Quarkyonic Phase

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Based on work in progress
with Pavel Slepov and Kristina Rannu

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- I.A, L. D. Faddeev and A. A. Slavnov,
  "Generating Functional for the S Matrix in Gauge Theories,"
  Teor. Mat. Fiz. **21**, 311-321 (1974)

$$
\begin{aligned}
Z(\phi_{cl}) &= \int e^{-iS(\phi+\phi_{cl})} d\phi \\
& \frac{\delta S}{\delta \phi}\Big|_{\phi=\phi_{cl}} = 0 \\
Z(J) &= \int e^{-i(S(\phi)+\int J\phi)} d\phi
\end{aligned}
$$

- I.A, L. D. Faddeev and A. A. Slavnov,
"Generating Functional for the S Matrix in Gauge Theories,"
Teor. Mat. Fiz. **21**, 311-321 (1974)
- *Abstract.* An expression for the generating functional of the elements of the S matrix is obtained. It is shown that in electrodynamics and in a Yang-Mills theory all the ultraviolet divergences can be eliminated by charge renormalization. In the formalism, gauge dependent counter terms do not arise at all.

$$Z(\phi_{cl}) = \int e^{-iS(\phi+\phi_{cl})} d\phi$$

$$\left.\frac{\delta S}{\delta \phi}\right|_{\phi=\phi_{cl}} = 0$$

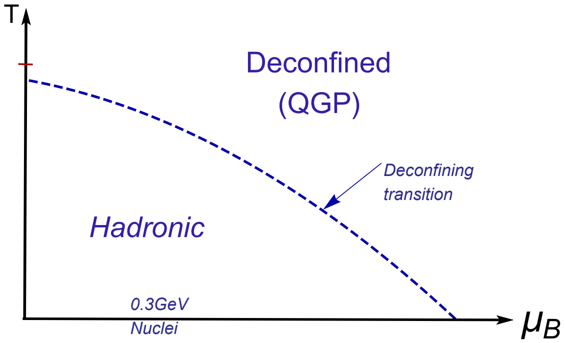
$$Z(J) = \int e^{-i(S(\phi)+\int J\phi)} d\phi$$

Outlook

- QCD Phase Diagram
- 1-st order phase transition in H(olographic)QCD
- How we can detect 1st order phase transition experimentally

QCD Phase Diagram: Early Conjecture

Cabibbo and Parisi, 1975



- μ a measure of the imbalance between quarks and antiquarks in the system

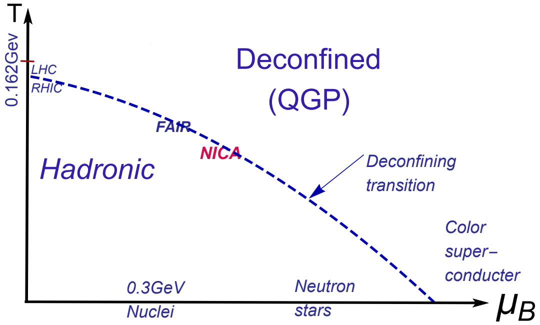
QCD Phase Diagram: Experiments

- LHC, RHIC (2005);
- FAIR (Facility for Antiproton and Ion Research),

  NICA (Nuclotron-based Ion Collider fAcility)

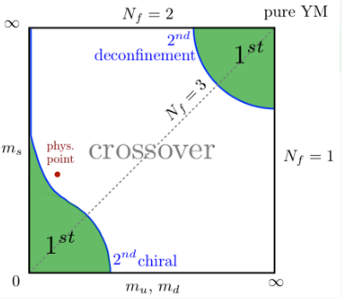
Main goals

- search for signs of the phase transition between hadronic matter and QGP;
- search for new phases of baryonic matter



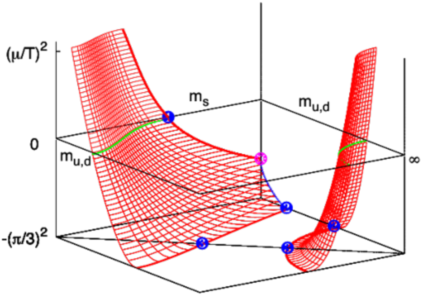
QCD Phase Diagram: Lattice

Phase diagram
on quark mass



Columbia plot
Brown et al., PRL (1990)

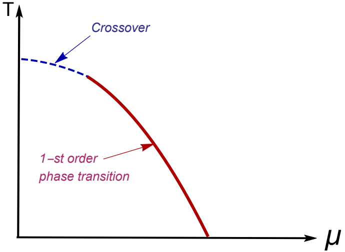
Main problem with $\mu \neq 0$
Imaginary chemical potential method



