Yang-Mills solitons in models with conformal symmetry breaking

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Avoiding Gribov ambiguity

A.A. Slavnov considered the existing procedures for quantising Yang-Mills fields as applicable only in perturbation theory, in particular, because of Gribov's ambiguity consisting in non-uniqueness of the choice of a representative of a class of gauge equivalent configurations in the Coulomb gauge. In 2009 he proposed a model with the bosonic part

$$L = -\frac{1}{4}F^a_{\mu\nu}F^{a\mu\nu} + \frac{1}{2}(D_\mu\varphi^a)(D^{\mu a}) - \frac{1}{2}(D_\mu\chi^a)(D^\mu\chi^a) + i(D_\mu b^a)(D^\mu e^a),$$

where the four scalar fields are in the adjoint representation of SU(2), φ, χ commuting and e, b anticommuting and χ having negative energy. This model was shown to be free from Gribov ambiguity.

Within this model he was able to find classical solitons of the magnetic monopole type, in spite of the absence of the scalar potential term.

Existence of finite energy particle-like solutions was possible due to violation of conformal invariance of the YM lagrangian by the scalar field. The aim of this talk is to review some other theories with different mechanisms of conformal symmetry breaking, leading to YM solitons of other types.

No-go theorems for YM solitons

In classical papers by Coleman, Deser and Pagels of mid 70-ies it was proven that finite energy static classical solutions of field theories with positive energy density and zero trace of the energy-momentum are impossible: "There are no classical glueballs".

Non-existence of static solutions can be related to conformal invariance of the Yang–Mills theory, which implies that the stress–energy tensor is traceless :

$$T^{\mu}_{\mu} = 0 = -T_{00} + T_{ii}, \quad \mu = 0, ..., 3, \; i = 1, 2, 3$$

Given the positivity of the energy density T_{00}, this means that the sum of the principal pressures T_{ii} is everywhere positive, *i.e.* the Yang–Mills matter is repulsive. This makes the mechanical equilibrium impossible. The Higgs field breaks the conformal invariance of the pure Yang–Mills theory and so in the spontaneously broken gauge theories particle-like solutions may exist. Two types of such solutions are known: magnetic monopoles and sphalerons. Topological criterion for the existence of monopoles is the non-triviality of the second homotopy group of the Higgs broken phase manifold $\pi_2(G/H)$ (this argument is absent in the Slavnov's case).

Magnetic monopoles and sphalerons

Thus topologically stable monopoles exist in the $SO(3)$ gauge theory with a real Higgs triplet, in which case $G/H = S^2$, but do not exist in the $SU(2)$ gauge theory with a complex Higgs doublet, where the symmetry is completely broken (the Higgs broken phase manifold is S^3).

However, in the theory with doublet Higgs another particle–like solution has been found by Dashen, Hasslacher and Neveu (74). Its existence was explained by Manton (83) as a consequence of non–triviality of the *third* homotopy group $\pi_3(S^3)$, indicating the presence of non–contractible loops in the configuration space. This solution is *the* sphaleron; it sits at the top of the potential barrier separating topologically distinct Yang–Mills vacua. Because of this position, the sphaleron is necessarily unstable. Still its rôle is very important, since in presence of fermions it can mediate transitions without the conservation of fermion number.

The aim of this talk is to review some other models with breaking of the conformal symmetry. We will discuss YM solitons in curved space with self-consistent gravity and solitons in flat space non-Abelian Born-Infeld theory motivated by string theory.

Einstein-Yang-Mills

It was a great surpirse when in 1988 Bartnik and McKinnon found that finite energy everywhere regular solutions exist in Einstein-Yang-Mills SU(2) coupled theory. The surprise was due to the fact that, in addition to the above no-go results, in pure gravity finite energy regular geons are forbidden too by Lichnerowitz theorem. But combination of gravitational attraction and YM repulsion turned out to be just what in needed for mechanical equilibrium in static solitons.

However, the mechanical equilibrium is only a neccessary condition for eistence of solitons. For 't Hooft-Polyakov monopoles topological arguments are equally important. In this case crucial for existence and stability is the topology of Higgs scalars manifold. Here we use term soliton in wide sence, including unstable sphalerons, and in presence of gravity, also black holes with regular horizons. Existence of flat space sphalerons is also based on topological argument related to Higgs fields. For the EYM system these arguments are no longer valid. Topological nature of BK solutions remained obscure till 1991, when we showed with M.S. Volkov that BK are just the EYM sphalerons (also, Sudarsky and Wald, 1992)

