On the significance of parameters in the comprehension schema in second-order arithmetic

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Conference of Mathematical Centers
November 2022
MSU and MIAN, Moscow

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This talk is devoted to the effect of parameters in the schema of comprehension in second-order arithmetic.

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1. Peano's axioms for numbers.
2. **Induction** $\Phi(0) \wedge \forall k \left(\Phi(k) \Longrightarrow \Phi(k+1)\right) \Longrightarrow \forall k \, \Phi(k)$, for every formula $\Phi(k)$ in $\mathcal{L}(\mathbf{PA}_2)$, and we allow parameters in $\Phi(k)$, *i. e.*, free variables other than k.

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$\mathbf{PA}_2^*$ is the subsystem of $\mathbf{PA}_2$ with **CA** replaced by $\mathbf{CA}^*$.

The problem we consider

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We study the problem in the context of **deductive strength**, and we obtain the opposite answer.

Theorem 1

$\mathbf{PA}_2^*$ *does not prove that every set has its complement.*

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Proof (sketch)

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Then $\langle \omega ; X \rangle$ is a model of $\mathbf{PA}_2^*$ in which sets a_n do not have their complements, and hence the full $\mathbf{CA}$ fails.

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Thus $\mathbf{PA}_2^*$ does not prove a simple consequence of the full $\mathbf{CA}$.

Second theorem

Theorem 2

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$\mathbf{PA}_2^* + \mathbf{CA}(\Sigma_2^1)$ *does not prove a certain instance of the full* $\mathbf{CA}$*.*

$\mathbf{CA}(\Sigma_2^1)$ is $\mathbf{CA}$ (with parameters) restricted to Σ_2^1 formulas.

Theorem 2 proof sketch

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Then $\langle \omega ; X \rangle$ proves Theorem 2.

Model 2: picture

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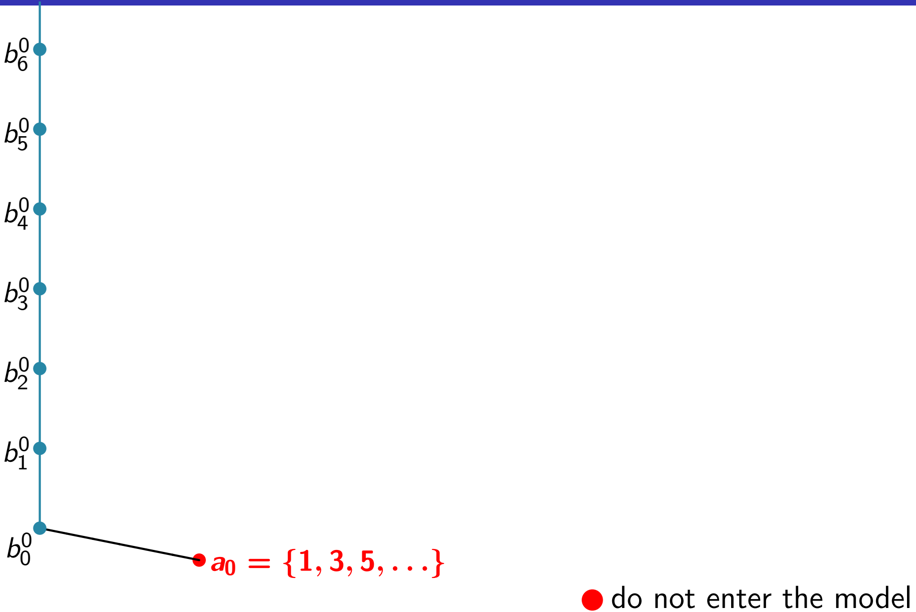
● do not enter the model


b_6^0
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b_3^0
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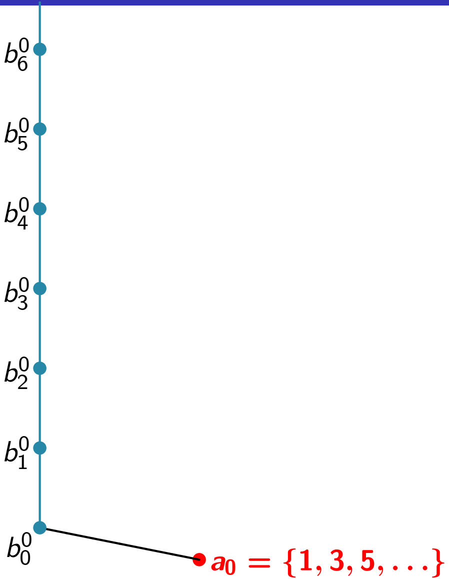
b_1^0
b_0^0
$a_0 = \{1, 3, 5, \ldots\}$
do not enter the model