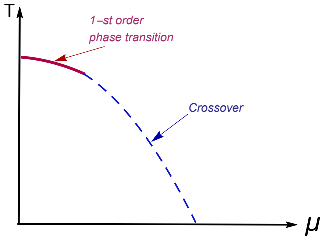
Philipsen, Pinke, PRD (2016)

“Heavy” and “light” quarks from Columbia plot

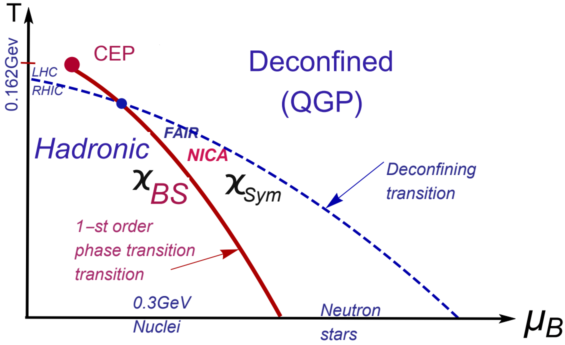
Light quarks



Heavy quarks

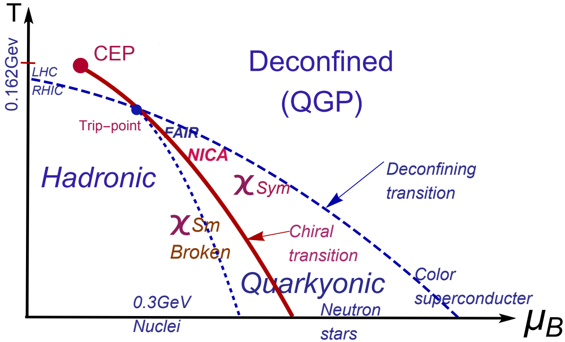


The expected QCD phase diagram



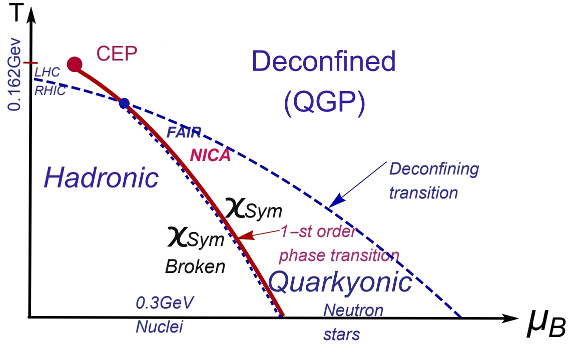
- Parameter of the chiral symmetry breaking $< \bar{\psi}\psi >$
  - $< \bar{\psi}\psi > = 0 \Longleftrightarrow \chi$-symmetry
  - $< \bar{\psi}\psi > \neq 0 \Longleftrightarrow$ broken χ-symmetry

The expected QCD phase diagram



- Quarkyonic phase: baryon free ⇒ dense baryons *McLerran, Pisarski 0706.2191*
  - Baryon density jumps

The expected QCD phase diagram



Holographic QCD

- Perturbation methods are not applicable to describe QCD phase diagram
- Lattice methods do not work, because of problems with the chemical potential.
- Holographic QCD - phenomenological model(s)

- One of goals of Holographic QCD – describe QCD phase diagram

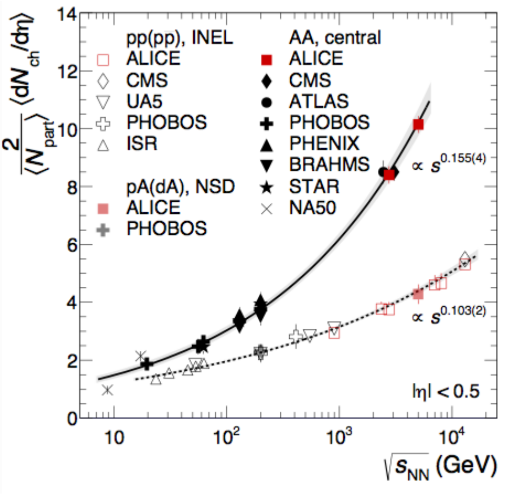
- **Requirements:**
  - reproduce the QCD results from perturbation theory at short distances
  - reproduce Lattice QCD results at large distances (~ 1 fm) and **small** μ_B

Holographic method - phenomenological approach

Motivated by AdS/CFT duality Maldacena,1998

- Temperature in QCD $\Longleftrightarrow$ black hole temperature in (deform.)AdS
- Thermalization in QCD $\Longleftrightarrow$ formation of black hole in (deform.)AdS5
- Thermalization models (black hole formation models):
  colliding shock waves; the area of the trapped surface determines the multiplicity

