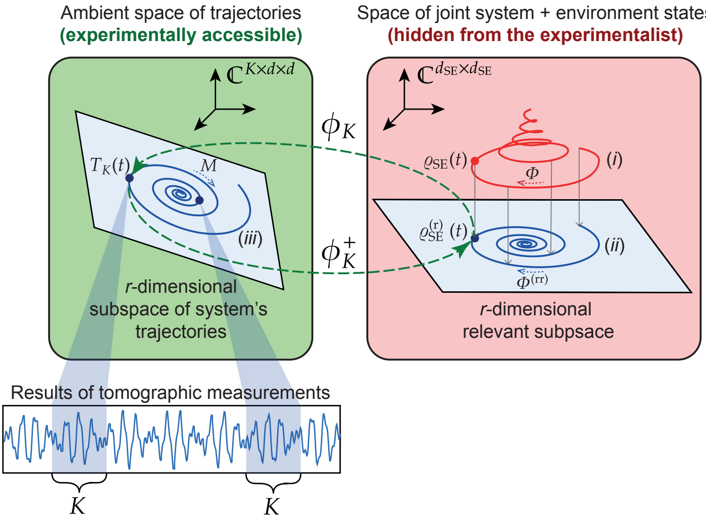
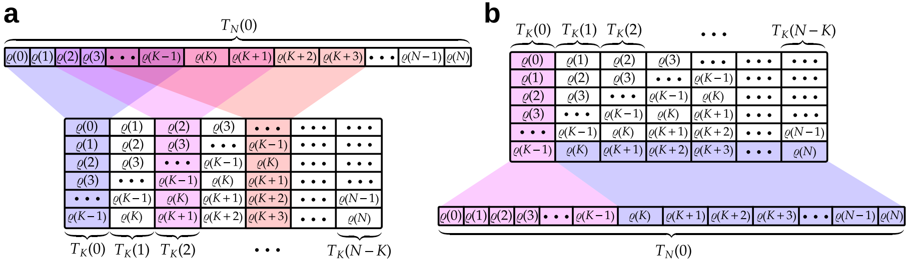
Gutknecht M. H. A brief introduction to Krylov space methods for solving linear systems. In Frontiers of computational science 2007 (pp. 53-62). Springer, Berlin, Heidelberg.

Summary: “trajectories” and state dynamics connection

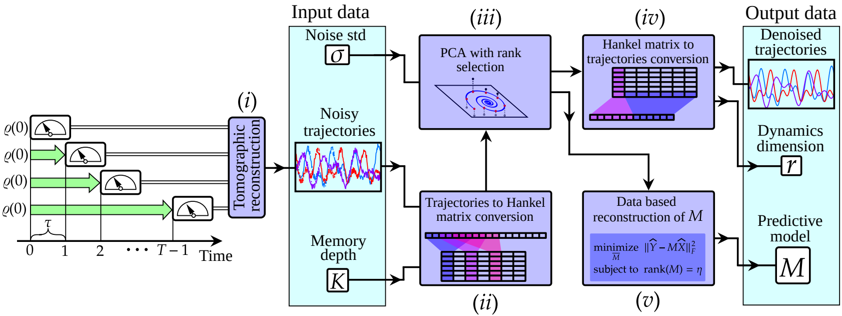


Data processing scheme



Key collocation: Time-delay embedding

Data processing scheme



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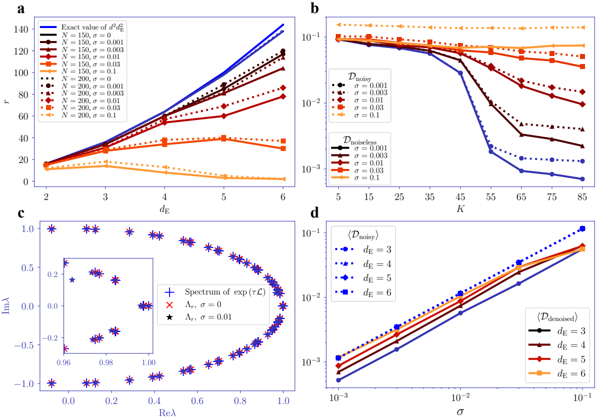
- Schmid P.J. Dynamic mode decomposition of numerical and experimental data. Journal of fluid mechanics. 2010 Aug;656:5-28.

