Неразрешимость модальных и суперинтуиционистских логик унарного предиката при двух переменных в языке

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Undecidability of modal and superintuitionistic logics of a single unary predicate in languages with two variables

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  - the quantifier prefix: $\exists^*\forall^*$ decidable, $\forall^3\exists^*$ undecidable;
  - the number of variables: 2 decidable, 3 undecidable;
  - the number and arity of predicate letters: any number of monadic decidable, a single binary undecidable.

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- F. Wolter and M. Zakharyaschev 2001 Monodic fragments are decidable.

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- M. Rybakov, D. Shkatov 2018 **QInt**, **QK**, as well as a number of related logics, including those containing the constant domain axiom, are undecidable in languages with two individual variables and a single monadic predicate letter.

This talk

In this talk, we concern the following:

- "Kripke trick" for modal and superintuitionistic logics.
- Simulating all unary predicate letters wih a single unary letter.
- Results.

Language

Intuitionistic predicate formulas:

$$\varphi \quad ::= \quad P(\bar{x}) \mid \bot \mid (\varphi \wedge \varphi) \mid (\varphi \vee \varphi) \mid (\varphi \to \varphi) \mid \forall x\, \varphi \mid \exists x\, \varphi$$

Modal predicate formulas:

$$\varphi \quad ::= \quad P(\bar{x}) \mid \bot \mid (\varphi \wedge \varphi) \mid (\varphi \vee \varphi) \mid (\varphi \to \varphi) \mid \forall x\, \varphi \mid \exists x\, \varphi \mid \Box\varphi$$

Standard abbreviations:

$$\begin{array}{lcl} \neg\varphi & = & (\varphi \to \bot); \\ (\varphi \leftrightarrow \psi) & = & ((\varphi \to \psi) \wedge (\psi \to \varphi)); \\ \Diamond\varphi & = & \neg\Box\neg\varphi. \end{array}$$

Kripke semantics

Kripke semantics

Kripke frame is a pair $\mathfrak{F} = \langle W, R\rangle$; for the intuitionistic language R is reflrxive, transitive, and antisymmetric.

Expanding domains. For a frame $\langle W, R\rangle$ consider a sysytem $(D_w)_{w\in W}$ of non-empty sets (domains) such that

$$(*) \quad wRw' \implies D_w \subseteq D_{w'}.$$

For every $w \in W$ define a classical model $\mathfrak{M}_w = (D_w, I_w)$.
For the intuitionistic case we aditionally claim:

$$wRw' \implies P^w \subseteq P^{w'}.$$

This gives us a first-order Kripke model $\mathfrak{M} = (W, R, D, I)$ is a Kripke model, where $D = (D_w)_{w\in W}$ and $I = (I_w)_{w\in W}$.

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(Locally) constant domains. Replace $(*)$ with cd-condition:

$$(**) \quad wRw' \implies D_w = D_{w'}.$$

Predicate Kripke frames: an example

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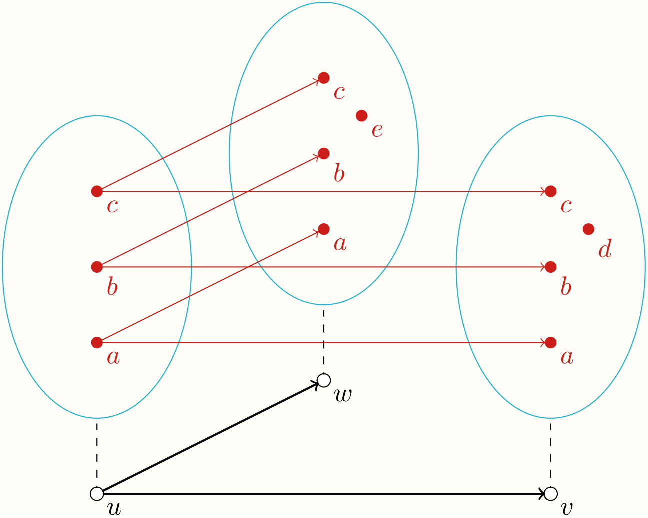

