

Non-Markovian quantum dynamics identification and control

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- Data-driven non-Markovian quantum dynamics identification
- Reduced-order modeling based control of non-Markovian quantum dynamics
- Data-driven control of non-Markovian quantum dynamics
- Outlook

Data-driven non-Markovian quantum dynamics identification

“Quantum trajectories” as alias of system-environment dynamics

Discrete in time system and
environment dynamics

$$\varrho_{\text{SE}}(t+1) = \Phi[\varrho_{\text{SE}}(t)]$$

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A “Quantum trajectory”

$$T_K(t) = (\varrho_S(t), \dots, \varrho_S(t+K))$$

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Connection between a “quantum trajectory” and a state

$$\phi_K : \varrho_{\text{SE}} \mapsto \left(\text{Tr}_{\text{E}}(\varrho_{\text{SE}}), \text{Tr}_{\text{E}}(\Phi[\varrho_{\text{SE}}]), \dots, \text{Tr}_{\text{E}}(\Phi^K[\varrho_{\text{SE}}]) \right)$$

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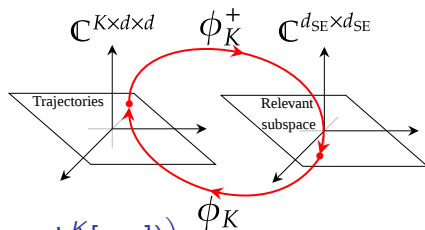
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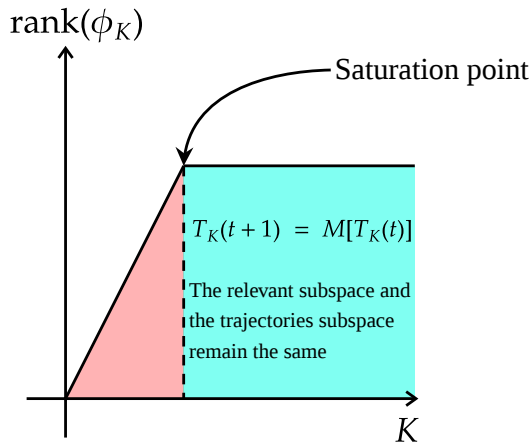
Relevant subspace projector

$$\pi_K = \phi_K^+ \phi_K;$$

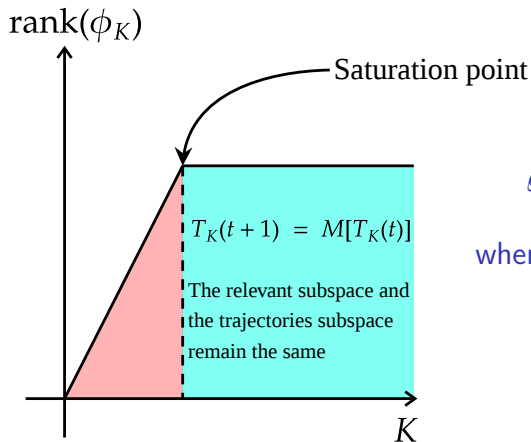
Relevant “part” of the density matrix
matrix $\varrho_{\text{SE}}^{(r)} = \pi_K[\varrho_{\text{SE}}]$.



What happens if we change K ?



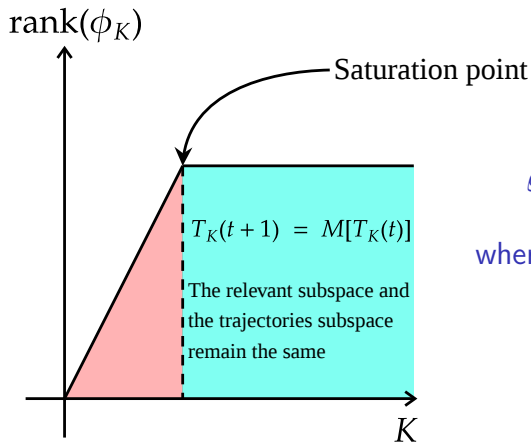
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$$\varrho_{\text{SE}}^{(r)}(t+1) = \Phi^{(rr)} [\varrho_{\text{SE}}^{(r)}(t)]$$

where $\Phi^{(rr)} = \pi_K \Phi \pi_K$;

