Non-abelian fermionic T-duality in supergravity

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Abstract

We make sense of certain non-abelian fermionic T-duals of solutions to type II supergravity, by embedding them in Double Field Theory. We describe the mechanism of obtaining these solutions and consider several particular examples of Minkowski, D_p-brane and fundamental string backgrounds. In each case we find Killing spinors and corresponding field transformations.

Set up of the problem

In our work we would like to present a neat trick that allows one to embed backgrounds resulting from non-abelian fermionic T-duality in Double Field Theory (DFT). This does not constitute a definition of non-abelian fermionic T-duality in full generality, however the resulting backgrounds are genuine solutions of DFT, which makes them valid backgrounds for the superstring propagation.

The abelian constraint of fermionic T-duality is given by the vanishing of the vector field

$$\tilde{K}^m = \left\{ \begin{matrix} \epsilon\bar{\gamma}^m\epsilon - \hat{\epsilon}\gamma^m\hat{\epsilon} & \text{(IIA)} \\ \epsilon\bar{\gamma}^m\epsilon + \hat{\epsilon}\bar{\gamma}^m\hat{\epsilon} & \text{(IIB)} \end{matrix} \right\} \overset{!}{=} 0$$

Here $\epsilon, \hat{\epsilon}$ is a pair of Killing spinors that fix a fermionic direction in the $\mathcal{N}=2$ $d=10$ superspace. The fermionic Buscher procedure assumes invariance of the sigma-model superfields under shifts in the direction of $\epsilon, \hat{\epsilon}$, which is equivalent to an unbroken supersymmetry. The resulting transformation of the supergravity background is simply

$$e^{\phi}F' = e^{\phi}F + 16\,i\,\frac{\epsilon\otimes\hat{\epsilon}}{C},$$
$$\phi' = \phi + \frac{1}{2}\log C.$$

The fields here are the RR bispinor F and the dilaton ϕ.

Define next convenient vector field

$$K_m \equiv \begin{cases} i(\epsilon\gamma^m\epsilon + \hat{\epsilon}\gamma^m\hat{\epsilon}) & \text{(IIA)}, \\ i(\epsilon\gamma^m\epsilon - \hat{\epsilon}\gamma^m\hat{\epsilon}) & \text{(IIB)}. \end{cases}$$

The scalar parameter C is then defined by the system of PDEs:

$$\partial_m C = iK_m - ib_{mn}\tilde{K}^n,$$
$$\tilde{\partial}^m C = i\tilde{K}^m.$$

This system of equations along with DFT constraints defines non-abelian fermionic T-dual solutions. In our work we find such C on different backgrounds.

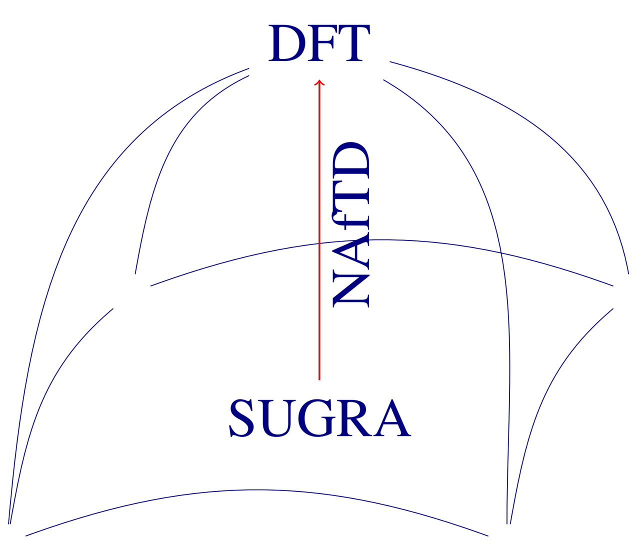

Figure 1: Non-abelian fermionic T-duality translate the supergravity solution into DFT solution.

Minkowski space

Consider flat Minkowski background of Type IIB theory. This is a maximally supersymmetric solution, i.e. one has 16 Killing spinors ϵ and 16 Killing spinors $\hat{\epsilon}$, all of which are constant. These span the full 32-dimensional vector space of spinors of $\mathcal{N}=(2,0)$ SUSY in $d=1+9$, where we choose the basis elements $\{\epsilon_i, \hat{\epsilon}_i\}$, $i \in \{1,\dots,16\}$ as

$$(\epsilon_i)^\alpha = \delta_i{}^\alpha, \quad (\hat{\epsilon}_i)^{\hat{\alpha}} = \delta_i{}^{\hat{\alpha}}.$$

Geometric example. Let us consider fermionic T-duality in the direction given by the spinors

$$\epsilon = \epsilon_1 - i\hat{\epsilon}_9, \quad \hat{\epsilon} = -\hat{\epsilon}_1 - i\hat{\epsilon}_9.$$

These Killing spinors leading to

$$C = 4(x^8 + i\tilde{x}_9).$$

One may recover the fermionic T-dual background, the metric is still flat and no b field is generated. We do get a nontrivial dilaton and the RR fluxes. This background is clearly not a supergravity solution due to the dual coordinate dependence of C. However, it can be shown to satisfy equations of motion of DFT.

Non-geometric example. Moreover, consider a fermionic T-duality transformation generated simply by a single spinor, for which the commutativity condition obviously does not hold:

$$\epsilon = \frac{1}{\sqrt{2}}(\epsilon_1 + i\epsilon_9), \quad \hat{\epsilon} = 0.$$

The corresponding

$$C = -x^8 - \tilde{x}_8 + i(x^9 + \tilde{x}_9)$$

produces the background with trivial $F_{(p)} = 0$ that cannot be T-dualized into any geometric background. The fields include exotic dependence on dual coordinates, i.e. on combinations of the type $x \pm \tilde{x}$, however the section constraint is satisfied.