Kripke semantics

Kripke semantics

Truth relation (intuitionistic language):

- $\mathfrak{M}, w \models^g P(x_1, \ldots, x_n)$ if $\langle g(x_1), \ldots, g(x_n) \rangle \in P^w$;
- $\mathfrak{M}, w \not\models^g \bot$;
- $\mathfrak{M}, w \models^g \varphi \wedge \psi$ if $\mathfrak{M}, w \models^g \varphi$ and $\mathfrak{M}, w \models^g \psi$;
- $\mathfrak{M}, w \models^g \varphi \vee \psi$ if $\mathfrak{M}, w \models^g \varphi$ or $\mathfrak{M}, w \models^g \psi$;
- $\mathfrak{M}, w \models^g \varphi \to \psi$ if $\mathfrak{M}, w' \models^g \varphi$ implies $\mathfrak{M}, w' \models^g \psi$, for any $w' \in R(w)$;
- $\mathfrak{M}, w \models^g \exists x\, \varphi$ if $\mathfrak{M}, w \models^{g'} \varphi$, for some g' s.t. $g' \stackrel{x}{=} g$ and $g'(x) \in D_w$;
- $\mathfrak{M}, w \models^g \forall x\, \varphi$ if $\mathfrak{M}, w' \models^{g'} \varphi$, for every $w' \in R(w)$ and every g' s.t. $g' \stackrel{x}{=} g$ and $g'(x) \in D_{w'}$.

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- $\mathfrak{M}, w \models^g \Box\varphi$ if $\mathfrak{M}, w' \models^g \varphi$, for every $w' \in R(w)$.

- $\mathfrak{M}, w \models \varphi(x_1, \dots, x_n)$ if $\mathfrak{M}, w \models^g \varphi(x_1, \dots, x_n)$, for every g such that $g(x_1), \dots, g(x_n) \in D_w$;
- $\mathfrak{M} \models \varphi$ if $\mathfrak{M}, w \models \varphi$, for every $w \in W$;
- $\mathfrak{F} \models \varphi$ if $\mathfrak{M} \models \varphi$, for every model $\mathfrak{M}$ based over $\mathfrak{F}$.

Logics

Logics

The logics under consideration are:

- **QInt**, the logic of all intuitionistic frames;
- **QKC**, the si-logic of convergent intuitionistic frames;
- **QK**, the modal logic of all frames;
- $\mathbf{QGL} = \mathbf{QK} \oplus \Box(\Box p \to p) \to \Box p$;
- $\mathbf{QGrz} = \mathbf{QK} \oplus \Box(\Box(p \to \Box p) \to p) \to p$;
- In general, for a normal modal propositional logic L, define $\mathbf{Q}L = \mathbf{QK} \oplus L$.

Kripke trick

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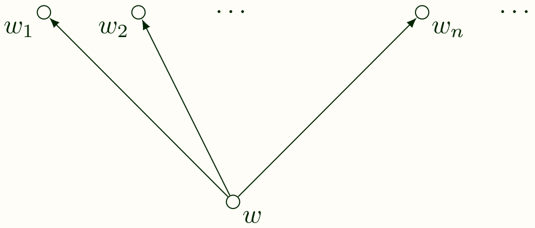
Modal language

Just substitute $\Diamond(Q_1(x) \wedge Q_2(y))$ instead of $P(x, y)$.



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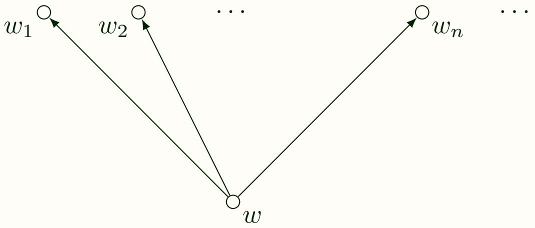

Intuitionistic language

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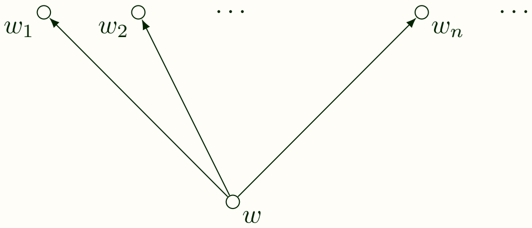

Intuitionistic language

Substitution: $(Q_1(x) \wedge Q_2(y) \to \bot) \vee q$ instead of $P(x, y)$.