Total multiplicity produced in heavy ions collision



Plot from PRL'16 (ALICE)
PbPb
$\mathcal{M} \sim s_{NN}^{0.15}$

The bulk of the particles are born immediately after the collision of heavy ions

Multiplicity

- Experiment

$$\mathcal{M} \sim s^{0.155}$$

- Macroscopic theory of high-energy collisions

$$Landau: \qquad \mathcal{M} \sim s^{0.25}$$

- Holographic approach
  - The simplest model gives (collision of shock waves)

$$AdS: \qquad \mathcal{M} \sim s^{0.33}$$

    Gubser et al, Phys.Rev. D, 2008; Gubser et al, JHEP, 2009; Alvarez-Gaume et al, PLB; 2009 Aref'eva et al, JHEP, 2009, 2010, 2012; Lin, Shuryak, JHEP, 2009, 2011; Kiritsis, Taliotis, JHEP, 2011
  - Anisotropic Lifshitz type background with exponent ν

$$\mathcal{M}_\nu \sim s^{\frac{1}{2+\nu}}, \qquad \textbf{Aref'eva, Golubtsova, JHEP, 2014}$$

$$\mathcal{M}_{LHC} \sim s^{0.155} \qquad \nu = 4.45$$

Holographic model of an anisotropic plasma in a magnetic field at a nonzero chemical potential

I.A, K. Rannu, P.Slepov, JHEP, 2021

$$S = \int d^5x \sqrt{-g}\left[R - \frac{f_1(\phi)}{4} F_{(1)}^2 - \frac{f_2(\phi)}{4} F_{(2)}^2 - \frac{f_B(\phi)}{4} F_{(B)}^2 - \frac{1}{2}\partial_M\phi\partial^M\phi - V(\phi)\right]$$

$$ds^2 = \frac{L^2}{z^2}\,\mathfrak{b}(z)\left[-g(z)\,dt^2 + dx^2 + \left(\frac{z}{L}\right)^{2-\frac{2}{\nu}} dy_1^2 + e^{c_B z^2}\left(\frac{z}{L}\right)^{2-\frac{2}{\nu}} dy_2^2 + \frac{dz^2}{g(z)}\right]$$

$$A_{(1)\mu} = A_t(z)\delta_\mu^0 \qquad F_{(2)} = dy^1 \wedge dy^2 \qquad F_{(B)} = dx \wedge dy^1$$

$$A_t(0) = \mu \qquad g(0) = 1 \qquad \text{Dudal et al., (2019)}$$

$$A_t(z_h) = 0 \qquad g(z_h) = 0 \qquad \phi(z_0) = 0 \;\rightarrow\; \sigma_{\text{string}}$$

Giataganas (2013), Aref'eva, Golubtsova (2014) *Gürsoy, Järvinen et al., (2019)*

$\mathfrak{b}(z) = e^{2\mathcal{A}(z)} \Leftrightarrow$ quarks mass "Bottom-up approach"

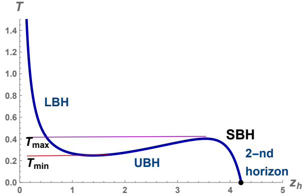
$\mathcal{A}(z) = -\,cz^2/4 \rightarrow$ heavy quarks background (b, t) *Andreev, Zakharov (2006)*

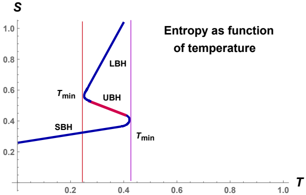
$\mathcal{A}(z) = -\,a\ln(bz^2+1) \rightarrow$ light quarks background (d, u) *Li, Yang, Yuan (2017)*

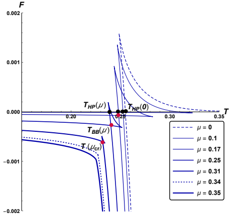
Origin of 1-st order phase transition in HQCD

- $g(z)$ blackenning function
- Due **non-monotonic** dependence of $T = T(z_h) = g'(z)/4\pi\Big|_{z=z_h}$
  the entropy $s = s(T)$ is **not monotonic**
- As a consequence the free energy $F = \int sdT$ undergoes the phase transition

1-st order phase transition (BB-PT) describes transition
from small black holes $\rightarrow$ large black holes





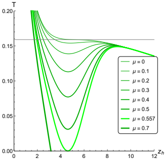


The swallow-tailed shape

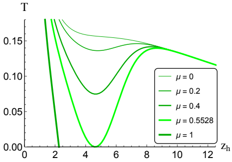
- Physical quantities that probe backgrounds are smooth relative to z_h
  $\Rightarrow$ their dependence on T should be taken from stable region
- BB-PT immediately provides the 1-st PT for corresponding characteristic of QCD