the structure of Sacksness

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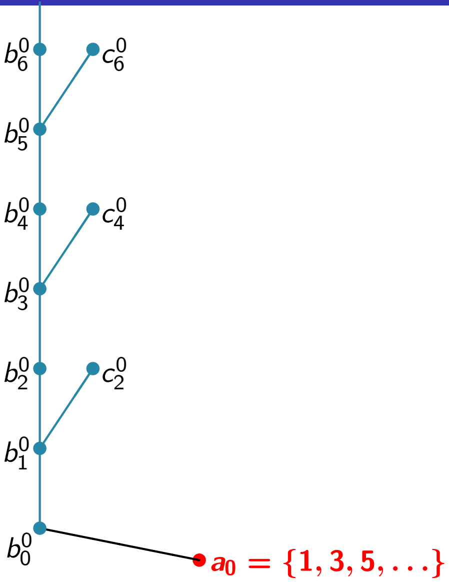

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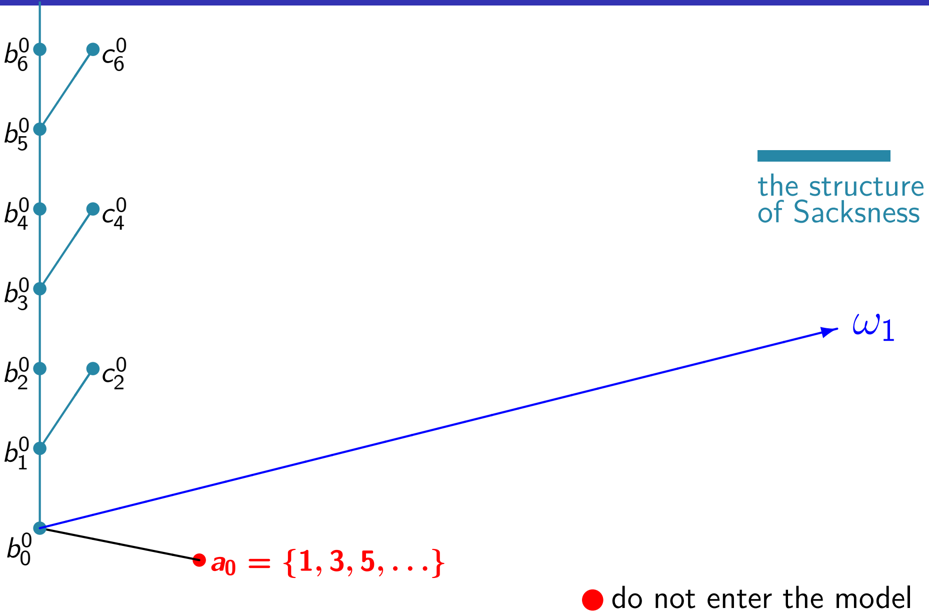