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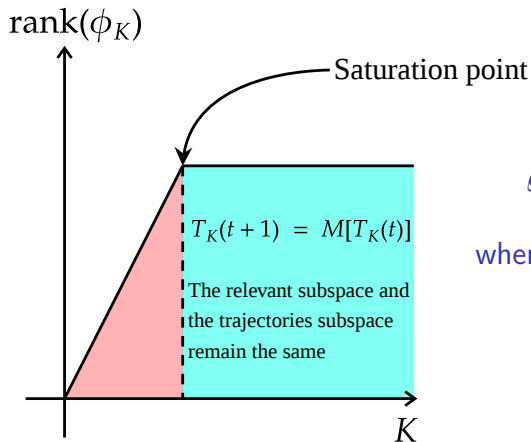


$$\varrho_{\text{SE}}^{(r)}(t+1) = \Phi^{(rr)} [\varrho_{\text{SE}}^{(r)}(t)]$$

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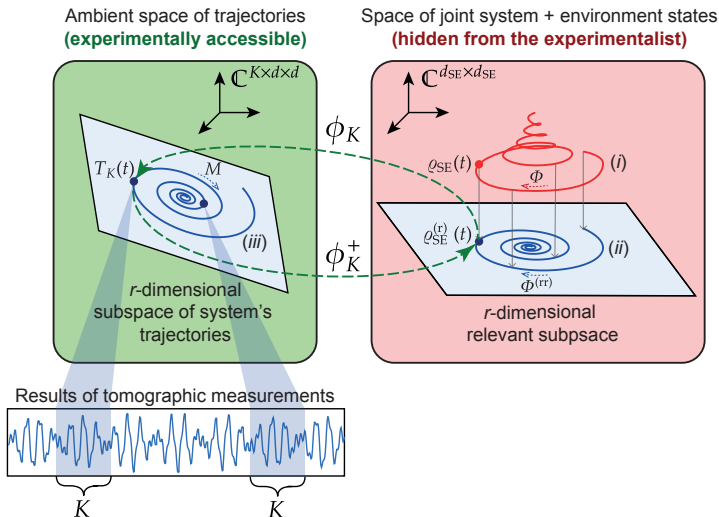
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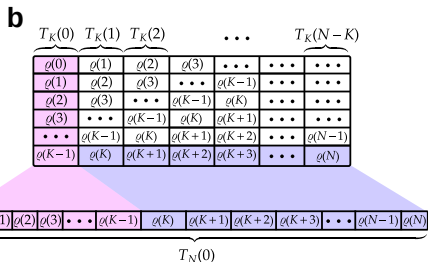
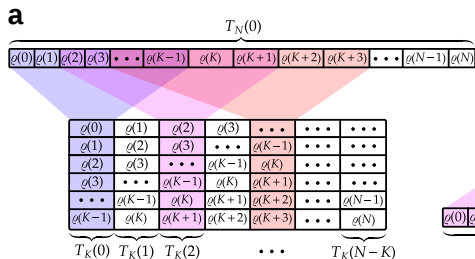
$$M = \phi_K \Phi \phi_K^+$$

Gutknecht M. H. A brief introduction to Krylov space methods for solving linear systems. In Frontiers of computational science 2007 (pp. 53-62). Springer, Berlin, Heidelberg.