Origin of 1-st order phase transition in HQCD

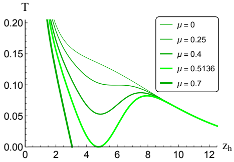
Light quarks, $\nu = 1$



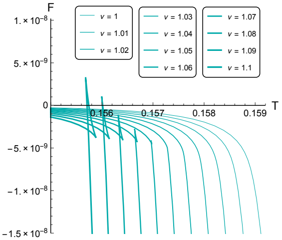
$c_B = 0$



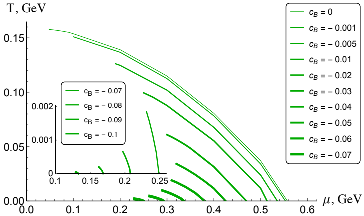
$c_B = 0.001$



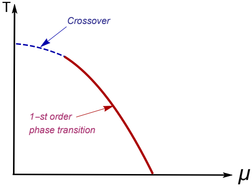
$c_B = 0.01$

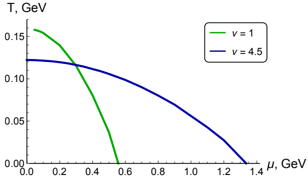


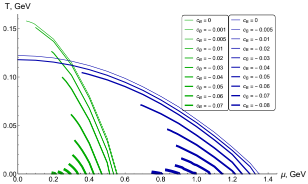
$\mu = 0$



Comparison of the 1-st order phase transition







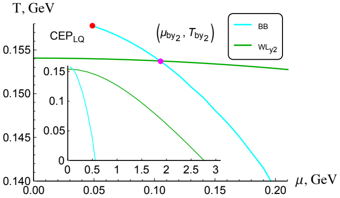
Schematic picture $\qquad B = 0 \qquad B \neq 0$

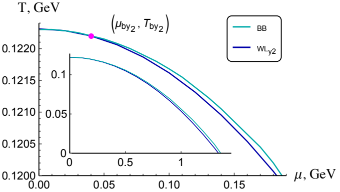
green lines $\nu = 1$ $\qquad$ blue lines $\nu = 4.5$

- For $B = 0$, the onset of the 1st order PTs moves towards $\mu = 0$ as ν increases
- As c_B increases (strong magnetic field) phase transition line lengths decrease

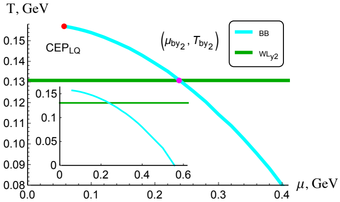
Phase transitions for light quarks

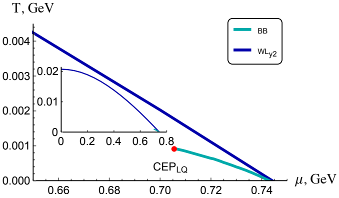
isotropic, $\nu = 1$ **anisotropic, $\nu = 4.5$**





$c_B = 0$ Quarkyonic phase (QP) appears during isotropization





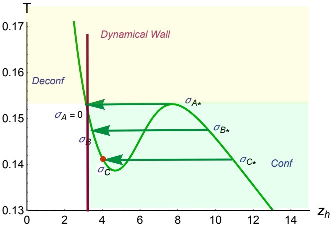
$c_B = -0.0000985$

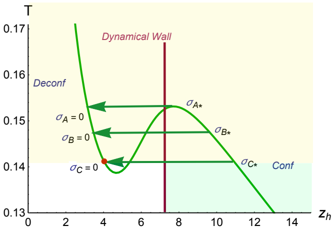
$c_B = -0.0858$

For $\nu = 4.5$ QP appears at large magnetic field and large μ

Background 1-st order phase transition ⇒ 1-st order phase transition for physical quantities

- Physical quantities that probe backgrounds are smooth relative to z_h ⇒ their dependence on T should be taken from stable region
- BB-PT immediately provides the 1-st PT for corresponding characteristic of QCD





The arrows show transitions from the unstable phases to the stable ones

Photon emission rate and electrical conductivity

The photon emission rate in thermal equilibrium

$$d\Gamma = -\frac{d\mathbf{k}}{(2\pi)^3}\frac{e^2 n_b(|\mathbf{k}|)}{|\mathbf{k}|}\,\mathrm{Im}\,[\eta_{\mu\nu}G_R^{\mu\nu}]_{k^0=|\mathbf{k}|}\,,$$