Colored black holes

The general spherically symmetric SU(2) gauge field can be written as

$$A = a_0 \ \mathbf{T}_3 \, dt + w \left(\mathbf{T}_2 \, d\vartheta - \mathbf{T}_1 \, \sin\vartheta \, d\varphi \right) + \mathbf{T}_3 \cos\vartheta \, d\varphi,$$

and the static spacetime metric is

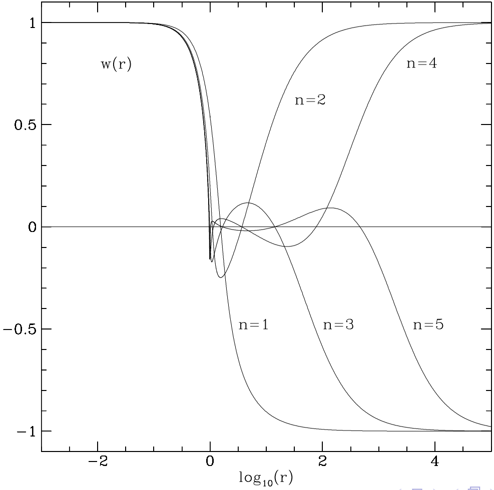
$$ds^2 = \sigma^2 N dt^2 - dr^2/N - r^2 \left(d\vartheta^2 + \sin^2\vartheta \, d\varphi^2 \right).$$

The amplitudes a_0, $w = w^*$, m, $N \equiv 1 - 2m/r$, and σ depend on r. The trivial solution of the Einstein-YM equations is $a_0 = 0$, $w = \pm 1$, $m = M$, $\sigma = 1$, which describes the Schwarzschild metric and a pure gauge YM field. Note that $w = \pm 1$ correspond to topologically inequivalent YM vacua, with neighboring Chern-Simons numbers.

A simple non-trivial solution describes "colored black holes" (Bais75...) $a_0 = a_0(\infty) + Q/r$, $w = 0$, $N = 1 - 2M/r + (Q^2+1)/r^2$, $\sigma = 1$. The metric is RN with the electric charge Q and unit magnetic charge. This solution is *embedded Abelian*. "Non-Abelian baldness of colored black holes" (DG and Ershov, 88) proves than no black hole solution with $w \neq 0$ is possible in this case. These are not fully equivalent to their U(1) counterparts: the Dirac string can be globally removed by passing to the regular gauge; also, unlike their U(1) counterparts, the colored black holes are unstable.

Bartnik-McKinnon

If $a = 0$ (no color charges) essentially non-Abelian magnetic part w can be turned on. A whole family of regular AF solutions were found (BK,88) oscillating between the vacua $w = \pm 1$:



Sphaleron interpretation

The BK solutions admit an interesting interpretation as EYM sphalerons DG and Volkov 91, Sudarsky 92, Gibbons93. This is based on some common features which they share with the sphaleron solution of the Weinberg-Salam model (Manton83,Klinkhamer84). In fact, sphalerons exist also in other gauge models with vacuum periodicity (Jackiw76). In such theories the potential energy is a periodic function of the winding number of the gauge field. The minima of the energy, which are called topological vacua, are separated by a potential barrier of a finite height, and "sitting" on the top of the barrier there is a classical field configuration called sphaleron.

Since its energy determines the barrier height, the sphaleron is likely to be important for the barrier transition processes when the system interpolates between distinct vacuum sectors, at least in the electroweak theory (Kuzmin85,Rubakov96). The winding number of the gauge field changes during such processes, which leads to the fermion number non-conservation due to the axial anomaly.

Topological vacua

Contrary to the case of sphalerons in YM-Higgs theory, in the EYM case, the topological argument applies to the YM field itself. Topological vacua in EYM theory are defined as fields $\{g_{\mu\nu}, A\}$ with zero ADM mass: $g_{\mu\nu} = \eta_{\mu\nu}$, and pure gauge $A = i\mathrm{U}d\mathrm{U}^{-1}$. Here $\eta_{\mu\nu}$ is Minkowski metric on R^4 and U=U(x^i). Imposing an asymptotic condition (Jackiw76)

$$\lim_{r\to\infty} \mathrm{U}(x^i) = 1,$$

any U(x^i) can be viewed as a mappings $S^3 \to$ SU(2), and the set of all U's falls into a countable sequence of disjoint homotopy classes characterized by an integer winding number

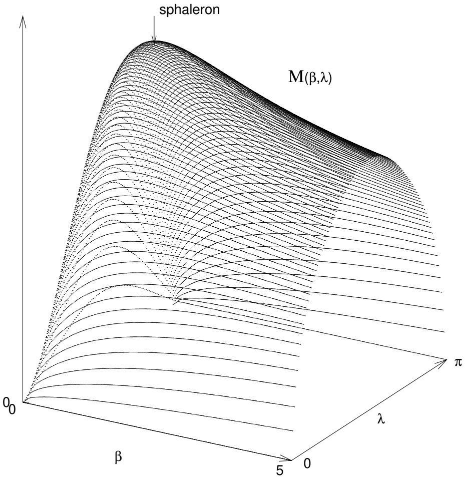
$$\mathbf{k}[\mathrm{U}] = \frac{1}{24\pi^2}\,\mathrm{tr}\int_{R^3} \mathrm{U}d\mathrm{U}^{-1}\wedge \mathrm{U}d\mathrm{U}^{-1}\wedge \mathrm{U}d\mathrm{U}^{-1}.$$

