Renormalizable Extension of the Abelian Higgs-Kibble Model with a Dim. 6 Derivative Operator

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Introduction

Gauge Symmetry + Spontaneous Symmetry Breaking
$\Rightarrow$ Successful description of Electroweak Physics
up to the TeV scale

2022: 10 years from the discovery of the Higgs resonance @LHC

What comes next?

- new particles (SUSY,...)
- exploring the Higgs potential (precision physics @HE-HL LHC)

  Higgs scalar doublet $\phi = \begin{pmatrix} \phi^+ \\ \frac{1}{\sqrt{2}}(v+\sigma+i\chi) \end{pmatrix}$, v.e.v. $\langle\phi\rangle = \frac{v}{2}$

  Anomalous trilinear Higgs coupling:

$$V(\phi) = \frac{\lambda}{2}\left(\phi^\dagger\phi - \frac{v^2}{2}\right)^2 \supset \frac{v\lambda}{2}\sigma^3 \to \kappa\frac{v\lambda}{2}\sigma^3\,, \quad \kappa_{SM} = 1$$

- ...

Effective Field Theories (EFTs)

Dim.4 Lagrangian plus higher dim. ops. arranged in powers of a large inverse energy scale Λ

$$\mathscr{L}_{BSM} = \mathscr{L}_4 + \frac{1}{\Lambda}\sum_i c_i^5 \mathscr{O}_i^5 + \frac{1}{\Lambda^2}\sum_i c_i^6 \mathscr{O}_i^6 + \dots$$

compatible with the low-energy symmetry pattern.

Power-counting renormalizability of $\mathscr{L}_4$ is lost,
since more and more UV divergences arise
as more and more loops are included: high price to pay ...

Instability of the radiative corrections,
UV completion unknown.

Renormalizability in the modern sense

Gomis and Weinberg, Nucl.Phys. B469 (1996) 473-487

- Power-counting (p.c.) renormalizabilty is lost
- Still locality of the counter-terms (as formal power series) holds provided that:
  1. non-linear field redefinitons are taken into account
  2. the renormalization of the gauge-invariant operators is carried out order by order in the perturbative loop expansion

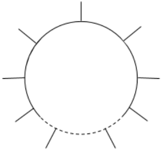
Effective parameterization of electroweak physics at energy well below the scale Λ (Warsaw basis for ops. up to dim. 6, ...)

Prototype dim.6 operator

$$\phi^\dagger\phi(D^\mu\phi)^\dagger D_\mu\phi \supset \sigma^2\partial^\mu\sigma\partial_\mu\sigma$$

- Power-counting maximally violated

The UV degree of divergence is always 4 irrespectively of the number of external legs



$$\delta_{UV} = 4 + (2-2) + (2-2) + \cdots = 4$$

(At $z \neq 0$ not) power-counting renormalizable Abelian HK model

$$\phi = \frac{1}{\sqrt{2}}(\sigma + v + i\chi)\,, \quad D_\mu \phi = \partial_\mu \phi - ieA_\mu \phi$$

$$S_{HK} = \int d^4x \Big[-\frac{1}{4}F_{\mu\nu}^2 + (D^\mu \phi)^\dagger D_\mu \phi - \frac{\lambda}{2}\Big(\phi^\dagger \phi - \frac{v^2}{2}\Big)^2 + \frac{z}{2v^2}\phi^\dagger \phi \Box \phi^\dagger \phi \Big]$$

Field coordinates

Some field coordinates are better suited than others
in order to study the UV properties of a given model.

Free massless theory in polar coordinates

$$\phi = \frac{1}{\sqrt{2}}(\rho + v)e^{i\frac{\vartheta}{v}}$$

$$S = \int d^4x \, \frac{1}{2}\partial^\mu \phi \partial_\mu \phi = \int d^4x \left[\frac{1}{2}\partial^\mu \rho \partial_\mu \rho + \frac{1}{2}\left(1 + \frac{\rho}{v}\right)^2 \partial^\mu \vartheta \partial_\mu \vartheta\right]$$

Goldstone modes ϑ's and the unphysical polarization of the gauge field are rotated away by an operatorial gauge transformation, only physical components left in the non-renormalizable Lagrangian

