On the unreasonable effectiveness of Mathematical Logic

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Abstract

Following many impressive and unexpected applications of Model Theory (and Logic, more generally) in mathematics and some in mathematical physics, we speculate that what is happening currently is a shift of paradigm in Maths similar to the one which happened in Physics when "the observer" became a part of physical model.

Observers and languages

Observer as part of physical universe ->
Observer as part of mathematical universe

Mathematical Logic, especially Model Theory, changes the paradigm of mathematics by focusing on the language in which a certain part of mathematics is being considered.

An illustration

Mathematics of the 19th century studied the new *complex numbers* as an object, without answering the question of **what is the theory of complex numbers?**
We now have:
- the field of complex numbers;
- the field of complex numbers with metric;
- the ring of Gaussian integers;
- the theory of functions in complex variables;
- the theory of contour integration and residues;
- ...

The logic observer pays attention to **what and how is definable** in the given structure (language).

The Bourbaki-style notion of a structure with its own langauge clarified the above point
(cf. also the category-theoretic approach).

New ways of doing maths

1. The new ways of understanding **infinity and limits**

(i) set theory;

(ii) non-standard analysis;

(iii) pseudo-finite structures;

...

2. **Classification of mathematical theories** with respect to the language/structure

(i) Algorithmic decidability and the complexity theory;

(ii) Stability-theoretic (Shelah's) classification theory
(cf. with the Mendeleev table of elements)

(iii) o-minimality theory

Observer as a problem

The presence of an obserever causes **self-reflection** (SLP) paradoxes.

Comparison: Self-reflection “paradoxes” in physics:
- the **measurements problem**
- the **uncertainty principle**
- ...

Self-reflection is also a feature of financial markets

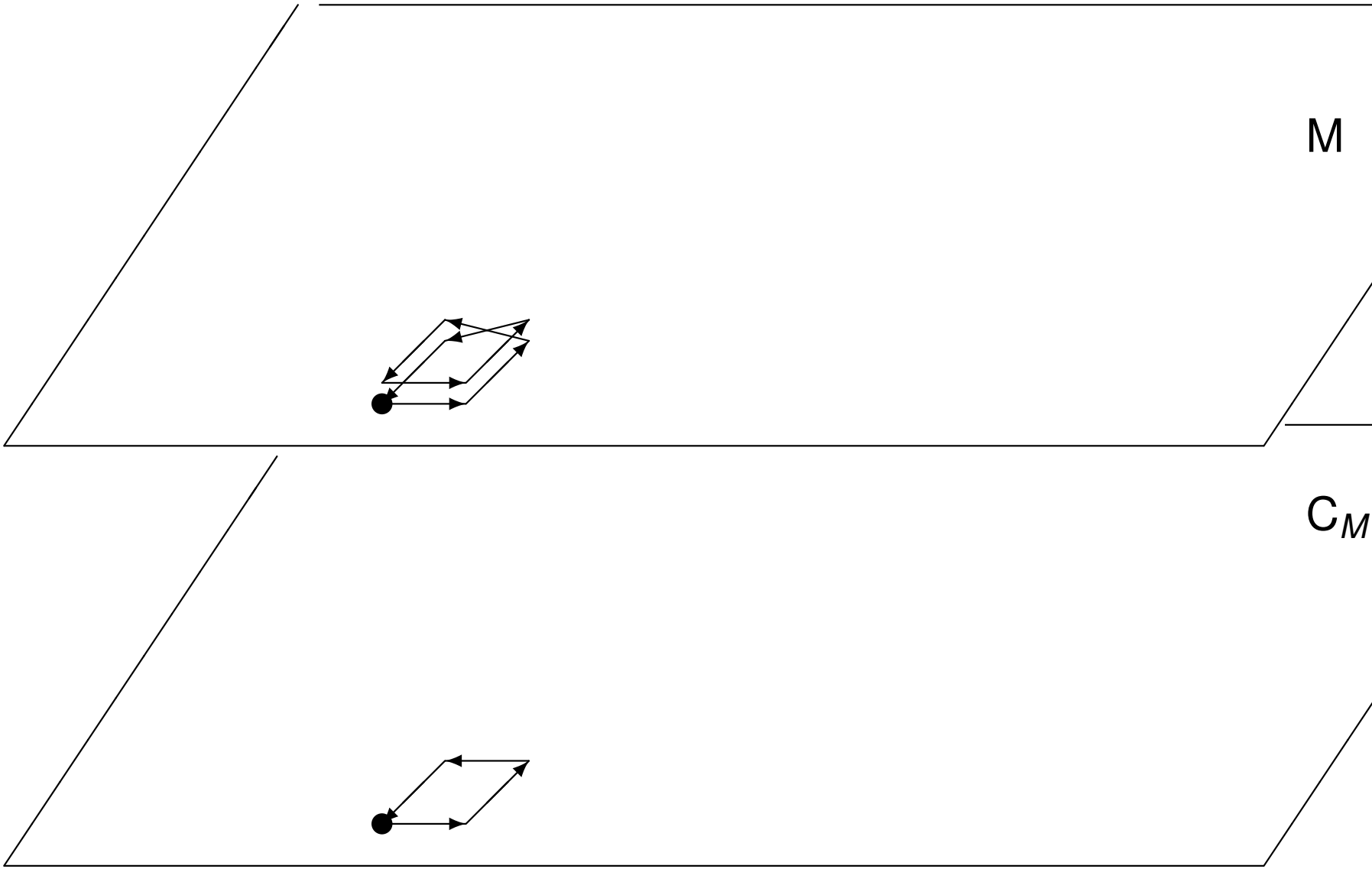
SLP and non-commutative geometry

Physics solution to SLP is through introducing non-commutativity in the *logic of physics* in the form of the **canonical commutation relation** (CCR)

$$PQ - QP = i\hbar$$

which leads to the geometric interpretation "points have structure", or

physical space is fibered
(in some “non-commutative” way)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Non-commutativity effects in Logic

The observer reflecting on his own state changes the state

The definition "*The smallest positive integer not definable in under sixty letters*" enhances the expressive power of the language, and so changes the state of the observer.

Non-commutativity effects in Logic

Is the **Homotopy Type Theory** the non-commutative version of Logic?
(Note the role that **fibrations** play in homotopy theory)