Numerical experiments



$$\frac{d\varrho_{\mathrm{SE}}}{dt} = \mathcal{L}(\varrho_{\mathrm{SE}}) = -ia_{\mathrm{unit}}[H, \varrho_{\mathrm{SE}}] + a_{\mathrm{diss}} \sum_{i,j=1}^{d^2 d_{\mathrm{E}}^2 - 1} \gamma_{ij} \left(F_i \varrho_{\mathrm{SE}} F_j^\dagger - \frac{1}{2}\{F_j^\dagger F_i, \varrho_{\mathrm{SE}}\} \right).$$

Numerical experiments

	$\sigma = 10^{-1}$		$\sigma = 10^{-2}$		$\sigma = 10^{-3}$	
	$N = 150$	$N = 200$	$N = 150$	$N = 200$	$N = 150$	$N = 200$
$d_{\mathrm{E}}^{\mathrm{eff}} = 2$	**2**	**2**	**2**	**2**	**2**	**2**
$d_{\mathrm{E}}^{\mathrm{eff}} = 3$	2	2	**3**	**3**	**3**	**3**
$d_{\mathrm{E}}^{\mathrm{eff}} = 4$	2	2	**4**	**4**	**4**	**4**
$d_{\mathrm{E}}^{\mathrm{eff}} = 5$	2	2	4	**5**	**5**	**5**
$d_{\mathrm{E}}^{\mathrm{eff}} = 6$	1	1	5	5	5	**6**

- Luchnikov I. A., Vintskevich S. V., Grigoriev D. A., Filippov S. N. Machine learning non-Markovian quantum dynamics. Physical Review Letters. 2020 Apr 9;124(14):140502.
- Luchnikov I. A., Vintskevich S. V., Ouerdane H., Filippov S. N. Simulation complexity of open quantum dynamics: Connection with tensor networks. Physical review letters. 2019 Apr 23;122(16):160401.
- Guo C., Modi K., Poletti D. Tensor-network-based machine learning of non-Markovian quantum processes. Physical Review A. 2020 Dec 17;102(6):062414.
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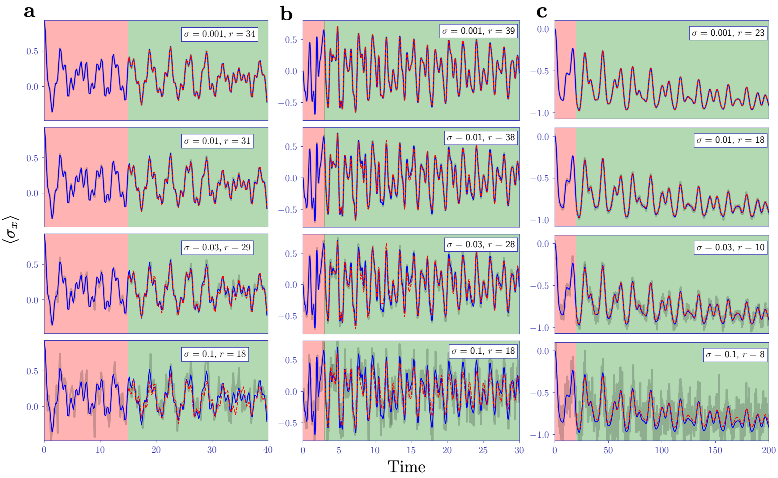
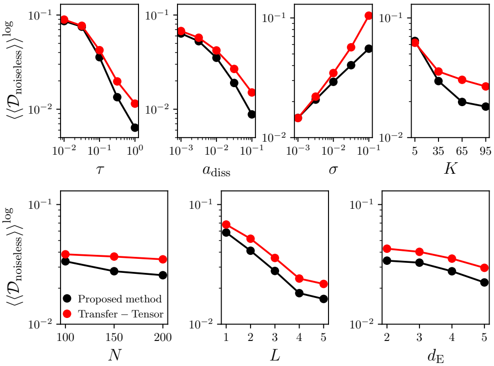
Numerical experiments



Numerical experiments



Cerrillo J., Cao J. Non-Markovian dynamical maps: numerical processing of open quantum trajectories. Physical review letters. 2014 Mar 17;112(11):110401.

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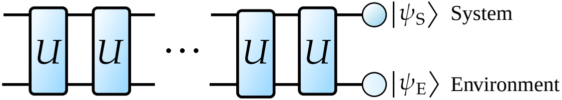
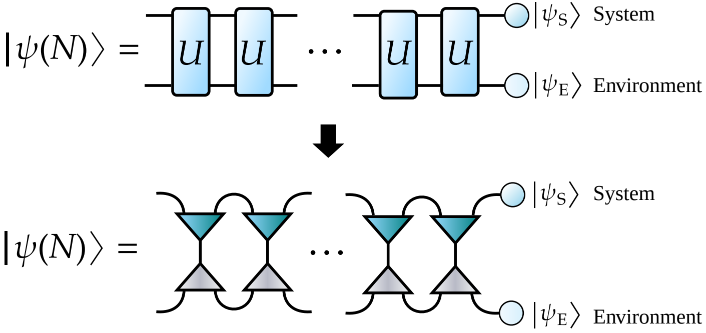
Reduced-order modeling based control of non-Markovian quantum dynamics