As a result, the vacuum fields split into equivalence classes with respect to the winding number of the gauge field $\mathbf{k}[\mathrm{U}]$. These are called topological vacua. Note that the gravitational field is assumed to be topologically trivial (for detail see Volkov and DG, Phys.Rept.99)

The vacua with different k cannot be continuously deformed into one another within the class of vacuum fields subject to asymptotic condition. However, it is possible to join them through a continuous sequence of non-vacuum fields, $\{g_{\mu\nu}[\lambda], A[\lambda]\}$, where the gauge field $A[\lambda]$ obeys the desired asymptotic for all values of the parameter λ, while $g_{\mu\nu}[\lambda]$ is an asymptotically flat metric on R^4. The crucial point is that there are such interpolating sequences that *pass through the BK field configurations*.

The BK solution with $n = 1$ is the EYM sphaleron sitting at the top of the potential barrier between $k = 0$ and $k = 1$ vacua. Its Chern-Simons number is $\nu = 1/2$. Higher-n BK solutions are excited states looking as non-linear superpositions of sphalerons and anti-sphalerons with $\nu = -1/2$. Solutions with even-n are topologically trivial. All odd-n solutions have $\nu = 1/2$.

Crucial for this interpretation is the number of odd-parity negative modes around sphaleron, which change the Chern-Simons number. Calculation shows that this number is n (Volkov and DG95). In addition, there are n even-parity negative modes, related to gravitational instability. Note also that these solutions admit a zero energy fermion bound state Gibbons93, Volkov94, and the spectrum of the Dirac operator for odd n exhibits the level crossing structure indicating on fermion non-conservation.


sphaleron
M(β,λ)
π
λ
0
5
β
0
0
0

EYM hairy black holes

The EYM system of equations admits also similar w-oscillating AF gravitating solutions with a regular event horizon (Volkov and DG89, Bizon90, Kunzle et al.91). This came as a surprise too, since these solutions violated the no-hair theorems known for vacuum and electrovacuum black holes. It became clear that non-linear gravitating field systems do not obey the uniqueness theorems known in vacuum and electrovacuum gravity and the corresponding black holes may admit non-trivial hair outside the horizon.

Sphaleronic interpretation of EYM black holes does not work, since construction of the vacuum-to-vacuum path is based on the energy constraint which is obscured in presence of gravity.

More complicated hairy solutions with rotations, cosmological constant, solutions for higher rank gauge groups, for dilatonic YM actions and black holes with Higgs fields were also found.

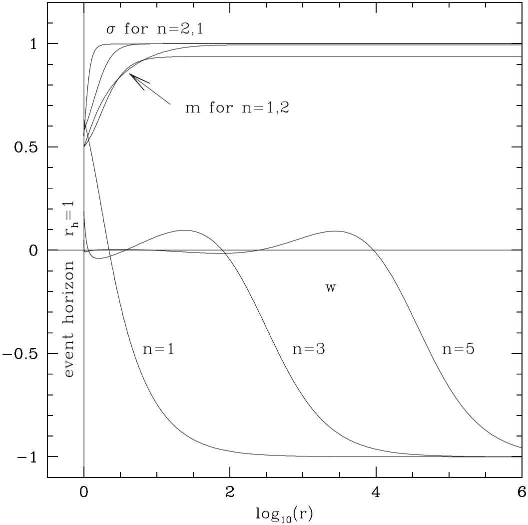


Figure 2: The amplitudes w, m and σ for the non-Abelian black hole solutions with $r_h = 1$.

Internal structure of EYM BH

Abelian solutions with color charges have Reissner-Nordstrom (or Kerr-Newman) interiors, endowed with the inner Cauchy horizons. Solutions with no-Abelian hair do not have such structure. Numerical integration reveals quite unusual generic behaviour of the solutions in the interior region (Donets, DG and Zotov97, Breitenlohner, Lavrelashvili and Maison97). Choosing an arbitrary external solution and integrating inwards from the event horizon, the metric functions m and σ and the derivative w' exhibit in the interior region violent oscillations, whose amplitude and frequency grow without bounds as the system approaches the singularity. At the same time, as this happens at the very short scale, the amplitude w is almost constant in the internal region. One can derive approximate two-dimensonal dynamical system describing oascillations analytically and revealing phenomenon called mass inflation at the Cauchi horizon. Actually, approach of such "would be" internal horizon one encounters violent oscillations transforming it into spacelike singularity.