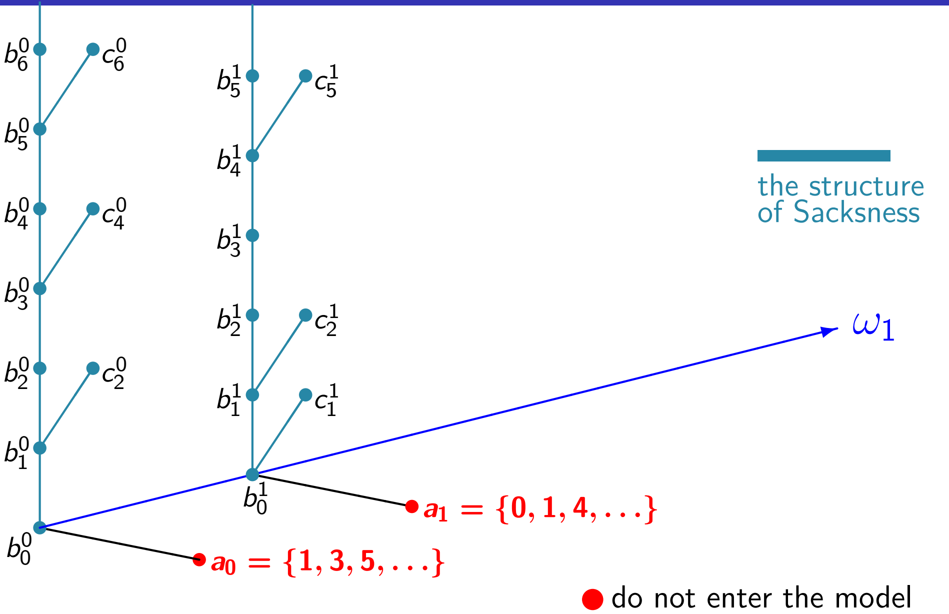
b_6^0
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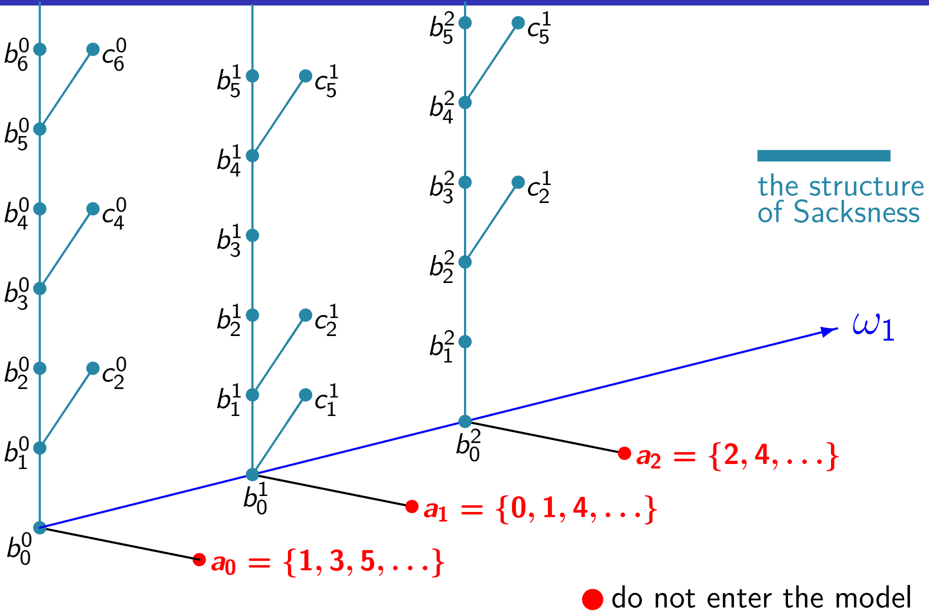

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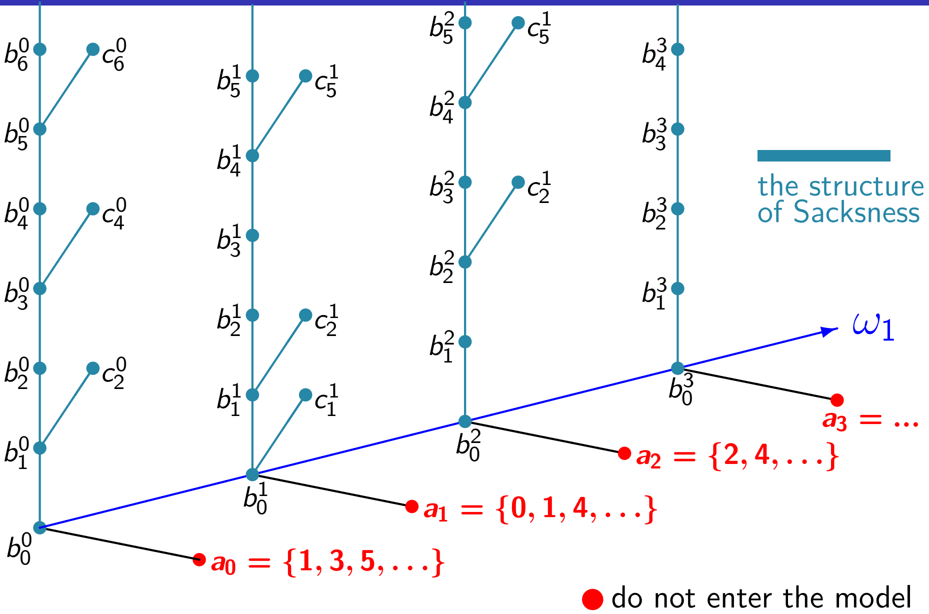

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A finite axiomatizability problem

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Work in progress.

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