Reduce-order modeling of non-Markovian quantum dynamics

$|\psi(N)\rangle =$



Reduce-order modeling of non-Markovian quantum dynamics



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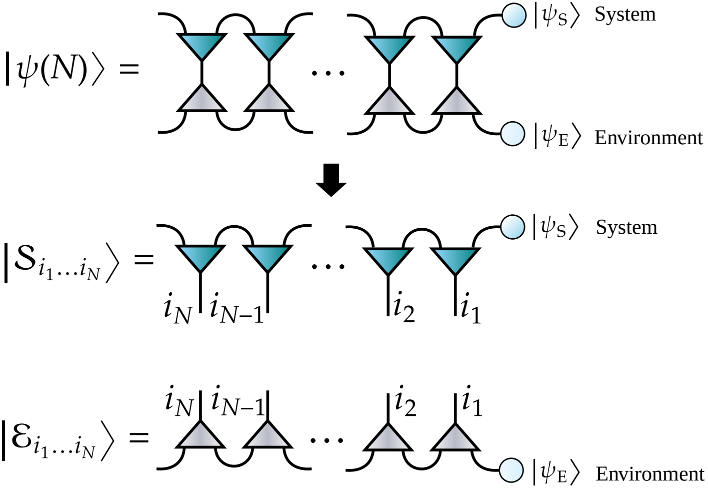
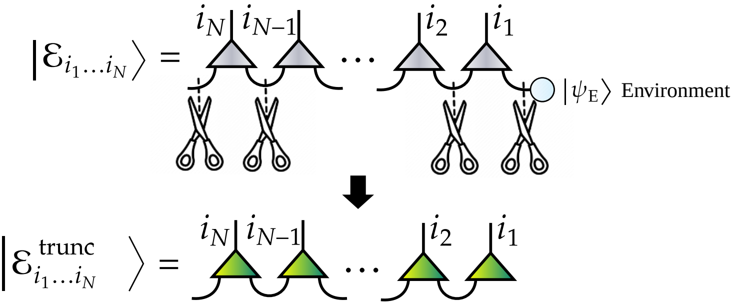
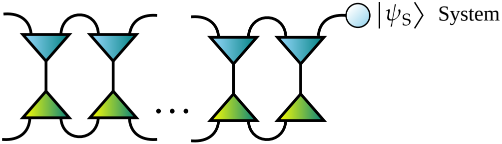


Reduce-order modeling of non-Markovian quantum dynamics

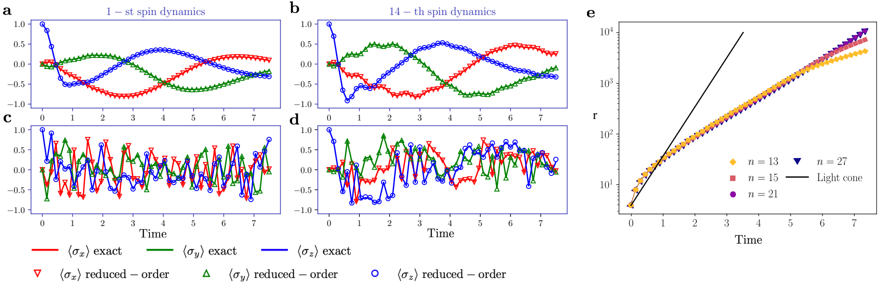


Reduce-order modeling of non-Markovian quantum dynamics

$$|\psi(N)\rangle =$$



Numerical results: dynamics of a single spin in a spin chain



Model parameters:

$J_x = 0.9, \;\; J_y = 1, \;\; J_z = 1.1$

$h_x = h_y = h_z = 0.2$

Optimization approaches

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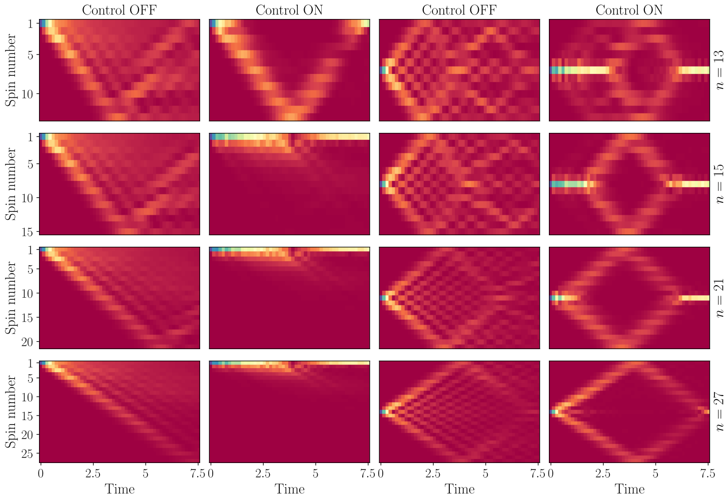
- Oza A., Pechen A., Dominy J., Beltrani V., Moore K., Rabitz H. Optimization search effort over the control landscapes for open quantum systems with Kraus-map evolution. Journal of Physics A: Mathematical and Theoretical. 2009 May 5;42(20):205305.
- Luchnikov I. A., Krechetov M. E., Filippov S. N. Riemannian geometry and automatic differentiation for optimization problems of quantum physics and quantum technologies. New Journal of Physics. 2021 Jul 2;23(7):073006.
- Luchnikov I., Ryzhov A., Filippov S., Ouerdane H. QGOpt: Riemannian optimization for quantum technologies. SciPost Physics. 2021 Mar 30;10(3):079.

Numerical results: controllable information spreading



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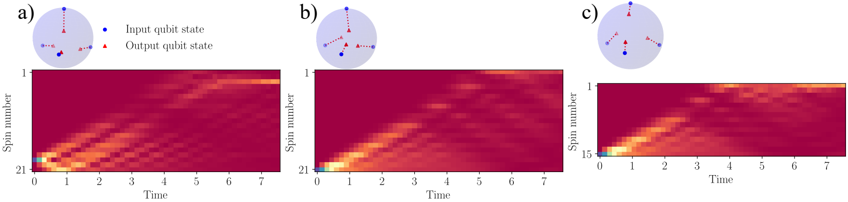
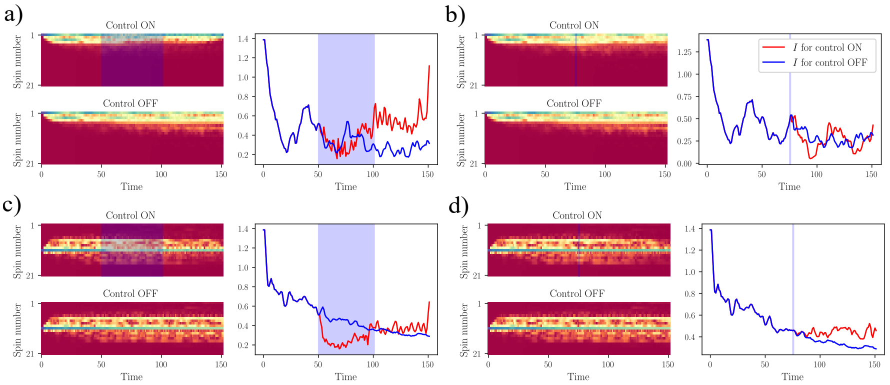


Numerical results: spin-echo in MBL systems



- Sonner M., Serbyn M., Papić Z., Abanin D. A. Thouless energy across the many-body localization transition in Floquet systems. Physical Review B. 2021 Aug 26;104(8):L081112.

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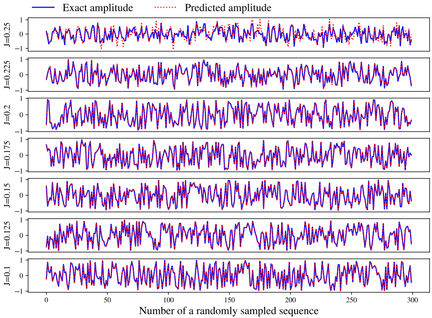
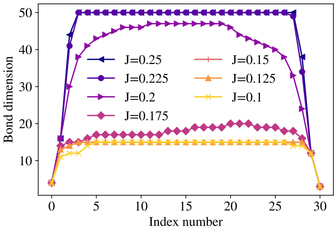
Data-driven control of non-Markovian quantum dynamics

Probbing a data driven reconstruction of MBL system response





10^6 vs 10^{19} evaluations

- Sonner M., Serbyn M., Papić Z., Abanin D. A. Thouless energy across the many-body localization transition in Floquet systems. Physical Review B. 2021 Aug 26;104(8):L081112.
- Oseledets I., Tyrtyshnikov E. TT-cross approximation for multidimensional arrays. Linear Algebra and its Applications. 2010 Jan 1;432(1):70-88.

Outlook

- Quantum chaos detection/analysis with "quantum trajectories"
- Reduced-order modeling beyond one-dimensional quantum environments
- Tensor-train based optimization of discrete control sequences for fully data-driven control of complex quantum systems [Chertkov A., Ryzhakov G., Novikov G., Oseledets I. Optimization of Functions Given in the Tensor Train Format. arXiv preprint arXiv:2209.14808. 2022 Sep 29.]

Thank you for your attention!