Note that while the exterior structure of the EYM solutions does not change considerably after adding a dilaton or a Higgs fields, the interior solutions change completely (DG et al.97) and exhibit a regular power-law behaviour near the singularity.

Horizons inside classical lumps

Soon after it was realized that a variety of hairy black holes can be constructed in theories admitting regular solitons in flat space simply placing the event horizon instead of regular center when gravity is switched on. The corresponding proofs can be provided using the implicit function theorem (Kastor et al.92).

Historically, the first investigation of such systems was motivated by a wish to understand the structure of very heavy magnetic monopoles (Nieuwenhuizen76). More systematic investigations of the problem were undertaken after the discovery of the BK solutions. It has been found that, for a large class of non-linear matter models, the flat space solitons can be generalized to curved spacetime, provided that the dimensionless gravitational coupling constant κ is small. What is less obvious less obvious, it turned out that under certain conditions gravity can be treated perturbatively even for black holes, provided that the event horizon radius r_h is small. As a result, the gravitating lumps can be further generalized by replacing the regular center by a black hole with a small radius r_h.

Adding the Higgs action

$$S_{\rm H} = \int \left(\frac{1}{2}\,{\rm tr}\, D_\mu \Phi\, D^\mu \Phi - \frac{\lambda}{4}\,(\Phi^a \Phi^a - v^2)^2 \right) \sqrt{-g}\, d^4 x\,,$$

where $D_\mu \Phi = \partial_\mu \Phi - i[A_\mu, \Phi]$, with The spherically symmetric Higgs field is $\Phi = v\phi \mathbf{T}_r,$, $\phi = \phi(t,r)$ being a real scalar, er get the 2D matter Lagrangian

$$L_m = L_{\rm YM} + \frac{r^2}{2}\,(\partial\phi)^2 - |w|^2\phi^2 - \frac{\epsilon^2}{4}\,r^2\,(\phi^2 - 1)^2.$$

Here the length scale is ${\rm L} = 1/M_{\rm W}$, $\epsilon = M_{\rm H}/\sqrt{2}M_{\rm W}$, where $M_{\rm W} = gv$ and $M_{\rm H} = \sqrt{2\lambda}v$ are the vector boson mass and the Higgs boson mass, respectively. The BPS limit is attended if $\epsilon = 0$ and is stable. For any given value of ϵ there is one solution with mass M increasing with ϵ such that $1 \leq M(\epsilon) < M(\infty) = 1.96$.

The rescaled gravitational coupling constant is $\kappa = 4\pi G v^2$. Switching gravity off, $\kappa \to 0$ we have flat space monopoles with $N = \sigma = 1$. Once gravity is switchwd on, N develops a minimum at some $r_m \sim 1$, $N(r_m) > 0$, one can call r_m the monopole radius. The solutions are characterized by κ and ϵ. The mass, $M(\kappa)$, decreases with growing κ due to the gravitational binding, while r_m does not change considerably.

As a result, the ratio of the gravitational radius of the monopole to its radius r_m is proportional to κ. For regular solutions κ must not exceed some critical value, otherwise the system becomes unstable to gravitational collapse. Correspondingly, the self-gravitating monopoles exist only for a finite range $0 < \kappa \leq \kappa_{\max}(\epsilon)$. One has

$$\kappa_{\max}(\infty) = 1/2 < \kappa_{\max}(\epsilon) \leq \kappa_{\max}(0) = 1.97$$

(Breitenlohner et al.95). As κ approaches the critical value, $N(r_m)$ tends to zero the functions w and ϕ reach their asymptotic values already at $r = r_m$, and the proper distance $l = \int_0^r \frac{dr}{\sqrt{N}}$ diverges for $r \to r_m$ As a result, the spatial geometry on the hypersurface $t =$const. develops an infinite throat separating the interior region with a smooth origin and non-trivial YMH field from the exterior region where $w = 0$, $\phi = 1$, and the metric is extreme RN. The throat is characterized by a constant radius, $r_o = \sqrt{\kappa_{\max}}$. The metric function $\sigma(r)$, normalized by $\sigma(0) = 1$, diverges for $r \to r_m$. The limiting solution thus splits into two independent solutions, which is similar to the behaviour of the BK solutions for $n \to \infty$. The interior solution is geodesically complete.

Secondly, in the non-linear regime, when gravity is not weak, the fundamental lumps and black holes admit a discrete spectrum of gravitational excitations. The excited solutions become infinitely heavy for $\kappa \to 0$ and, remarkably, reduce then to the rescaled BK solutions. The excitations thus can be thought of as gravitating solitons with small BK particles or EYM black holes in the center.