Abelian case

$$\phi \to e^{-i\frac{\vartheta}{v}}\phi\,, \quad A'_\mu = A_\mu - \frac{1}{ev}\partial_\mu \vartheta$$

Non-abelian case
(T^a generators of the gauge group, g coupling constant)

$$\phi = (v+\rho)\Omega\,, \quad \Omega = \exp\left(iT_a\frac{\vartheta_a}{v}\right)$$

$$\phi' = \Omega^\dagger \phi\,, \;\; A'_\mu = \Omega^\dagger A_\mu \Omega + \frac{i}{g}\Omega^\dagger \partial_\mu \Omega$$

The X-formalism

Use a gauge-invariant scalar
to parameterize the physical Higgs field,
leave the Goldstone fields linearly realized

Implementation via (au)X(iliary) fields $X_{1,2}$

$$X_2 \sim \frac{1}{v}\Big(\phi^\dagger\phi - \frac{v^2}{2}\Big)\,, \quad X_1 \text{ Lagrange multiplier}$$

Power-counting renormalizability is preserved (at $z = 0$)

The $X_{1,2}$-system

We add to the classical action the term

$$\int d^4x \, \frac{1}{v}(X_1 + X_2)(\Box + m^2)\left(\phi^\dagger\phi - \frac{v^2}{2} - vX_2\right)$$

Going on-shell with X_1 yields a Klein-Gordon equation

$$(\Box + m^2)\left(\phi^\dagger\phi - \frac{v^2}{2} - vX_2\right) = 0 \Rightarrow X_2 = \frac{1}{v}\left(\phi^\dagger\phi - \frac{v^2}{2}\right) + \eta$$

η being a scalar field of mass m.

In perturbation theory the correlators of the mode η with any gauge-invariant operators vanish, so that one can safely set $\eta = 0$.

$$\Gamma^{(0)} \supset \int \mathrm{d}^4x \Bigg[-\frac{1}{4}F^{\mu\nu}F_{\mu\nu} + (D^\mu\phi)^\dagger(D_\mu\phi)$$
$$- \frac{M^2 - m^2}{2}X_2^2 - \frac{m^2}{2v^2}\left(\phi^\dagger\phi - \frac{v^2}{2}\right)^2 + \frac{z}{2}\partial^\mu X_2 \partial_\mu X_2$$
$$\underbrace{-\bar{c}(\square + m^2)c + \frac{1}{v}(X_1 + X_2)(\square + m^2)\left(\phi^\dagger\phi - \frac{v^2}{2} - vX_2\right)}_{s-\text{invariant}}\Bigg]$$

Constraint U(1) BRST symmetry s
(decoupling of $X_1, \bar{c}, c$ from the physical spectrum):

$$sX_1 = vc\,, \quad sc = 0\,, \quad s\bar{c} = \frac{1}{v}\left(\phi^\dagger\phi - \frac{v^2}{2} - vX_2\right)$$

Going on-shell with X_1 the second line becomes

$$
\begin{aligned}
& -\frac{M^2-m^2}{2}X_2^2-\frac{m^2}{2v^2}\left(\phi^\dagger\phi-\frac{v^2}{2}\right)^2+\frac{z}{2}\partial^\mu X_2\partial_\mu X_2 \\
& \sim -\frac{M^2}{2v^2}\left(\phi^\dagger\phi-\frac{v^2}{2}\right)^2+\frac{z}{2v^2}\partial^\mu\left(\phi^\dagger\phi-\frac{v^2}{2}\right)\partial_\mu\left(\phi^\dagger\phi-\frac{v^2}{2}\right)
\end{aligned}
$$

m has disappeared (it never contributes to physical quantities after going on-shell - a powerful check of the radiative computations)

R_ξ-gauge-fixing and external sources

$$\Gamma^{(0)} \supset \int d^4x \left[\frac{\xi b^2}{2} - b\,(\partial A + \xi e v \chi) + \bar{\omega}\left(\Box\omega + \xi e^2 v(\sigma + v)\omega\right) \right.$$

$$\left. + \underbrace{\bar{c}^* \left(\phi^\dagger \phi - \frac{v^2}{2} - v X_2\right)}_{\text{Constraint BRST antifield}} + \underbrace{\sigma^*(-e\omega\chi) + \chi^* e\omega(\sigma + v)}_{\text{Gauge BRST antifields}} \right].$$

