Loop corrections to a cosmological particle creation

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Introduction

Let us consider dynamics of interacting fields in the background of an expanding geometry. We investigate the question whether quantum effects should substantially change the background geometry. We expect that the result obtained for the toy model considered here will show how to deal with the fields in the early Universe.

Consider the geometry given by the line element [1], [2]:

$$ds^2 = C(\eta)(d\eta^2 - dx^2), \quad (1)$$

where the conformal factor $C(\eta)$ is given by:

$$C(\eta) = A + B\tanh(\rho\eta). \quad (2)$$

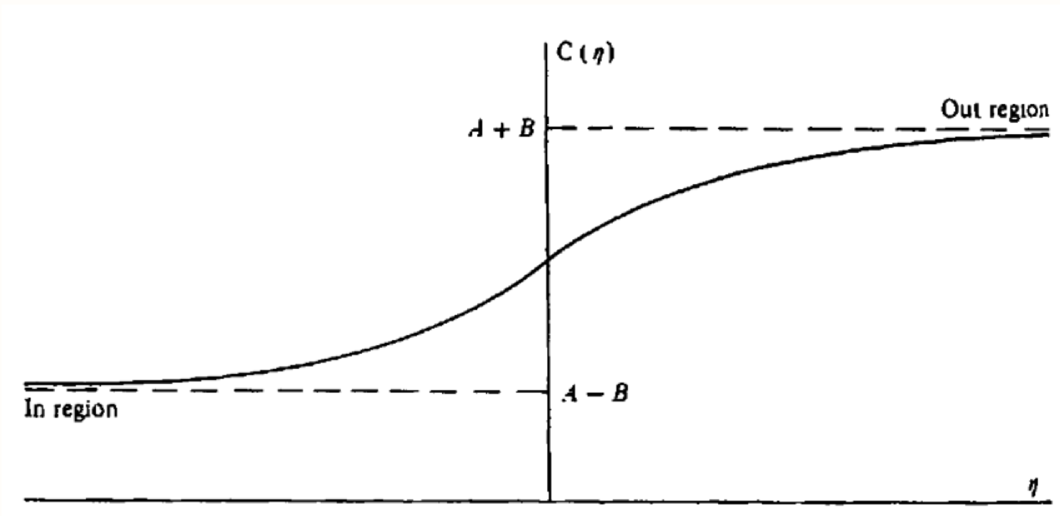

Figure 1: η variable dependence of the conformal factor.

Let us consider massive minimally coupled scalar field in the background (1):

$$S = \int d^2x\sqrt{-g}\left[\frac{1}{2}(\partial_\mu\phi)^2 - \frac{m^2}{2}\phi^2 - \frac{\lambda}{4!}\phi^4\right]. \quad (3)$$

For the free field theory with $\lambda = 0$, we obtain that the total number of particles created in the process of system's evolution is

$$N = \int dk\, \frac{\omega_{in}}{\omega_{out}} \frac{\sinh^2(\pi\omega_-/\rho)}{\sinh(\pi\omega_{in}/\rho)\sinh(\pi\omega_{out}/\rho)}, \quad (4)$$

i.e. the occupation number of the in-modes in the distant future is a finite non-zero number.

It is generally believed that the tree-level result (4) is not sufficintly affected by the quantum corrections as the latter are suppressed by the coupling constant λ. Let us check that this is actually the case in this scenario.

We study loop corrections to the occupation numbers using non-stationary diagrammatic technique (Schwinger-Keldysh technique), i.e. we consider evoultion of the system on the closed time contour:

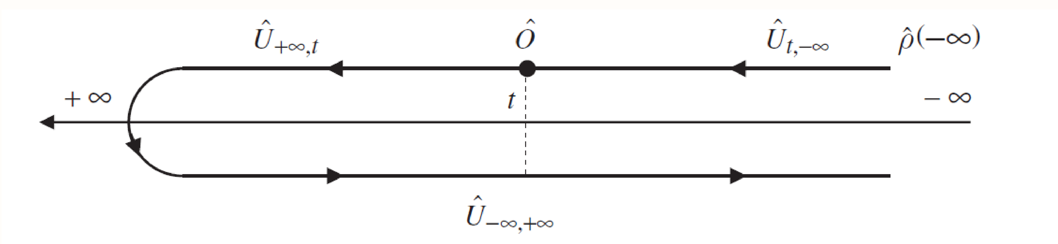

Figure 2: Closed time contour

If we consider our system in closed time contour, we effectively have three types of propagators, one which is Keldysh propagator defined as

$$G^K(\eta, x|\eta', x') = \langle\{\phi(\eta, x)\phi(\eta', x')\}\rangle. \quad (5)$$

In terms of modes, the Keldysh function is written as follows:

$$G^K(x|, x') = \int dk \left\{\left(\frac{1}{2} + \langle a_k^\dagger a_k\rangle\right) u_k^{in}(\eta, x)\, u_k^{in*}(\eta', x') + \langle a_k a_{-k}\rangle\, u_k^{in}(\eta, x)\, u_{-k}^{in}(\eta', x') + \text{c.c.}\right\}, \quad (6)$$

i.e. Keldysh function contains information about the ocupation number $n_k = \langle a_k^\dagger a_k\rangle$ and anomalous quantum average $\langle a_k a_{-k}\rangle$.