Finally, the fundamental and excited lumps and black holes cannot exist beyond certain maximal values of κ and r_h. Regular monopoles have masses and radii depending on Higgs coupling constant The existence of a bound for κ is easy to understand. As κ is proportional to the ratio of the gravitational radius of the object to its typical size, it cannot be too large, since otherwise the system becomes unstable with respect to the gravitational collapse. The solutions, however, do not collapse as κ approaches the critical value. Instead they either become gravitationally closed or coalesce with the excited solutions.

The existence of the bound for r_h is quite interesting. It seems as if a small black hole could not swallow up a soliton which is larger than the black hole itself. As a result, a hairy black hole appears. However, a big black hole can completely absorb all non-trivial hair.

Classical glueballs in non-Abelian Born-Infeld theory

The superstring theory gives rise to one important modification of the standard Yang-Mills quadratic Lagrangian suggesting the action of the Born-Infeld (BI) type (Tseytlin97). Such a modification breaks the scale invariance, and the non-existence of classical particle-like solutions can be overruled. This is particularly intriguing since now neither gravity, nor scalar fields are involved, so one is thinking about the genuine *classical glueballs*. Our investigation showed that the $SU(2)$ BIYM classical glueballs indeed do exist and display a remarkable similarity with the BK solutions of the EYM theory.

Non–Abelian generalisation of the Born–Infeld action presents an ambiguity in specifying how the trace over the the matrix–valued fields is performed in order to define the Lagrangian. Simpler version adopts the ordinary trace prescription prviding a simple closed form for the action. However, another trace prescription is favored in the superstring context, namely, the *symmetrized* trace (Tseytlin97). In what follows we consider both version observing that the difference for our problems is not big.

The flat space BIYM action with the ordinary trace prescription reads

$$S = \frac{\beta^2}{4\pi} \int (1 - \mathcal{R})\, d^4x, \quad \mathcal{R} = \sqrt{1 + \frac{1}{2\beta^2} F^a_{\mu\nu} F^{\mu\nu}_a - \frac{1}{16\beta^4} (F^a_{\mu\nu} \tilde{F}^{\mu\nu}_a)^2}.$$

The "critical" field strength parameter is β of dimension L^{-2} . The trace of the stress–energy tensor is non-zero:

$$T^\mu_\mu = \mathcal{R}^{-1} \left[4\beta^2 (1 - \mathcal{R}) - F^a_{\mu\nu} F^{\mu\nu}_a \right] \neq 0$$

and vanishes in the limit $\beta \to 0$ when the action reduces to usual one. Assuming the monopole ansatz for the YM field

$$A^a_0 = 0, \quad A^a_i = \epsilon_{aik} \frac{n^k}{r} (1 - w(r)),$$

where $n^k = x^k / r,\ r = (x^2 + y^2 + z^2)^{1/2}$, after the integration over the sphere one obtains a two-dimensional action from which β can be eliminated by the coordinate rescaling $\sqrt{\beta} t \to t,\ \sqrt{\beta} r \to r$:

$$S = \int L dr,\ L = r^2 (1 - \mathcal{R}),\ \mathcal{R} = \sqrt{1 + 2\frac{w'^2}{r^2} + \frac{(1 - w^2)^2}{r^4}},$$

where prime denotes the derivative with respect to r. The total energy (mass) of the configuration is $M = \int_0^\infty (\mathcal{R} - 1) r^2 dr$.

A trivial solution to this theory $w \equiv 0$ corresponds to the pointlike magnetic BI-monopole with the unit magnetic charge (embedded $U(1)$ solution). In the Born–Infeld theory it has a finite self-energy:

$$M = \int \left(\sqrt{r^2 + 1} - r^2 \right) dr = \frac{\pi^{3/2}}{3\Gamma(3/4)^2} \approx 1.23604978.$$

Let us look now for essentially non–Abelian solutions of finite mass (DG and Kerner99). In order to assure the convergence of the integral the quantity $\mathcal{R} - 1$ must fall down faster than r^{-3} as $r \to \infty$. Thus, far from the core the BI corrections have to vanish and the EOM reduces to the ordinary YM equation. The latter is equivalent to the following two-dimensional autonomous system

$$\dot{w} = u, \quad \dot{u} = u + (w^2 - 1)w,$$

where a dot denotes the derivative with respect to $\tau = \ln r$. This dynamical system has three non-degenerate stationary points $(u = 0, w = 0, \pm 1)$, from which $u = w = 0$ is a focus, while two others $u = 0,\ w = \pm 1$ are saddle points with eigenvalues $\lambda = -1$ and $\lambda = 2$. The separatices along the directions $\lambda = -1$ start at infinity and after passing through the saddle points go to the focus with the eigenvalues $\lambda = (1 \pm i\sqrt{3})/2$.