Gauge U(1) BRST symmetry

$$sA_\mu = \partial_\mu \omega;\ s\phi = ie\omega\phi;\ s\sigma = -e\omega\chi;\ s\chi = e\omega(\sigma + v);\ s\bar{\omega} = b;\ sb = 0$$

Diagonalize by redefining $\sigma = \sigma' + X_1 + X_2$, $b' = b - \frac{1}{\xi}\partial A - ev\chi$:

$$\Delta_{\sigma'\sigma'} = \frac{i}{p^2 - m^2}; \quad \Delta_{X_1X_1} = -\frac{i}{p^2 - m^2}; \quad \Delta_{X_2X_2} = \frac{i}{(1+z)p^2 - M^2}$$

$$\Delta_{\mu\nu} = -i\left(\frac{1}{p^2 - M_A^2}T_{\mu\nu} + \frac{1}{\frac{1}{\xi}p^2 - M_A^2}L_{\mu\nu}\right); \qquad M_A = ev;$$

$$\Delta_{b'b'} = \frac{i}{\xi}; \quad \Delta_{\chi\chi} = \frac{i}{p^2 - \xi M_A}; \quad \Delta_{\bar{\omega}\omega} = \frac{i}{p^2 - \xi M_A^2}; \quad \Delta_{\bar{c}c} = \frac{-i}{p^2 - m^2}.$$

In the diagonal mass eigenstate basis the dependence on the parameter z only arises via the X_2-propagator

$$\Delta_{X_2X_2}(k^2, M^2) = \frac{i}{(1+z)k^2 - M^2}$$

Define

$$\mathscr{D}_z^{M^2} = (1+z)\partial_z + M^2\partial_{M^2} \,.$$

Then $\Delta_{X_2X_2}$ is an eigenvector of $\mathscr{D}_z^{M^2}$ with eigenvalue -1:

$$\mathscr{D}_z^{M^2}\Delta_{X_2X_2}(k^2, M^2) = -\Delta_{X_2X_2}(k^2, M^2).$$

Take an amplitude and decompose it according to the number ℓ of internal X_2-lines:

$$\Gamma^{(n)}_{\Phi_1\cdots\Phi_r} = \sum_{\ell\geq 0} \Gamma^{(n;\ell)}_{\Phi_1\cdots\Phi_r} .$$

$$\mathscr{D}_z^{M^2}\Gamma^{(n;\ell)}_{\Phi_1\cdots\Phi_r} = -\ell\Gamma^{(n;\ell)}_{\Phi_1\cdots\Phi_r} \Longrightarrow \mathscr{D}_z^{M^2}\Gamma^{(n)}_{\Phi_1\cdots\Phi_r} = -\sum_{\ell\geq 0}\ell\Gamma^{(n;\ell)}_{\Phi_1\cdots\Phi_r}.$$

The most general solution reads

$$\Gamma^{(n;\ell)}_{\Phi_1\cdots\Phi_r}(z, M^2) = \frac{1}{(1+z)^\ell}\Gamma^{(n;\ell)}_{\Phi_1\cdots\Phi_r}(0, M^2/1+z).$$

Thus, amplitudes at $z \neq 0$ in each ℓ-sector are obtained from those at $z = 0$ by dividing them by the $(1+z)^\ell$ factor and rescaling by $(1+z)$ the square of the Higgs mass M^2.

The z-flow

Boundary conditions (BCs) at $z = 0$: amplitudes of the power-counting renormalizable HK model

Amplitudes at $z \neq 0$ are the solutions to the z-differential equation satisfying the above BCs

Compare with the usual effective field theory approach:

- dim. 6 interaction vertices induced by $\sim z\ \phi^\dagger \phi \Box \phi^\dagger \phi$ generate an infinite number of seemingly unrelated UV divergent amplitudes already at one loop order;
- usually amplitudes are evaluated in the linearized approximation with respect to $z \Rightarrow$ here fully resummed amplitudes, **exact** dependence on z;
- the result holds true to all orders in the loop expansion.

