Generating cosmological perturbations in non-singular Horndeski cosmologies

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Abstract

- We construct a concrete model of Horndeski bounce (see Fig. 1) with the strong gravity in the past;
- The correct spectra of cosmological perturbations may be generated at early contracting epoch within considered model;
- The background evolution and perturbations are legitimately described within classical field theory and weakly coupled quantum theory.

Null Energy Condition

Non-singular cosmology Realization of non-singular evolution within classical field theory requires the violation of the Null Energy Condition (NEC) $T_{\mu\nu}n^\mu n^\nu > 0$ (or Null Convergence Condition (NCC) $R_{\mu\nu}n^\mu n^\nu > 0$ for modified gravity).

$$T_{00} = \rho, \quad T_{ij} = a^2\gamma_{ij}p,$$
$$\dot{H} = -4\pi G(\rho + p) + \text{curvature term}.$$

Let us use $n_\mu = (1, a^{-1}\nu^i)$ with $\gamma_{ij}\nu^i\nu^j = 1$ and then NEC leads to

$$T_{\mu\nu}n^\mu n^\nu > 0 \to \rho + p \geq 0 \to \dot{H} \leq 0.$$

Penrose theorem: singularity in the past.

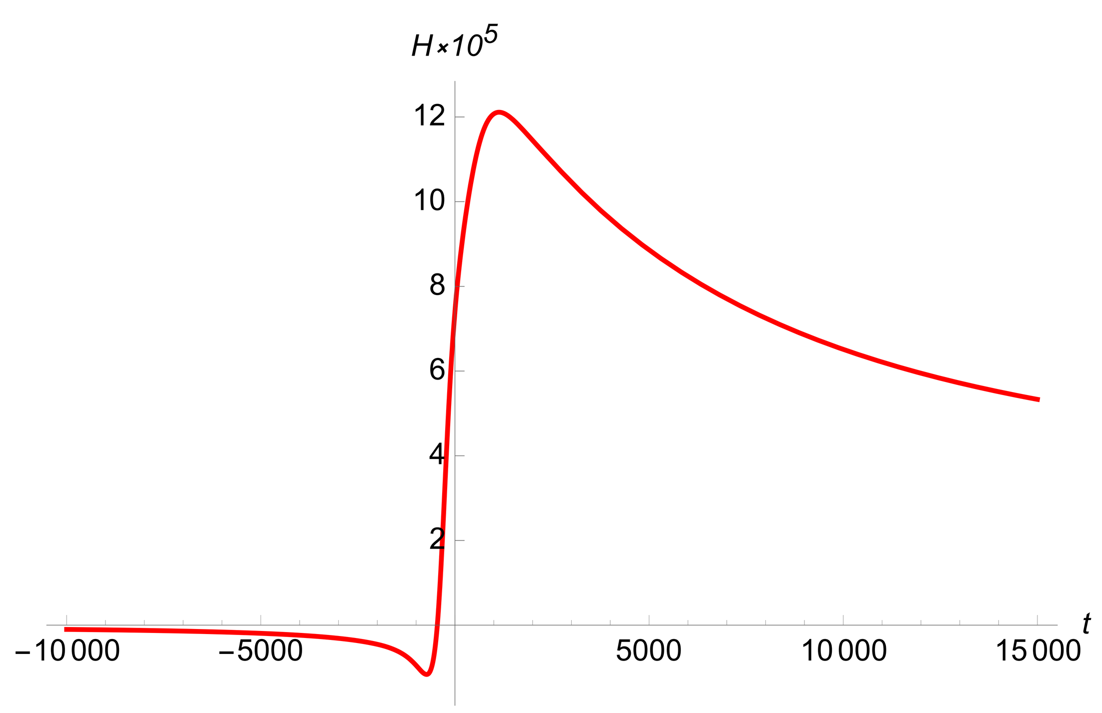

Figure 1: Evolution of the Hubble parameter: bounce. [Qui'2011,2013; Easson'2011; Cai'2012; Osipov'2013; Koehn'2013; Battarra'2014; Ijjas'2016]

Horndeski theory

Violation of NEC/NCC without obvious pathologies is possible in the class of Horndeski theories [Horndeski'74]:

$$\mathcal{L}_H = G_2(\phi, X) - G_3(\phi, X)\Box\phi + G_4(\phi, X)R + G_{4,X}\left[(\Box\phi)^2 - (\nabla_\mu\nabla_\nu\phi)^2\right] + G_5(\phi, X)G^{\mu\nu}\nabla_\mu\nabla_\nu\phi - \frac{1}{6}G_{5,X}\left[(\Box\phi)^3 - 3\Box\phi(\nabla_\mu\nabla_\nu\phi)^2 + 2(\nabla_\mu\nabla_\nu\phi)^3\right].$$