The function $w(\tau)$ approaching the focus as $\tau \to \infty$ is unbounded. Two other separatices, passing through saddle points along the directions specified by $\lambda = 2$, go to infinity in both directions. Since there are no limiting circles, generic phase curves go to infinity or approach the focus, unless $w = 0$ identically. All of them produce a divergent mass integral. The only trajectories remaining bound as $\tau \to \infty$ are those which go to the saddle points along the separatrices specified by $\lambda = -1$.
From this reasoning one finds that *the only finite-energy configurations with non-vanishing magnetic charge are the embedded U(1) BI-monopoles*. Indeed, such solutions should have asymptotically $w = 0$, which does not correspond to bounded solutions unless $w \equiv 0$.

The remaining possibility is $w = \pm 1, \dot{w} = 0$ asymptotically, which corresponds to zero magnetic charge. Coming back to r-variable one finds

$$w = \pm 1 + \frac{c}{r} + O(r^{-2}),$$

where c is a free parameter. This gives a convergent integral as $r \to \infty$. Note that two values $w = \pm 1$ correspond to two neighboring topologically distinct YM vacua.

Now consider local solutions near the origin $r = 0$. For convergence of the total energy, w should tend to a finite limit as $r \to 0$. Then using the EOM one finds that the only allowed limiting values are $w = \pm 1$ again. In view of the symmetry of under reflection $w \to \pm w$, one can take without loss of generality $w(0) = 1$. Then the following Taylor expansion can be checked to satisfy the EOM:

$$w = 1 - br^2 + \frac{b^2(44b^2 + 3)}{10(4b^2 + 1)} r^4 + O(r^6),$$

with b being the only free parameter. Then, analysing the dynamical system, one can prove that any finite energy solution should start at the origin with the above expansion. Thus the global finite energy solution starting with this should meet some solution from the family at infinity. Since both these local solutions are non–generic, one can at best match them for some discrete values of parameters. To complete the existence proof one has to show that this discrete set of parameters is non-empty. For this we present the EOM in the resolved form

$$\ddot{w} = \gamma \dot{w} + w(w^2 - 1),$$

where the “negative friction coefficient” is

$$\gamma = 1 + \frac{\dot{\mathcal{R}}}{\mathcal{R}} = 1 - \frac{\left[\dot{w} + w(1-w^2)\right]^2 + (1-w^2)^3}{r^4 + (1-w^2)^2}.$$

Details of the proof can be found in DG and Kerner99. Then one finds the allowed values of the parameter b numerically. Actual solutions have behavior similar to BK solutions of EYM

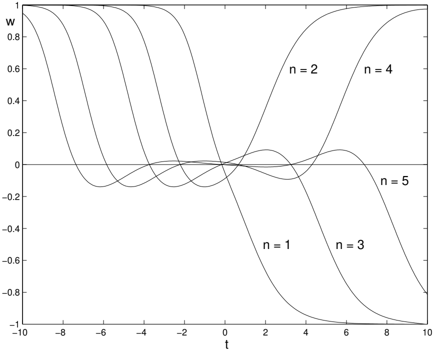


Figure 3: Solutions $w(\tau)$ for $n = 1, ..., 5$.

Symmetrized trace and gravity

Introducing matrix-valued YM field $F_{\mu\nu} = F^a_{\mu\nu} t_a, \quad \mathrm{tr}\, t_a t_b = \delta_{ab}$ one has to expand the lagrangian in terms of t_a and compute

$$\mathrm{Str}(t_{a_1} \dots t_{a_p}) \equiv \frac{1}{p!} \mathrm{Str}\left(t_{a_1} \dots t_{a_p} + \text{all permutations}\right),$$

where the gauge algebra can not be applied, i.e. $\tau_a^2 \neq 1$ until the symmetrization of the expansion is completed. The result depends on particular ansatz for YM field, in our case, on two scalar functions

$$V^2 = \frac{(1 - w^2(r))^2}{2\beta^2 r^4}, \quad K^2 = \frac{w'^2(r)}{2\beta^2 r^2}$$

. One obtains

$$L_{NBI} = \frac{\beta^2}{4\pi}\left(1 - \frac{1 + V^2 + K^2 \mathcal{A}}{\sqrt{1 + V^2}}\right),$$

where

$$\mathcal{A} = \sqrt{\frac{1 + V^2}{V^2 - K^2}}\, \mathrm{arctanh} \sqrt{\frac{V^2 - K^2}{1 + V^2}}.$$

When $\beta \to \infty$, the standard YM lagrangian is recovered. In the strong field region this expression differs essentially from the square root/ordinary trace lagrangian.