Compact representation of the z-differential equation

Let us define a modified 1-PI Green's function depending on an auxiliary parameter t:

$$\Gamma^{(n)}_{\Phi_1\cdots\Phi_r}(t) = \Gamma^{(n;0)}_{\Phi_1\cdots\Phi_r} + \sum_{\ell\geq 1} t^{\ell-1}\Gamma^{(n;\ell)}_{\Phi_1\cdots\Phi_r}; \quad \Gamma^{(n)}_{\Phi_1\cdots\Phi_r}(1) = \Gamma^{(n)}_{\Phi_1\cdots\Phi_r}$$

$$\begin{aligned}\mathscr{D}_z^{M^2}\int_0^1 \mathrm{d}t\,\Gamma^{(n)}_{\Phi_1\cdots\Phi_r}(t) &= \sum_{\ell\geq 1}\int_0^1 \mathrm{d}t\;\, t^{\ell-1}\;\mathscr{D}_z^{M^2}\Gamma^{(n;\ell)}_{\Phi_1\cdots\Phi_r} \\ &= -\sum_{\ell\geq 1}\int_0^1 \mathrm{d}t\;\ell\, t^{\ell-1}\Gamma^{(n;\ell)}_{\Phi_1\cdots\Phi_r} \\ &= -\sum_{\ell\geq 1}\Gamma^{(n;\ell)}_{\Phi_1\cdots\Phi_r} = -\Gamma^{(n)}_{\Phi_1\cdots\Phi_r} + \Gamma^{(n;0)}_{\Phi_1\cdots\Phi_r}.\end{aligned}$$

Collecting finally the Green's functions
in the t-dependent generating functional

$$\Gamma(t) = \sum_{n,\Phi,r} \int \mathrm{d}^D p_1 \ldots \mathrm{d}^D p_r \quad \underbrace{w_{\Phi_1 \cdots \Phi_r}}_{\text{comb. factors}} \; \Gamma^{(n)}_{\Phi_1 \cdots \Phi_r}(t) \; \Phi_1 \cdots \Phi_r,$$

we arrive at

$$\int_0^1 \mathrm{d}t \, \mathscr{D}_z^{M^2} \Gamma(t) = -\Gamma(1) + \Gamma_0$$

where the subscript 0 denotes the Stückelberg sector
(no internal X_2-lines):

$$\Gamma_0 = \sum_{n,\Phi,r} \int \mathrm{d}^D p_1 \ldots \mathrm{d}^D p_r \; w_{\Phi_1 \cdots \Phi_r} \Gamma^{(n;0)}_{\Phi_1 \cdots \Phi_r} \Phi_1 \cdots \Phi_r$$

Taylor expansion around $z = 0$

$$\Gamma(t) = \sum_k z^k \Gamma_{[k]}(t)\,, \quad \Gamma_{[0]} \text{ p.c. renormalizable theory}$$

Projection of the z-differential equation

$$\mathcal{O}(1): \qquad \int_0^1 \mathrm{d}t \, \left[\Gamma_{[1]}(t) + M^2 \partial_{M^2} \Gamma_{[0]}(t)\right] = -\Gamma_{[0]}(1) + \Gamma_0;$$

$$\mathcal{O}(z): \qquad \int_0^1 \mathrm{d}t \, \left[2\Gamma_{[2]}(t) + \Gamma_{[1]}(t) + M^2 \partial_{M^2} \Gamma_{[1]}(t)\right] = -\Gamma_{[1]}(1);$$

$$\mathcal{O}(z^2): \qquad \int_0^1 \mathrm{d}t \, \left[3\Gamma_{[3]}(t) + 2\Gamma_{[2]}(t) + M^2 \partial_{M^2} \Gamma_{[2]}(t)\right] = -\Gamma_{[2]}(1);$$

$$\mathcal{O}(z^k): \int_0^1 \mathrm{d}t \, \left[(k+1)\Gamma_{[k+1]}(t) + k\Gamma_{[k]}(t) + M^2 \partial_{M^2} \Gamma_{[k]}(t)\right] = -\Gamma_{[k]}(1).$$