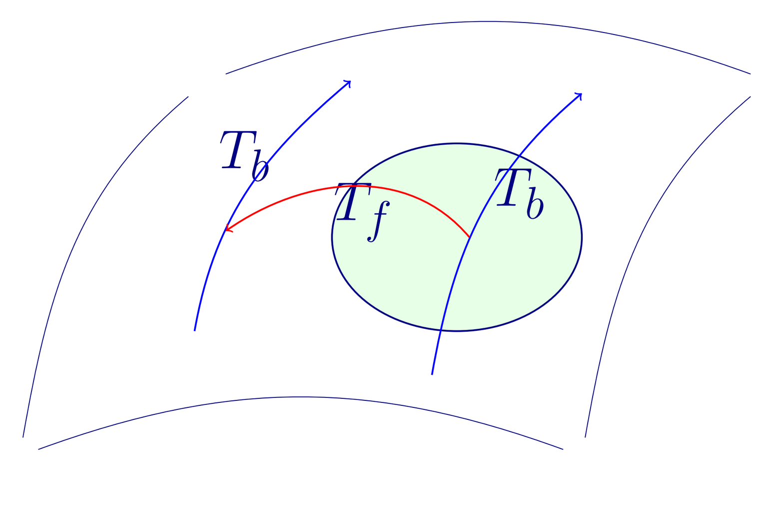

Figure 2: Schematic depiction of bosonic and non-abelian fermionic orbits of T-duality.

Dp-brane

The solitonic type IIB Dp-brane, $p<7$, has background metric

$$g_{\mu\nu} = \left(H_{D_p}^{-\frac{1}{2}}\eta_{ij}, H_{D_p}^{\frac{1}{2}}\delta_{mn}\right),$$
$$H_{D_p} = 1 + \frac{Q}{(\delta_{mn}x^m x^n)^{\frac{7-p}{2}}},$$

where i,j and m,n denotes brane and outer coordinates correspondingly.

Because of 1/2BPS condition there are 16 independent Killing spinors, parametrized by constant ϵ_0:

$$\epsilon = H_{D_p}^{-\frac{1}{8}}\epsilon_0, \quad \hat{\epsilon} = -\gamma^{0\bar{1}..p}\epsilon = -H_{D_p}^{-\frac{1}{8}}\gamma^{0\bar{1}..p}\epsilon_0.$$

For the general case of Dp-brane we can choose particular Killing spinor ϵ_0 to consider the nontrivial C such as

$$C = 2(x_m + i\tilde{x}_j), \tag{1}$$

where m must be from $p+1$ to 10 and j must be from 0 to $p+1$.

Type II fundamental string

Type II fundamental string has background metric, dilaton and Kalb-Ramond field

$$g_{\mu\nu} = \left(H_{F1}^{-1}\eta_{ij}, \delta_{mn}\right), \quad e^{-2\phi} = He^{-2\phi_0},$$
$$H_{F1} = 1 + \frac{h}{(\delta_{mn}x^m x^n)^3}, \quad b_{ty} = H_{F1}^{-1} - 1$$

where i,j and m,n denotes two string and eight outer coordinates correspondingly. Same as for Dp-brane after using the 1/2BPS condition we find the appropriate Killing spinors

$$\epsilon_0 = (1-\gamma^{0\bar{1}})\eta = (1-\gamma^0\gamma^1)\eta \quad \text{IIA, IIB},$$
$$\hat{\epsilon}_0 = \begin{cases} (1+\gamma^{\bar{0}1})\bar{\eta} = (1-\gamma^0\gamma^1)\bar{\eta} & \text{IIA}, \\ (1+\gamma^{\bar{0}1})\eta = (1+\gamma^0\gamma^1)\eta & \text{IIB}, \end{cases}$$

where $(\eta, \bar{\eta})$ are an arbitrary constant spinors.

We obtain the general expression for the function C in both IIA and IIB theories:

$$C = \frac{1}{2}(A+B)(x^1 + \tilde{x}_0) + \frac{1}{2}(A-B)(x^0 - \tilde{x}_1),$$

Where A and B are the sums of squared components of $(\eta, \bar{\eta})$.

IIA geometric example with zero mass. Make $A = -B = 1$, then

$$C = x^0 - \tilde{x}_1.$$

The T-duals are

$$e^{-2\phi} = \frac{He^{-2\phi_0}}{x^1 + \tilde{x}_0}, \qquad m = 0,$$
$$F_{(2)} = -i\frac{e^{-\phi_0}}{2C^{3/2}}\left[dx^{67} + dx^{38} + dx^{49} - dx^{25}\right],$$
$$F_{(4)} = i\frac{e^{-\phi_0}}{2C^{3/2}}\left[\frac{1}{H}dx^{01}(dx^{67} - dx^{25} + dx^{38} + dx^{49}) + (dx^{89} - dx^{34})(dx^{26} + dx^{57}) + (dx^{39} - dx^{48})(dx^{27} - dx^{56})\right].$$

Non-geometric example. If we set $A=1$ together with $B=0$ we obtain

$$C = x^1 + x^0 + \tilde{x}_0 - \tilde{x}_1.$$

This solution can have vanishing or not R-R fields and cannot be bosonically T-dualized in to the supergravity solution.

References

L. Astrakhantsev, I. Bakhmatov and E. Musaev, *"Non-abelian fermionic T-duality in supergravity"*, arXiv: 2101.08206

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