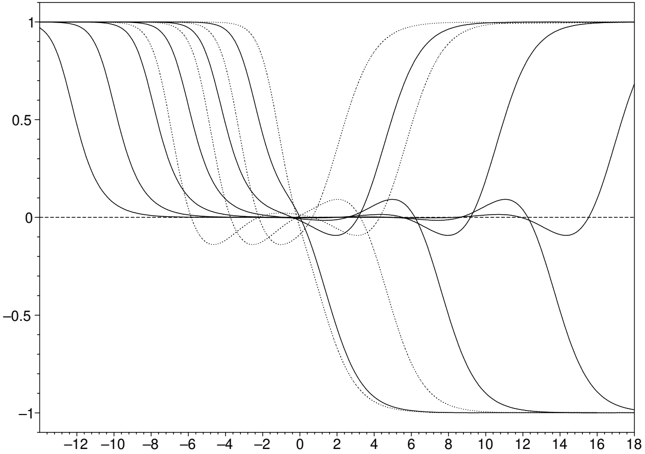


Figure 4: First six solutions $w_n(r)$ for flat space glueballs in the NBI theory with symmetrized trace (solid line) and their four ordinary trace counterparts (dotted).

$$ds^2 = N\sigma^2 dt^2 - \frac{dr^2}{N} - r^2(d\theta^2 + \sin^2\theta d\phi).$$

After suitable rescaling, two dimensional parameters of the theory G, β combine in one dimensionless coupling constant $g = G\beta$, Regular gravitating solutions start at the origin with

$$w = 1 - b\,r^2, \quad N = 1 - \frac{2}{3}\frac{g(1 + 8b^2 - \sqrt{1+4b^2})}{\sqrt{1+4b^2}} r^2$$

At infinity one must have $w \to \pm 1,\ N \sim 1 - 2M/r,$ For weak gravity regular solutions are similar to those in the flat case, the b_n unbounded while the number of w-nodes tends to infinity. The metric function $N(t)$ with increasing n approaches the metric of the abelian solution at some interval which moves more and more close to the origin.

But if $g > g_{cr} = 1/2$ the parameters b_n tend to a limiting value and the metric functions tend to the metric of the limiting abelian solution with a degenerate horizon. This situation resembles that of the EYM model and indeed this model could be recovered in the limit $g \to \infty,\ \sqrt{g}r \to r$. The strong gravity limit of purely YM NBI sphalerons are thus the BK solutions (in this limit tr and Str models coincide):

$$\lim_{g\to\infty} M_n(g)/\sqrt{g} = M_n^{BK},$$

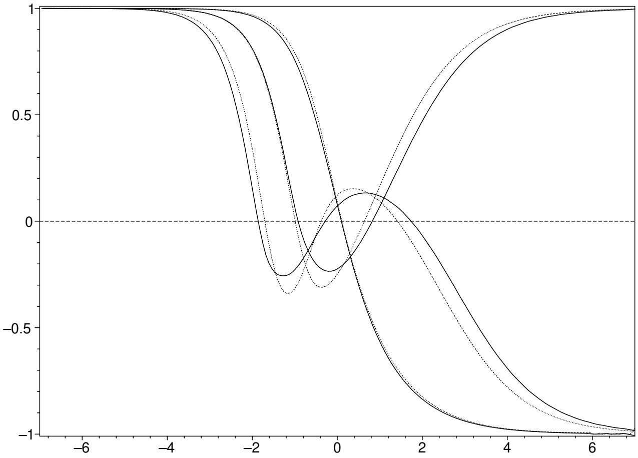


Figure 5: Functions $w_n(r)$, $n = 1, 2, 3$ for gravitating glueballs in the Str (solid line) and Tr (dotted line) NBI models with $g = g_{cr} = \frac{1}{2}$. Solutions are practically indistinguishable for $n = 1$, becoming slightly different for $n = 2, 3$.

NBI black holes are possible too. For them one starts with the expansion at the horizon $r = r_h$ (new parameter):

$$N = N_h'(r - r_h) + O((r - r_h)^2), \quad w = w_h + \frac{w_h(w_h^2 - 1)}{\mathcal{A} r_h^2 N_h'}(r - r_h) + O((r - r_h)^2),$$

NBI black holes exist for all horizon radii r_h with a discrete sequence of w_h. For non-small r_h the function N is monotonous outside the horizon, while for smaller r_h one observes a local minimum near the black hole.

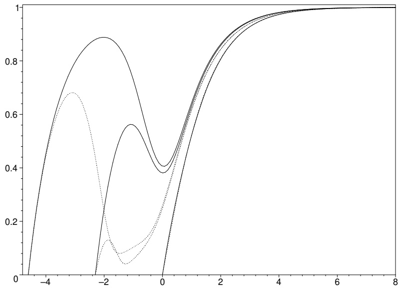


Figure 6: $N_n(r)$ for black holes in the Str model for $n = 1$ (solid) and $n = 3$ (dotted) with $r_h = 1, 0.1, 0.01$ and $g = 1/2$.