Summary: “trajectories” and state dynamics connection

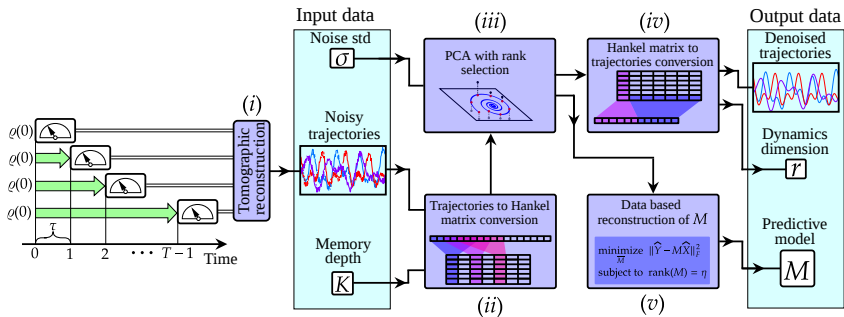


Data processing scheme



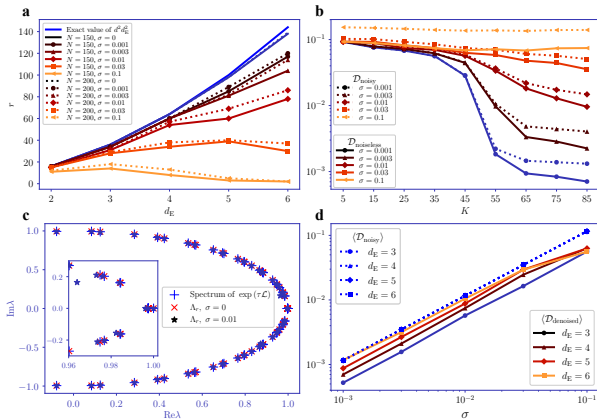
Key collocation: Time-delay embedding

Data processing scheme



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- Schmid P.J. Dynamic mode decomposition of numerical and experimental data. Journal of fluid mechanics. 2010 Aug;656:5-28.

Numerical experiments



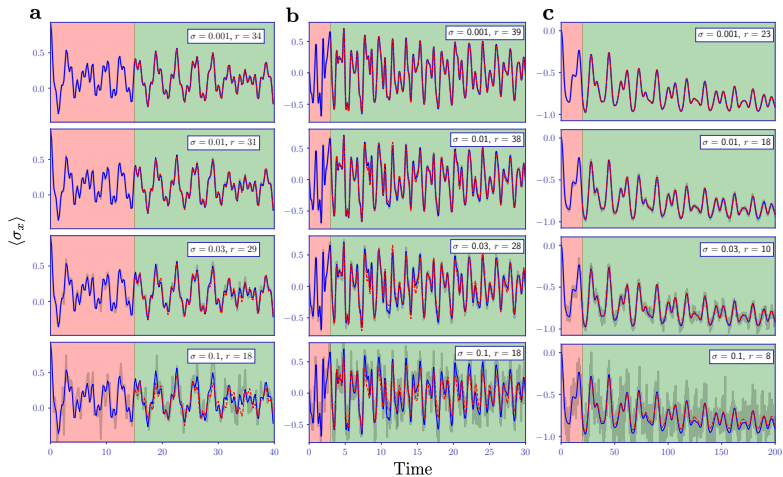
$$\frac{d\varrho_{\text{SE}}}{dt} = \mathcal{L}(\varrho_{\text{SE}}) = -i a_{\text{unit}}[H, \varrho_{\text{SE}}] + a_{\text{diss}} \sum_{i,j=1}^{d_E^2-1} \gamma_{ij} \left(F_i \varrho_{\text{SE}} F_j^\dagger - \frac{1}{2} \{F_j^\dagger F_i, \varrho_{\text{SE}}\} \right).$$

Numerical experiments

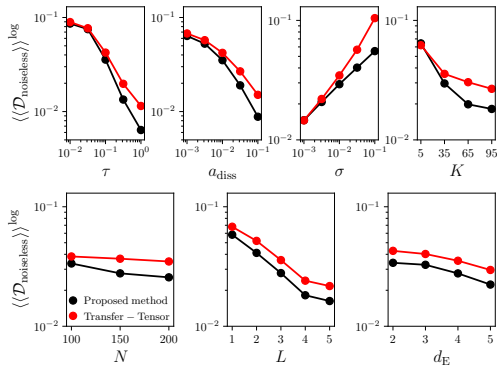
	$\sigma = 10^{-1}$		$\sigma = 10^{-2}$		$\sigma = 10^{-3}$	
	$N = 150$	$N = 200$	$N = 150$	$N = 200$	$N = 150$	$N = 200$
$d_E^{\text{eff}} = 2$	2	2	2	2	2	2
$d_E^{\text{eff}} = 3$	2	2	3	3	3	3
$d_E^{\text{eff}} = 4$	2	2	4	4	4	4
$d_E^{\text{eff}} = 5$	2	2	4	5	5	5
$d_E^{\text{eff}} = 6$	1	1	5	5	5	6