For our purposes it is enough to study

$$\mathcal{L}_H = G_2(\phi, X) - G_3(\phi, X)\Box\phi + G_4(\phi)R.$$

In the framework of this theory one can (quite straightforwardly) obtain healthy bounce epoch. It is convenient to work in ADM formalism:

$$ds^2 = -N^2dt^2 + \gamma_{ij}\left(dx^i + N^i dt\right)\left(dx^j + N^j dt\right),$$
$$\gamma_{ij} = a^2e^{2\zeta}(\delta_{ij} + h_{ij} + \ldots), \quad N = N_0(1+\alpha), \quad N_i = \partial_i\beta.$$

Here α and β are not physical. We work with unitary gauge $\delta\phi = 0$.

No-Go theorem

Another problem arises if one considers the whole evolution ($-\infty < t < +\infty$) of such a singularity-free universe: instabilities show up at some moment in the history → No-Go theorems [M. Libanov, S. Mironov, V. Rubakov'2016; T. Kobayashi'2016; S. Mironov, V. Rubakov, V. Volkova'2018]. The quadratic action for ζ (and the same form is for h_{ij}) is given by:

$$\mathcal{L}_{\zeta\zeta} = a^3\left[\mathcal{G}_S\frac{\dot{\zeta}^2}{N^2} - \frac{\mathcal{F}_S}{a^2}\zeta_{,i}\zeta_{,i}\right].$$

Remind that bounce solution is $a(t) \to \infty$ as $t \to -\infty$. No-Go works if

$$\int_{-\infty}^{t} a(t)(\mathcal{F}_T + \mathcal{F}_S)dt = \infty,$$
$$\int_{t}^{+\infty} a(t)(\mathcal{F}_T + \mathcal{F}_S)dt = \infty,$$

and $\mathcal{F}_{S,T} < 0$ at some moment of time.

- The way to avoid No-Go theorem is to obtain such a model/solution that $\mathcal{F}_{S,T} \to 0$ as $t \to -\infty$, where $\mathcal{F}_T = 2G_4$.
- Effective Planck mass goes to zero → we may have strong coupling at $t \to -\infty$.

Concrete bounce model

With the appropriate choice of lagrangian functions, the bounce solution is given by

$$N = \text{const}, \qquad a = d(-t)^\chi,$$

where $\chi > 0$ is a constant and $Nt \to t$ is cosmic time, so that $H = \chi/t$. Coefficients from quadratic actions are

$$\mathcal{G}_T = \mathcal{F}_T = \frac{g}{(-t)^{2\mu}},$$

$$\mathcal{G}_S = g\frac{g_S}{2(-t)^{2\mu}}, \qquad \mathcal{F}_S = g\frac{f_S}{2(-t)^{2\mu}},$$

$$u_T^2 = \frac{\mathcal{F}_T}{\mathcal{G}_T} = 1, \quad u_S^2 = \frac{\mathcal{F}_S}{\mathcal{G}_S} = \frac{f_S}{g_S} \neq 1.$$

To avoid No-Go:

$$1 > \chi > 0, \; 2\mu > \chi + 1.$$

To avoid SC regime ($t \to -\infty$):

$$\mu < 1.$$

Power spectrum

Spectra are given by

$$\mathcal{P}_\zeta \equiv \mathcal{A}_\zeta\left(\frac{k}{k_*}\right)^{n_s-1}, \qquad \mathcal{P}_T \equiv \mathcal{A}_T\left(\frac{k}{k_*}\right)^{n_T},$$

where k_* is pivot scale, the spectral tilts are

$$n_S - 1 = n_T = 2\cdot\left(\frac{1-\mu}{1-\chi}\right), \; n_S = 0.9649.$$

The amplitudes in our model are

$$\mathcal{A}_\zeta = \frac{C}{g}\frac{1}{g_S u_S^{2\nu}}, \; \mathcal{A}_T = \frac{8C}{g}, \; \nu \approx \frac{3}{2}.$$

and approximate flatness is ensured in our set of models by choosing $\mu \approx 1$ (the red-tilted spectrum is for $\mu > 1$).

The problem №1: red-tilted spectrum requires $\mu > 1$, while absence of strong coupling $\mu < 1$!