Check on the UV divergences at one loop order

$$\Gamma^{(1)}_{[\cdots]\Phi_1\Phi_2}(t) = \Gamma^{(1;0)}_{[\cdots]\Phi_1\Phi_2} + \Gamma^{(1;1)}_{[\cdots]\Phi_1\Phi_2} + t\,\Gamma^{(1;2)}_{[\cdots]\Phi_1\Phi_2}$$

Consider e.g. the 2-point $\bar{c}^*$ function:

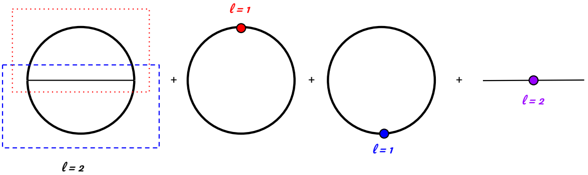
$$\overline{\Gamma}^{(1;0)}_{\bar{c}^*\bar{c}^*} = \frac{1}{16\pi^2}\frac{1}{\epsilon}; \qquad \overline{\Gamma}^{(1;1)}_{\bar{c}^*\bar{c}^*} = 0; \qquad \overline{\Gamma}^{(1;2)}_{\bar{c}^*\bar{c}^*} = \frac{1}{16\pi^2}\frac{1}{(1+z)^2}\frac{1}{\epsilon}.$$

One can easily check the z-differential equation
on the UV divergent parts $\overline{\Gamma}^{(1)}$:

$$\mathcal{O}(1): \int_0^1 \mathrm{d}t\; t\,\overline{\Gamma}^{(1;2)}_{[1]\bar{c}^*\bar{c}^*} = -\overline{\Gamma}^{(1;2)}_{[0]\bar{c}^*\bar{c}^*},$$

$$\mathcal{O}(2): \int_0^1 \mathrm{d}t\; t\left(2\,\overline{\Gamma}^{(1;2)}_{[2]\bar{c}^*\bar{c}^*} + \overline{\Gamma}^{(1;2)}_{[1]\bar{c}^*\bar{c}^*}\right) = -\overline{\Gamma}^{(1;2)}_{[1]\bar{c}^*\bar{c}^*}\,.$$

The z-differential equation constrains the counter-terms since $\mathscr{D}_z^{M^2}$ acts also on the counter-terms themselves. Consider for instance the following 2-loop amplitude



$$\mathscr{D}_z^{M^2} \Gamma^{(2;2)}_{\Phi_1\Phi_2} = -2\Gamma^{(2;2)}_{\Phi_1\Phi_2}$$

A general n-loop 1-PI Green's function can be decomposed as (after insertion of counter-terms of loop order $j < n$)

$$\Gamma^{(n)}_{\Phi_1\cdots\Phi_r} = \sum_{\ell\geq 0}\left[\Gamma^{(n;\ell)}_{\Phi_1\ldots\Phi_r} - \sum_{k=1}^{n}\frac{1}{\epsilon^k}\overline{\Gamma}^{(n;\ell)}_{k;\Phi_1\ldots\Phi_r} + \underbrace{F^{(n;\ell)}_{\Phi_1\ldots\Phi_r}}_{\text{finite cts}}\right]$$

with

$$\mathscr{D}^{M^2}_z\overline{\Gamma}^{(n;\ell)}_{k;\Phi_1\ldots\Phi_r} = -\ell\overline{\Gamma}^{(n;\ell)}_{k;\Phi_1\ldots\Phi_r}; \qquad \mathscr{D}^{M^2}_z F^{(n;\ell)}_{\Phi_1\ldots\Phi_r} = -\ell F^{(n;\ell)}_{\Phi_1\ldots\Phi_r},$$

and thus possess the structure:

$$\overline{\Gamma}^{(n;\ell)}_{k;\Phi_1\ldots\Phi_r}(z,M^2) = \frac{1}{(1+z)^\ell}\overline{\Gamma}^{(n;\ell)}_{k;\Phi_1\ldots\Phi_r}(0,M^2/1+z),$$

$$F^{(n;\ell)}_{\Phi_1\ldots\Phi_r}(z,M^2) = \frac{1}{(1+z)^\ell}F^{(n;\ell)}_{\Phi_1\ldots\Phi_r}(0,M^2/1+z).$$