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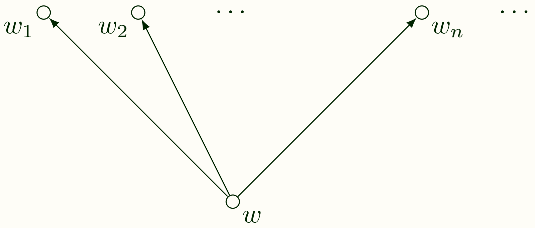

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Single unary letter: modal language

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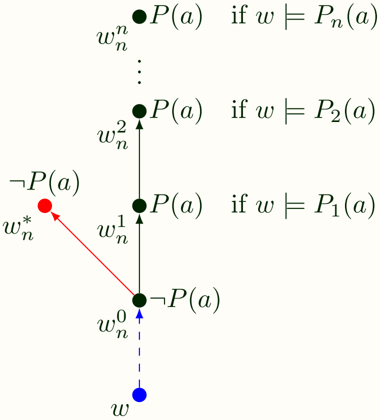

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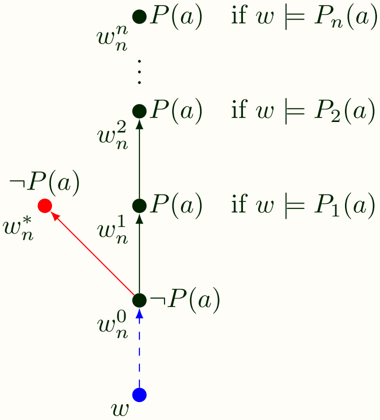

Let $\qquad A_k(x) \quad = \quad \Diamond\Box\bot \wedge \Diamond^n\Box\bot \wedge \Diamond^k P(x)$

Then the formula $\Diamond A_k(x)$ simulates $P_k(x)$ at the world w.

Some results

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- Let L be a logic containing $\mathbf{QK}$ and contained in $\mathbf{QGL} \oplus \boldsymbol{bf}$ or $\mathbf{QGrz} \oplus \boldsymbol{bf}$ or $\mathbf{QKTB} \oplus \boldsymbol{bf}$. Then L is Σ^0_1-hard in the language with a single unary predicate letter and two individual variables.

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- Let L be a logic containing **QK** and contained in **QS5**. Then L is Σ^0_1-hard in the language with a two unary predicate letters, two individual variables, and infinitely many proposition letters.

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- Let L be a logic containing **QK** and contained in **QS5**. Then L is Σ^0_1-hard in the language with a two unary predicate letters, two individual variables, and infinitely many proposition letters.
- Let $\mathfrak{F} = \langle \mathbb{N}, R \rangle$, where R is a relation between $<$ and $\leqslant$. Then the logic of $\mathfrak{F}$ is Π^1_1-hard in the language with a single unary predicate letter, single proposition letter, and two individual variables.

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- The logic of finite frames of a logic contained in $\mathbf{QGL} \oplus \boldsymbol{bf}$, $\mathbf{QGrz} \oplus \boldsymbol{bf}$ or $\mathbf{QKTB} \oplus \boldsymbol{bf}$ is Π^0_1-hard in the language with a single unary predicate letter and three individual variables.

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- Let L be a logic containing $\mathbf{QwGrz}$ and contained in $\mathbf{QGL.3} \oplus \boldsymbol{bf}$ or $\mathbf{QGrz.3} \oplus \boldsymbol{bf}$. Then the logic of L-frames is Π^1_1-hard in the language with a single unary predicate letter, single proposition letter, and two individual variables.