Result and discussions

Let us start discussion of loop correction starting from tadpole diagrams

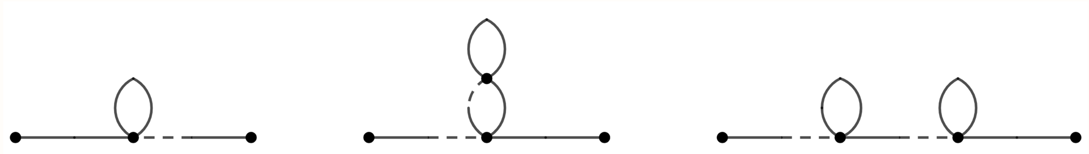

Figure 3: Different types of tadpole diagrams

In the IR limit $\frac{\eta_1+\eta_2}{2} \equiv \eta \gg \eta_1 - \eta_2$ the Dyson-Schwinger equation for the "exact" Keldysh function:

$$\tilde{G}^K_{12} = G^K_{12} - \frac{i\lambda}{2}\int_{\eta_0}^{\infty} d\eta_3\, dx_3\, C(\eta_3) \times \left(G^K_{13}G^K_{33}\tilde{G}^A_{32} + G^R_{13}G^K_{33}\tilde{G}^K_{32}\right). \quad (7)$$

Applying the differential operator $(\Box + m^2)$ to the both sides of this equation, one obtains

$$\left(\Box + m^2 + \frac{\lambda}{2}G^K_{11}\right)\tilde{G}^K_{12} = 0, \quad (8)$$