Reconstructing the $X_{1,2}$ 1-PI amplitudes

Two functional identities allow to fix the amplitudes involving external $X_{1,2}$-legs in terms of amplitudes without:

$$\frac{\delta\Gamma}{\delta X_1} = \frac{1}{v}(\square + m^2)\frac{\delta\Gamma}{\delta\bar{c}^*} \, ,$$
$$\frac{\delta\Gamma}{\delta X_2} = \frac{1}{v}(\square + m^2)\frac{\delta\Gamma}{\delta\bar{c}^*} - (\square + m^2)X_1 - ((1+z)\square + M^2)X_2 - v\bar{c}^*$$

In particular we can limit ourselves to amplitudes with zero external X_2-lines while studying the Slavnov-Taylor identities.

Rescale the parameters z, M^2 according to

$$1 + z \to \frac{1+z}{t}\,, \quad M^2 \to \frac{M^2}{t}\,.$$

in the vertex functional of the complete theory $\Gamma(1)$.
The new vertex functional is graduated w.r.t. ℓ.
At order n in the loop expansion

$$\begin{aligned}\widehat{\Gamma}(t)^{(n)} &\equiv \Gamma_0^{(n)}\Big|_{X_2=0} + t\left[\Gamma(t)^{(n)} - \Gamma_0^{(n)}\right]\Big|_{X_2=0} \\ &= \Gamma_0^{(n)}\Big|_{X_2=0} + \sum_{\ell\geq 1} t^\ell \Gamma^{(n;\ell)}\Bigg|_{X_2=0}\,.\end{aligned}$$

Since X_2 is BRST invariant, $\widehat{\Gamma}(t)$ is Slavnov-Taylor invariant:

$$\mathcal{S}(\widehat{\Gamma}(t)) = \int d^4x \left[\partial_\mu \omega \frac{\delta \widehat{\Gamma}(t)}{\delta A_\mu} + \frac{\delta \widehat{\Gamma}(t)}{\delta \sigma^*}\frac{\delta \widehat{\Gamma}(t)}{\delta \sigma} + \frac{\delta \widehat{\Gamma}(t)}{\delta \chi^*}\frac{\delta \widehat{\Gamma}(t)}{\delta \chi} + b\frac{\delta \widehat{\Gamma}(t)}{\delta \bar{\omega}}\right] = 0$$

We expand w.r.t. t and get a tower of identities holding in the number ℓ of internal X_2-lines:

$$\mathcal{S}_0(\Gamma^{(n;\ell)}) + \sum_{j=1}^{n-1}\sum_{i=0}^{\ell}(\Gamma^{(j;i)}, \Gamma^{(n-j;\ell-i)}) = 0\,,$$

$$(\Gamma, \Gamma) \equiv \int d^4x \left[\frac{\delta \Gamma}{\delta \sigma^*}\frac{\delta \Gamma}{\delta \sigma} + \frac{\delta \Gamma}{\delta \chi^*}\frac{\delta \Gamma}{\delta \chi}\right].$$

Major simplifications arise in the Landau gauge since the gauge BRST-antifield-dependent amplitudes do not receive radiative corrections. Hence

$$\mathcal{S}_0(\Gamma^{(n;\ell)}) = s(\Gamma^{(n;\ell)})$$

i.e. each sector with a given number of internal X_2-lines is separately BRST invariant.

Example: one-loop 3-point σ 1-PI Green's function
(diagrams with up to 3 internal X_2-lines)

$$\Gamma^{(1)}_{\sigma_1\sigma_2\sigma_3} = \sum_{\ell=0}^{3} \left[\Gamma^{(1;\ell)}_{\sigma_1\sigma_2\sigma_3} - \frac{1}{\epsilon}\overline{\Gamma}^{(1;\ell)}_{1;\sigma_1\sigma_2\sigma_3} + F^{(1;\ell)}_{\sigma_1\sigma_2\sigma_3} \right],$$