- Luchnikov I. A., Vintskevich S. V., Grigoriev D. A., Filippov S. N. Machine learning non-Markovian quantum dynamics. Physical Review Letters. 2020 Apr 9;124(14):140502.
- Luchnikov I. A., Vintskevich S. V., Ouerdane H., Filippov S. N. Simulation complexity of open quantum dynamics: Connection with tensor networks. Physical review letters. 2019 Apr 23;122(16):160401.
- Guo C., Modi K., Poletti D. Tensor-network-based machine learning of non-Markovian quantum processes. Physical Review A. 2020 Dec 17;102(6):062414.
- Aloisio IA, White GA, Hill CD, Modi K. On the sampling complexity of open quantum systems. arXiv preprint arXiv:2209.10870. 2022 Sep 22.

Numerical experiments



Numerical experiments



Cerrillo J., Cao J. Non-Markovian dynamical maps: numerical processing of open quantum trajectories. Physical review letters. 2014 Mar 17;112(11):110401.

Luchnikov I.A., Kiktenko E.O., Gavreev M.A., Ouerdane H., Filippov S.N., Fedorov A.K. Probing non-Markovian quantum dynamics with data-driven analysis: Beyond “black-box” machine-learning models. Physical Review Research. 2022 Oct 3;4(4):043002.

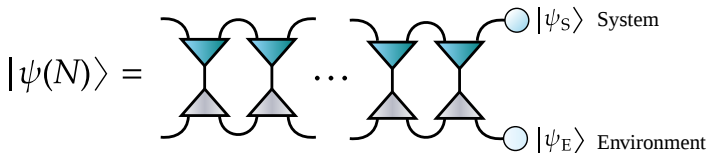
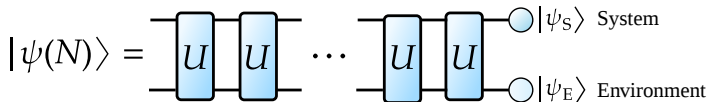
Reduced-order modeling based control of non-Markovian quantum dynamics

Reduce-order modeling of non-Markovian quantum dynamics

$$|\psi(N)\rangle = \begin{array}{c} \text{---} \\ \text{---} \end{array} \begin{array}{|c|} \hline U \\ \hline \end{array} \begin{array}{|c|} \hline U \\ \hline \end{array} \dots \begin{array}{|c|} \hline U \\ \hline \end{array} \begin{array}{|c|} \hline U \\ \hline \end{array} \begin{array}{c} \text{---} \circ |\psi_S\rangle \text{ System} \\ \text{---} \circ |\psi_E\rangle \text{ Environment} \end{array}$$

The diagram illustrates a quantum circuit for the evolution of a system and its environment. It consists of two horizontal lines representing the System and Environment. The System line starts at the top and ends at a circle labeled $|\psi_S\rangle$ System. The Environment line starts at the bottom and ends at a circle labeled $|\psi_E\rangle$ Environment. The circuit is composed of a sequence of unitary operations U , represented by blue rectangular blocks. The first two blocks are connected to both lines, followed by an ellipsis, and then two more blocks connected to both lines. The final state is $|\psi(N)\rangle$.

Reduce-order modeling of non-Markovian quantum dynamics



Reduce-order modeling of non-Markovian quantum dynamics

$$|\psi(N)\rangle = \text{Diagram} \quad \begin{array}{l} |\psi_S\rangle \text{ System} \\ |\psi_E\rangle \text{ Environment} \end{array}$$

The diagram for $|\psi(N)\rangle$ shows a sequence of interaction blocks. Each block consists of two teal triangles pointing down (System) and two grey triangles pointing up (Environment), connected by curved lines. The sequence is represented by an ellipsis in the middle. The final block has two light blue circles on its right side, labeled $|\psi_S\rangle$ System and $|\psi_E\rangle$ Environment.