$n_b(|\mathbf{k}|) = \frac{\mathcal{A}}{e^{-|\mathbf{k}|/T}-1}$ Bose-Einstein thermal distribution function

$k^\mu = (k^0, \mathbf{k})$ photon 4-momentum

$G_R^{\mu\nu}$ the retarded Green's function is related to the electric conductivity through the Kubo relation

$$\sigma^{\mu\nu} = -\frac{G_R^{\mu\nu}}{iw}, \qquad \text{see REFs}$$

The spectral density $\chi^{\mu\nu}(k) = -2\ \mathrm{Im}[G_{\mathrm{R}}^{\mu\nu}(k)]$

Electrical conductivity for light quarks

To find the electric conductivity, we add a probe Maxwell field to action

$$S_{out} = -\frac{1}{4}\int d^5x\sqrt{-g}f_0 F_{MN}F^{MN},$$

$f_0 = f_0(\phi)$ is the function of coupling of the Maxwell field to the dilaton.
A plane wave propagating in x_3 direction

$$A_M(t, x_3, z){=}\psi_M(z)\exp(-i(wt - kx_3)), \qquad M = 0, ...4.$$

Using the Kubo formula $\sigma^{\mu\nu} = -G_R^{\mu\nu}/iw$ we obtain:

$$\sigma^{11} = \frac{2f_0(z_h)}{z_h}\sqrt{\frac{\mathfrak{b}(z_h)\mathfrak{g}_3(z_h)\mathfrak{g}_2(z_h)}{\mathfrak{g}_1(z_h)}}, \qquad \sigma^{22} = \frac{2f_0(z_h)}{z_h}\sqrt{\frac{\mathfrak{b}(z_h)\mathfrak{g}_3(z_h)\mathfrak{g}_1(z_h)}{\mathfrak{g}_2(z_h)}}$$

$$\sigma^{33} = \frac{2f_0(z_h)}{z_h}\sqrt{\frac{\mathfrak{b}(z_h)\mathfrak{g}_1(z_h)\mathfrak{g}_2(z_h)}{\mathfrak{g}_3(z_h)}}$$

I.A, Ermakov, Rannu, Slepov, EPJC22, arXiv:2203.12539

Electrical conductivity for light quarks

Polarization

$$\frac{d\Gamma_s}{d\vec{k}} \propto \epsilon_m^{(s)} \epsilon_n^{(s)} \sigma^{mn}$$

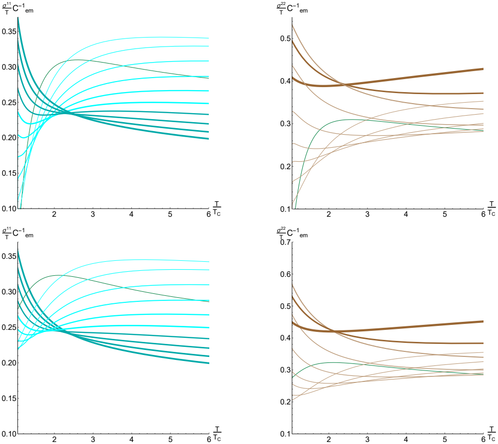
for $\vec{k} = (0, 0, k_3)$

$\epsilon_m^{(1)} = (\cos\varphi, \sin\varphi, 0)$

$\epsilon_m^{(2)} = (-\sin\varphi, \cos\varphi, 0)$

$$\frac{d\Gamma_1}{d\vec{k}} \propto \cos^2\varphi\, \sigma^{11} + \sin^2\varphi\, \sigma^{22}$$

σ^{11} σ^{22}



$\mu = 0$, $c_B = 0$

$\mu = 0.5$, $c_B = 0.05$

$\nu = 1$(darker green),
$\nu = 1.05,\ 1.1,\ 1.15,\ 1.2,\ 1.3,\ 1.5,\ 2,\ 3,\ 4.5$

- μ и magnetic field B for all σ^{ii} reduce the "spreading" in anisotropy

Conclusion

- **The hadronic matter - quarkyonic matter** phase transition $\Longleftrightarrow$ the first order phase transition for HQCD with light quarks (2009.05562, 2203.12539).
- A characteristic feature of quarkyonic matter is a small, compared with the confinement potential, a linear potential between quarks, which is not sufficient to keep quarks inside hadrons.
- Transverse-longitudinal anisotropy and magnetic field essentially influence on location of the quarkyonic phase
- We have observed **a jump of photon emission rate** on the hadronic - quarkyonic phase transition.
- We expect **a jump of jet quenching** on the hadronic - quarkyonic phase transition