$\overline{\Gamma}^{(1;\ell)}_{1;\sigma_1\sigma_2\sigma_3}, F^{(1;\ell)}_{\sigma_1\sigma_2\sigma_3}$ polynomials up to degree two
in the independent external momenta $p_{1,2}$ ($p_3 = -p_1 - p_2$):

$$\overline{\Gamma}^{(1;\ell)}_{1;\sigma_1\sigma_2\sigma_3} = \gamma^{0(1;\ell)}_{1;\sigma_1\sigma_2\sigma_3} + \gamma^{1(1;\ell)}_{1;\sigma_1\sigma_2\sigma_3}(p_1^2 + p_2^2 + p_1 \cdot p_2),$$

$$F^{(1;\ell)}_{\sigma_1\sigma_2\sigma_3} = f^{0(1;\ell)}_{\sigma_1\sigma_2\sigma_3} + f^{1(1;\ell)}_{\sigma_1\sigma_2\sigma_3}(p_1^2 + p_2^2 + p_1 \cdot p_2).$$

Sum rule at $z = 0$

Summing overthe different layers in ℓ (like one would do in a "standard" approach) gives for the coefficient of the quadratic term in the independent momenta

$$\gamma^{1(1)}_{1;\sigma_1\sigma_2\sigma_3} = \sum_{\ell=0}^{3} \gamma^{1(1;\ell)}_{1;\sigma_1\sigma_2\sigma_3} = \frac{z}{8\pi^2 v^2 (1+z)^4} \left[2M^2(1-2z) + m^2(1+z)\right].$$

At $z = 0$ $\gamma^{1(1)}_{1;\sigma_1\sigma_2\sigma_3}$ vanishes as expected, since the 3-point σ amplitude has UV dim. 1 in the p.c.-renormalizable theory.
Therefore we get the sum rule for the finite parts:

$$f^{1(1)}_{1;\sigma_1\sigma_2\sigma_3}\Big|_{z=0} = \sum_{\ell=0}^{3} f^{1(1;\ell)}_{1;\sigma_1\sigma_2\sigma_3}\Big|_{z=0} = 0.$$

Comparison with the standard EFT approach

At $z \neq 0$ in the standard effective field theory approach one would need to fix $f^{1(1)}_{1;\sigma_1\sigma_2\sigma_3}$ by an appropriate normalization condition.

However one needs to do this in a way consistent with the z-differential equation.

I.e. one would need to choose in some way the individual coefficients $f^{1(1;\ell)}_{1;\sigma_1\sigma_2\sigma_3}\Big|_{z=0}$ and then lift them up according to the prescription to solve the z-differential equation in the specific ℓ-sector.

Sum rule at $z \neq 0$

Without loss of generality one can define amplitudes at $z = 0$ in the MS scheme (all other schemes can be obtained by a finite redefinitions of the couplings and the field renormalizations).

One can require that each ℓ-sector flows into its $z = 0$ counter-part. This is consistent since each sector is separately ST-invariant.

In the MS scheme $f^{1(1;\ell)}_{1;\sigma_1\sigma_2\sigma_3}\Big|_{z=0} = 0$; then the lifted $f^{1(1;\ell)}_{1;\sigma_1\sigma_2\sigma_3}$ at $z \neq 0$ are also vanishing.

The same result is obtained by lifting the sum rule at $z = 0$ to $z \neq 0$:

$$f^{1(1)}_{1;\sigma_1\sigma_2\sigma_3} = \sum_{\ell=0}^{3} f^{1(1;\ell)}_{1;\sigma_1\sigma_2\sigma_3} = 0 \Rightarrow f^{1(1;\ell)}_{1;\sigma_1\sigma_2\sigma_3} = 0 \quad \forall\, \ell$$

as can be immediately seen by repeated application of the operator $\mathscr{D}^{M^2}_z$ to both sides of the above equation.

Some new features arise:

- a novel differential equation controls the UV divergences of a non-renormalizabile theory in terms of those of a power-counting renormalizable one
- separate Slavnov-Taylor invariance of each sector with a given number of internal X_2-lines

1) A mathematically consistent ‘cousin’ of the Higgs theory
even though not power-counting renormalizable
(possibly in the spirit of the reduction of couplings?)

It the answer is in the affirmative, one can apply the formalism to predictions in the $SU(2) \times U(1)$ electroweak theory deformed by the dim.6 operator

$$z \ \phi^\dagger \phi \Box \phi^\dagger \phi$$

2) Does this technique generalize to other gauge-invariant composite operators in spontaneously broken gauge effective field theories?