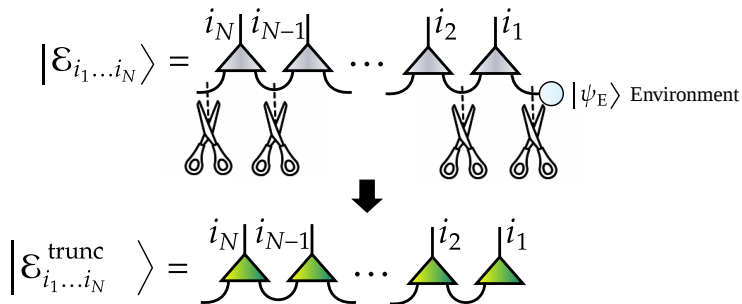
$$|S_{i_1 \dots i_N}\rangle = \text{Diagram} \quad \begin{array}{l} |\psi_S\rangle \text{ System} \end{array}$$

The diagram for $|S_{i_1 \dots i_N}\rangle$ shows a sequence of interaction blocks. Each block consists of two teal triangles pointing down (System) and two grey triangles pointing up (Environment). The teal triangles are labeled with indices $i_N, i_{N-1}, \dots, i_2, i_1$ from left to right. The sequence is represented by an ellipsis in the middle. The final block has a light blue circle on its right side, labeled $|\psi_S\rangle$ System.

$$|\mathcal{E}_{i_1 \dots i_N}\rangle = \text{Diagram} \quad \begin{array}{l} |\psi_E\rangle \text{ Environment} \end{array}$$

The diagram for $|\mathcal{E}_{i_1 \dots i_N}\rangle$ shows a sequence of interaction blocks. Each block consists of two teal triangles pointing down (System) and two grey triangles pointing up (Environment). The grey triangles are labeled with indices $i_N, i_{N-1}, \dots, i_2, i_1$ from left to right. The sequence is represented by an ellipsis in the middle. The final block has a light blue circle on its right side, labeled $|\psi_E\rangle$ Environment.

Reduce-order modeling of non-Markovian quantum dynamics

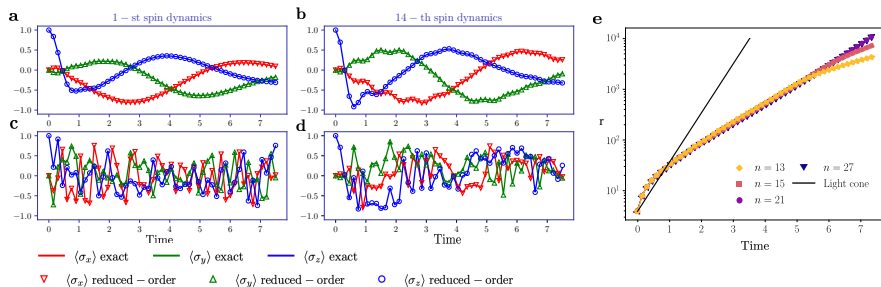


Reduce-order modeling of non-Markovian quantum dynamics

$$|\psi(N)\rangle =$$

The diagram illustrates a tensor network for a quantum state $|\psi(N)\rangle$. It consists of a sequence of pairs of tensors, each pair connected by a vertical line. Each pair is composed of a blue downward-pointing triangle and a green upward-pointing triangle. The top legs of the blue triangles are connected by a horizontal line, and the bottom legs of the green triangles are connected by a horizontal line. The rightmost blue triangle is connected to a light blue circle, which is labeled $|\psi_S\rangle$ System.

Numerical results: dynamics of a single spin in a spin chain



Model parameters:

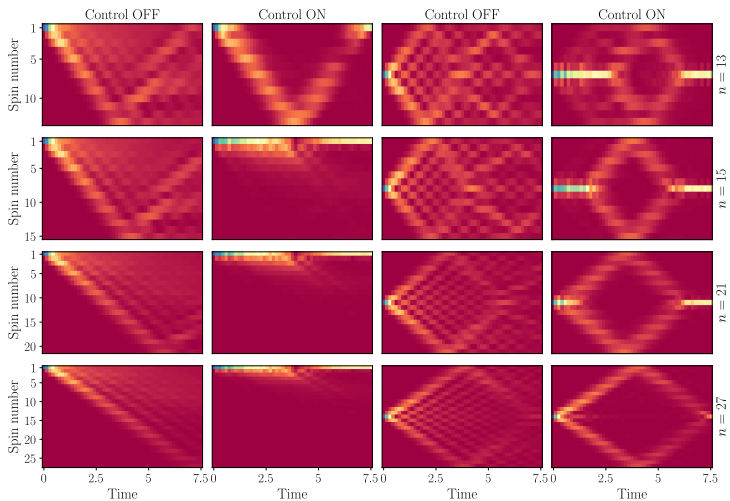
$$J_x = 0.9, \quad J_y = 1, \quad J_z = 1.1$$