NBI+Higgs

The flat space NBI and curved space EYM theories have a common feature to admit sphaleron solutions indicating that NBI provides effective attraction similar to gravitational attraction. To explore this further, it is useful to consider NBI coupled to Higgs and explore the magnetic monopoles (Dyadichev and DG,02).

In this theory regular monopoles may have sphaleron excitations quite similar to gravitating YMH monoples. Surprisingly enough, the analogy with gravitation goes farther, and one observes some analog of gravitational collapse in the flat space NBI theory. The size of monopoles (including excited states) is rapidly decreasing when $\beta \to \beta_{\rm cr}$ with the limiting configuration being an Abelian pointlike monopole. An analysis of stresses inside monopoles near the criticality shows that both radial and tangential pressure become negative in the core and rapidly increase. It is expected that in the dynamical picture these stresses will force the regular configuration to shrink to the pointlike structure.

Collapse of NBI monopoles in flat space

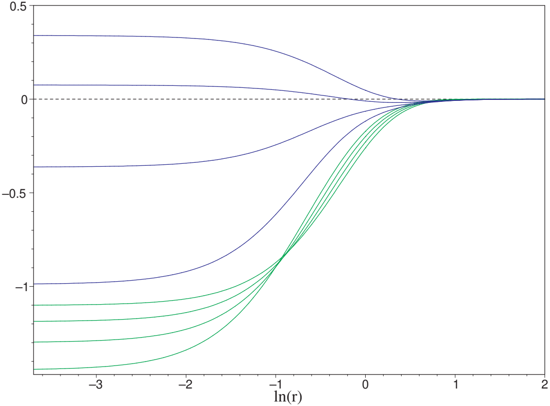


Figure 7: The radial pressure p_r for the ground state BI-monopole for $\beta = 0.9, 1.0, 1.2, 2.0$. The blue line — the pressure of the NBI field, the green line — the pressure of the Higgs field. The left asymptotic values go down with decreasing β for both fields.

EYM vortex

There exists cosmic string type solution of the EYM equations with cylindrically symmetric metric

$$ds^2 = l^2\{N(r)^2 dt^2 - H(r)^2 dr^2 - L(r)^2 d\varphi^2 - K(r)^2\, dz^2\},$$

and purely magnetic gauge field $A = \tau_2\, R(r)\, dz + \tau_3\, P(r)\, d\varphi$.

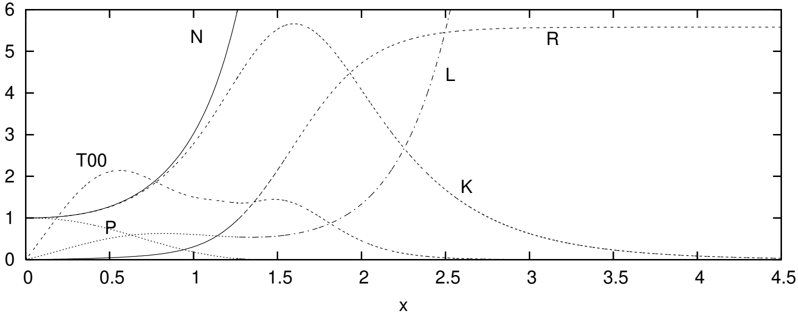


Figure 8: The globally regular solution. The central core extends up to $\xi \approx 1.2$, after which the configuration practically coincides with the Bonnor Abelian solution. For $\xi \geq 3$ the gauge field dies out and the solution becomes Kasner.

Strings have a compact central core dominated by a longitudinal magnetic field and endowed with an approximately Melvin geometry. This magnetic field component gets color screened in the exterior part of the core, outside of which the fields approach exponentially fast those of the electrovacuum Bonnor solutions with a circular magnetic field.

In the far field zone the solutions are not asymptotically flat but tend to vacuum Kasner metrics. They are endowed with a constant current along their symmetry axis, and can be considered as analogs of Witten's superconducting cosmic strings. In the latter case the superconductivity arises due to the presence of two complex scalars in the theory. Here their role is played by two color components of the self-interacting Yang-Mills field, which behave exactly in the same way as the scalars in Witten's model. One develops a non-zero condensate in the vortex core and vanishes at infinity, while the other one behaves the other way round.

Since the gauge symmetry of our theory is not broken, the gauge field coupled to the current is long-range. The slow fall-off of the energy density in the direction orthogonal to the string then does not let the solutions to be asymptotically flat, although they are asymptotically Ricci flat. This is the principal difference of the EYM strings as compared to Abelian gravitating cosmic strings of Nielsen-Olesen type (Grafinkle85).

Thank you for attention!