i.e. sum of the tadpole diagrams leads to the mass renormalization

$$m^2_{ren} = m^2 + \frac{\lambda}{2}G^K_{11}. \quad (9)$$

The next type of diagrams we need to consider is sunset diagrams

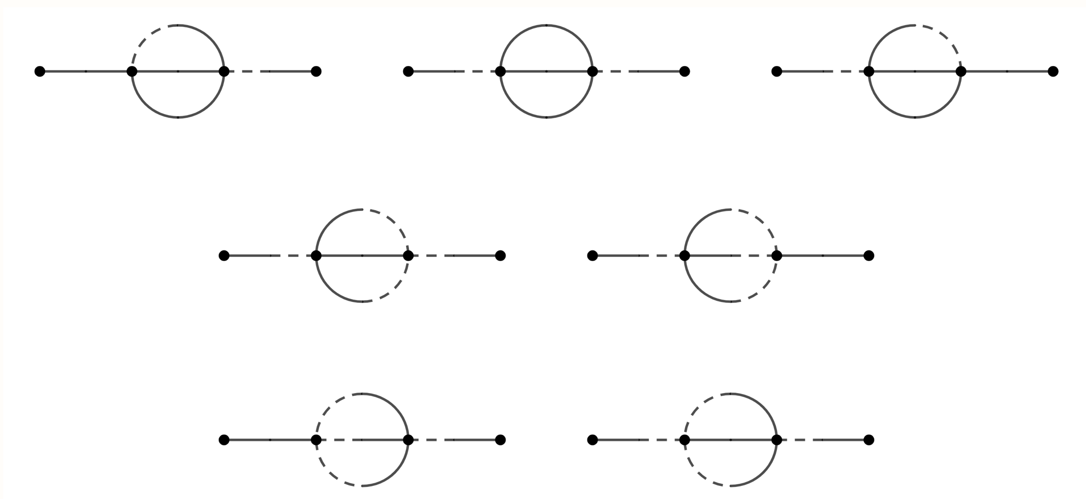

Figure 4: Sunset diagram corrections to the Keldysh propagator

In the IR limit after evaluation of the integrals over η_3 η_4 one obtains

$$n_p^{(2)} \approx \frac{\lambda^2(A+B)^2\eta}{64\pi}\int\frac{dq\,dr\,ds\,\delta^{(2)}\big(p-q-r-s\big)}{\omega_{in}(p)\omega_{in}(q)\omega_{in}(r)\omega_{in}(s)} \Big\{\mathcal{N}_1\big[(1+n_p)(1+n_q)(1+n_r)(1+n_s) - n_p n_q n_r n_s\big] + \mathcal{N}_2\big[(1+n_p)(1+n_q)(1+n_r)n_s - n_p n_q n_r(1+n_s)\big] + \mathcal{N}_3\big[(1+n_p)n_q(1+n_r)(1+n_s) - n_p(1+n_q)n_r n_s\big] + \mathcal{N}_4\big[(1+n_p)n_q n_r(1+n_s) - n_p(1+n_q)(1+n_r)n_s\big] + \mathcal{N}_5\big[(1+n_p)n_q n_r n_s - n_p(1+n_q)(1+n_r)(1+n_s)\big]\Big\}, \quad (10)$$

Analogously, one obtains for the anomalous quantum averages

$$\kappa_p^{(2)} \approx -\frac{\lambda^2(A+B)^2\eta}{64\pi}\int\frac{dq\,dr\,ds\,\delta\big(p-q-r-s\big)}{\omega_{in}(p)\omega_{in}(q)\omega_{in}(r)\omega_{in}(s)} \times \Big\{\mathcal{K}_1\big[(1+n_q)(1+n_r)(1+n_s) + n_q n_r n_s\big] + \mathcal{K}_2\big[(1+n_q)(1+n_r)n_s + n_q n_r(1+n_s)\big] + \mathcal{K}_3\big[n_q(1+n_r)(1+n_s) + (1+n_q)n_r n_s\big] + \mathcal{K}_4(1+2n_p)\big[(1+n_q)(1+n_r)(1+n_s) - n_q n_r n_s\big] + \mathcal{K}_5(1+2n_p)\big[(1+n_q)(1+n_r)n_s - n_q n_r(1+n_s)\big]\Big\}. \quad (11)$$

Hence, we obtain that loop corrections to both occupation number and anomalous quantum averages grow with time, and therefore in the IR limit quantum corrections can be comparable to the tree-level result (4).

Stress-energy tensor

Let us investigate how loop corrections to occupation number and anomalous quantum average change the behavior of the regularized stress-energy tensor. Regularizing the stress-energy tensor using Pauli-Villars method, one obtains in the distant past

$$\langle T^{\mu\nu}\rangle \xrightarrow[\eta\to-\infty]{} 0, \quad (12)$$

while in the distant future

$$\langle in|T^{\mu\nu}|in\rangle = \langle in|T_0^{\mu\nu}|in\rangle + \langle in|T^{\mu\nu}_{loop}|in\rangle,$$

where

$$\langle in|T_0^{\mu\nu}|in\rangle \xrightarrow[\eta\to+\infty]{} \int_{-\infty}^{\infty}\frac{dk}{4\pi\tilde{\omega}_{out}}k^\mu k^\nu \times \left(\frac{\tilde{\omega}_{out}}{\tilde{\omega}_{in}}(|C_1|^2 + |C_2|^2) - 1\right),$$

$$\langle in|T^{\mu\nu}_{loop}|in\rangle \xrightarrow[\eta\to+\infty]{} \int_{-\infty}^{\infty}\frac{dk}{2\pi\tilde{\omega}_{in}}k^\mu k^\nu \times \left[n_k^{(2)}(|C_1|^2 + |C_2|^2) + \kappa_k^{(2)}C_1C_2 + \kappa_k^{(2)*}C_1^*C_2^*\right], \quad (13)$$

where we have

$$k^\mu = \left(\frac{\tilde{\omega}_{out}}{A+B}, \frac{k}{A+B}\right). \quad (14)$$

Hence, we obtain that the regularized stress-energy tensor also contain secularly growing terms, and if one restricts the analysis at the two-loop level, the backreaction of matter fields on the background geometry should be substantial.

Summary and conclusions

- Therefore, it has been shown that in the interacting theory initial occupation numbers and anomalous quantum averages acquire secularly growing contributions. Moreover, it has been shown that these secularly growing contribution are non-zero even if the initial state is the Fock vacuum;
- It is shown that the stress-energy tensor also acquires secularly growing contributions;
- The analysis can be generalised to the arbitrary number of dimensions of spacetime. Ultimately, all the results stay the same;
- The next step is to sum all the secularly growing contributions in the leading order and to find the "exact" propagators in this limit. With the "exact" propagator it is interesting to calculate regularized stress-energy tensor and with the use of the semiclassical Einstein equations estimate whether the quantum corrections should change the background geometry.

References

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[2] C. W. Bernard and A. Duncan, Annals Phys. **107** (1977), 201 doi:10.1016/0003-4916(77)90210-X