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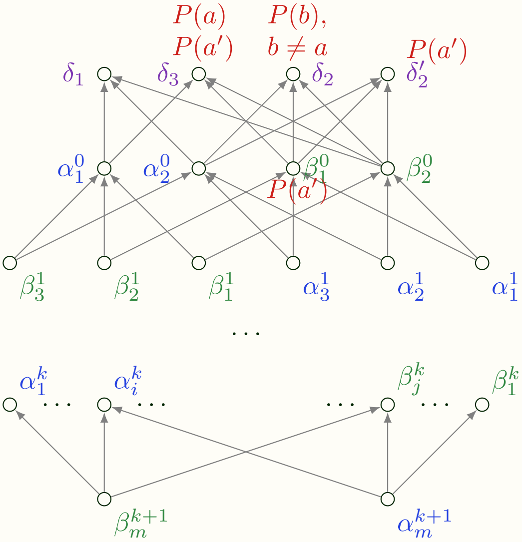
- Predicate counterparts of $\mathbf{CTL}^*$, **CTL**, **LTL**, $\mathbf{ATL}^*$, **ATL** are Π^1_1-hard in the language with a single unary predicate letter and two individual variables.

Single unary letter: intuitionistic language



Single unary letter: intuitionistic language

$$
\begin{array}{lcl}
D_1 & = & \exists x\, P(x); \\
D_2(x) & = & \exists x\, P(x) \to P(x); \\
D_3(x) & = & P(x) \to \forall x\, P(x); \\
\\
A_1^0(x) & = & D_2(x) \to D_1 \vee D_3(x); \\
A_2^0(x) & = & D_3(x) \to D_1 \vee D_2(x); \\
B_1^0(x) & = & D_1 \to D_2(x) \vee D_3(x); \\
B_2^0(x) & = & A_1^0(x) \wedge A_2^0(x) \wedge B_1^0(x) \to D_1 \vee D_2(x) \vee D_3(x); \\
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B_3^1(x) & = & B_1^0(x) \wedge B_2^0(x) \to A_1^0(x) \vee A_2^0(x); \\
\\
A_m^{k+1}(x) & = & A_1^k(x) \to B_1^k(x) \vee A_i^k(x) \vee B_j^k(x); \\
B_m^{k+1}(x) & = & B_1^k(x) \to A_1^k(x) \vee A_i^k(x) \vee B_j^k(x).
\end{array}
$$

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Let $\mathfrak{M}_a$ be the model with a constant domain $\mathcal{A}$ constructed for an individual a as on the picture.

Lemma

$$\mathfrak{M}_a, w \not\models A^k_m(a) \iff wR_0\alpha^k_m;$$
$$\mathfrak{M}_a, w \not\models B^k_m(a) \iff wR_0\beta^k_m.$$

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Let $b \in \mathcal{A} - \{a\}$ и $k \geqslant 2$.

Lemma

$\mathfrak{M}_a, w \models A^k_m(b)$ and $\mathfrak{M}_a, w \models B^k_m(b)$.

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Then, the formula $A^n_k(x) \vee B^n_k(x)$ simulates $P_k(x)$ at w: to refute $P_k(a)$, make α^n_k and β^n_k of $\mathfrak{M}_a$ with the domain D_w be accessible from w.
Here, n depends on the number of predicate letters we simulate.

Some results

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- Let L be a logic such that $\mathbf{QInt} \subseteq L \subseteq \mathbf{QKC} + \boldsymbol{cd}$. Then, the positive fragment of L is Σ^0_1-hard in the language with a single unary letter and two individual variables.

Some results

- Let L be a logic such that $\mathbf{QInt} \subseteq L \subseteq \mathbf{QKC} + \boldsymbol{cd}$. Then, the positive fragment of L is Σ^0_1-hard in the language with a single unary letter and two individual variables.
- Let L be a logic such that $\mathbf{QInt} \subseteq L \subseteq \mathbf{QKC} + \boldsymbol{cd}$. Then, the positive fragment of the logic of finite L-frames is Π^0_1-hard in the language with a single unary letter and three individual variables.

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Thank you!