Solution: consider time-dependent μ: changes from $\mu < 1$ to $\mu > 1$ (time runs as $-\infty < t < \infty$).

The problem №2: r-ratio is small:

$$r = \frac{\mathcal{A}_T}{\mathcal{A}_\zeta} \approx 8g_S u_S^3 < 0.032. \text{ [Tristram'2022]}$$

Solution: choose $u_S \ll 1$. [Mukhanov'1999, k-inflation]

Strong coupling

Cubic action for scalars

$$\mathcal{S}^{(3)}_{\zeta\zeta\zeta} = \int dt\, d^3x \Lambda_\zeta \partial^2\zeta\left(\partial_i\zeta\right)^2,$$

$$E^{\zeta\zeta\zeta}_{strong} \sim \Lambda_\zeta(\mathcal{G}_S)^{-3/2}u_S^{-11/2} \sim \frac{1}{|t|}\left(\frac{g^{1/2}u_S^{11/2}}{|t|^{\mu-1}}\right)^{1/3},$$

thus we obtain for $E^{\zeta\zeta\zeta}_{strong} > E_{cl}$:

$$\left(\frac{g u_S^{11}}{|t|^{2(\mu-1)}}\right)^{1/6} > 1.$$

Scalars exit (effective) horizon:

$$t_f^{2(\mu-1)} \sim g\mathcal{A}_\zeta u_S^3.$$

$$\left(\frac{g u_S^{11}}{|t_f(k_{min})|^{2(\mu-1)}}\right)^{1/6} \sim \left(\frac{u_S^8}{\mathcal{A}_\zeta}\right)^{1/6} \sim \left(\frac{r^{8/3}}{\mathcal{A}_\zeta}\right)^{1/6},$$

$$\left(\frac{r^{8/3}}{\mathcal{A}_\zeta}\right)^{1/6} > 1.$$

By the same logic we obtain constraint in tensor sector

$$\frac{1}{g}\left(\frac{d}{k}\right)^{2\frac{\mu-1}{1-\chi}} \sim \mathcal{A}_T \ll 1.$$

Conclusion

1. We construct the model of bounce, within one can generate nearly flat power spectrum of scalar perturbations;
2. The requirement of strong coupling absence leads to the fact that the r-ratio cannot be arbitrarily small.

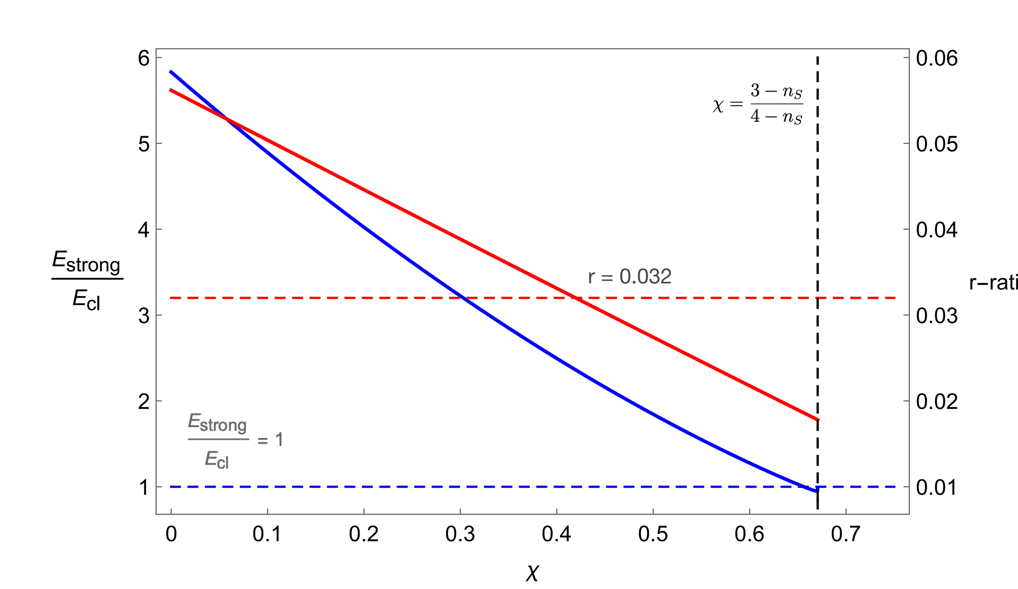

Figure 2: The r-ratio (red line) and ratio $E_{strong}(k_*)/E_{cl}(k_*)$ (blue line) as functions of χ for the central value $n_S = 0.9649$.