$$h_x = h_y = h_z = 0.2$$

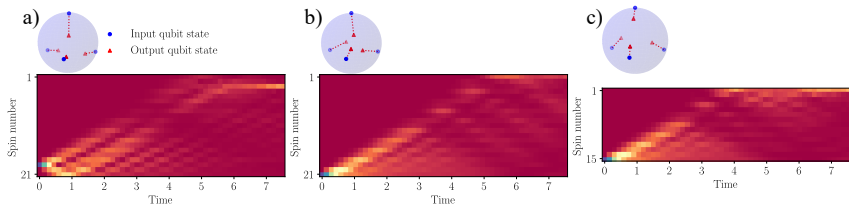
Optimization approaches

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- Oza A., Pechen A., Dominy J., Beltrani V., Moore K., Rabitz H. Optimization search effort over the control landscapes for open quantum systems with Kraus-map evolution. *Journal of Physics A: Mathematical and Theoretical*. 2009 May 5;42(20):205305.
- Luchnikov I. A., Krechetov M. E., Filippov S. N. Riemannian geometry and automatic differentiation for optimization problems of quantum physics and quantum technologies. *New Journal of Physics*. 2021 Jul 2;23(7):073006.
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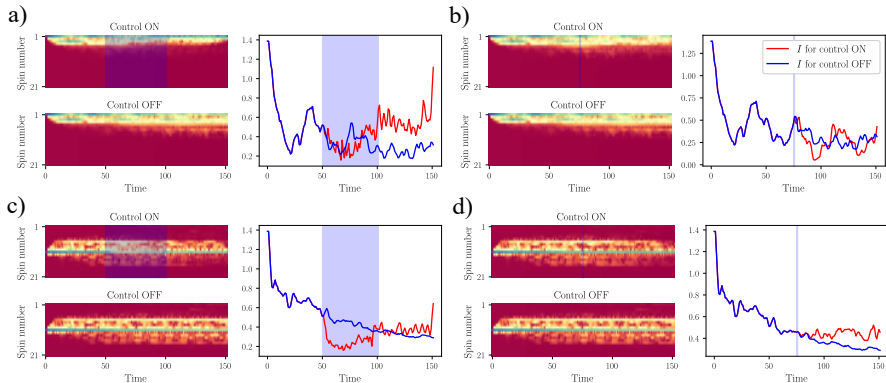
Numerical results: controllable information spreading



Numerical results: controllable information spreading



Numerical results: spin-echo in MBL systems

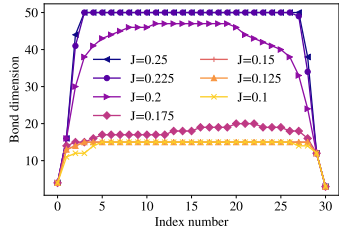
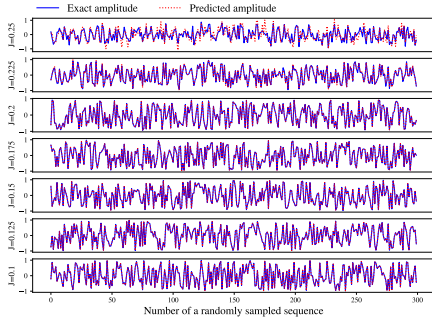


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Data-driven control of non-Markovian quantum dynamics

Probing a data driven reconstruction of MBL system response



10^6 vs 10^{19} evaluations

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Outlook

- Quantum chaos detection/analysis with “quantum trajectories”
- Reduced-order modeling beyond one-dimensional quantum environments
- Tensor-train based optimization of discrete control sequences for fully data-driven control of complex quantum systems [Chertkov A., Ryzhakov G., Novikov G., Oseledets I. Optimization of Functions Given in the Tensor Train Format. arXiv preprint arXiv:2209.14808. 2022 Sep 29.]

Thank you for your attention!