An Effective Field Theory for Large Oscillons

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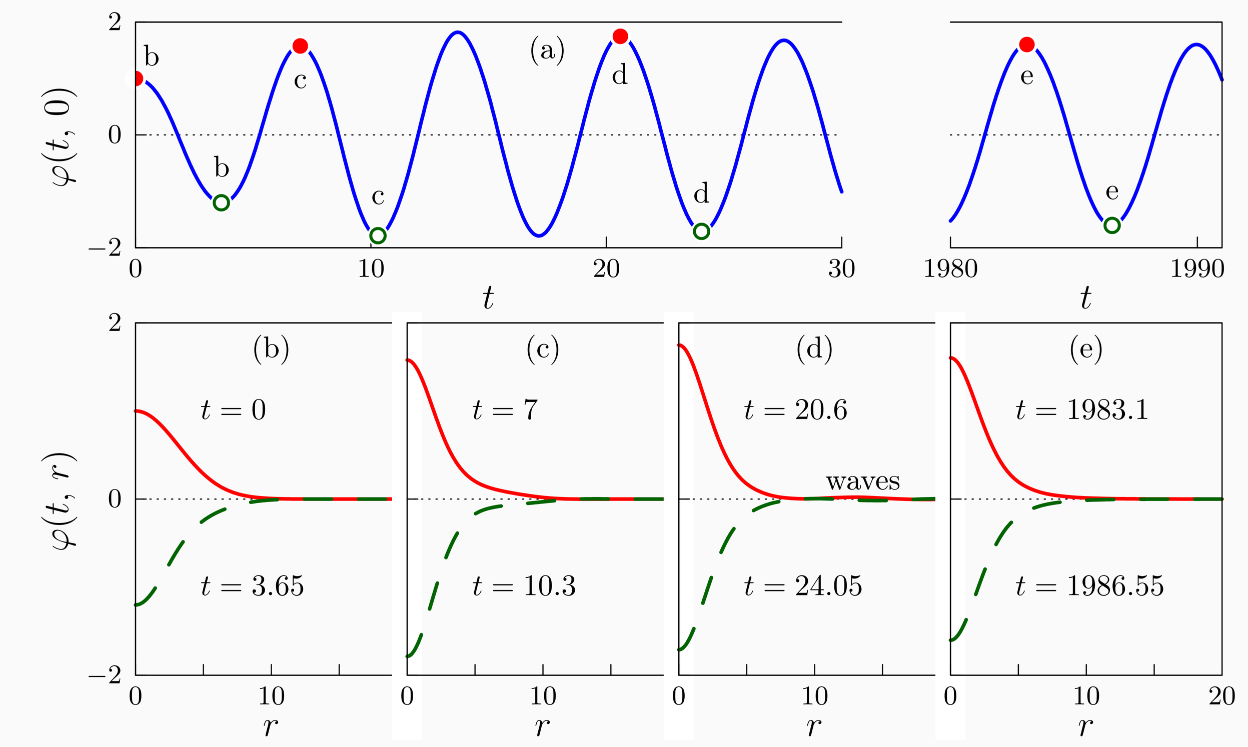
Oscillons: introduction

Oscillons are compact, almost periodic, and long-lived classical solutions in models with real bosonic fields, e.g., one real scalar field $\varphi(t, \mathbf{x})$,

$$\mathcal{L} = (\partial_t \varphi)^2/2 - (\partial_i \varphi)^2/2 - V(\varphi).$$

Example: Plateau potential $V(\varphi) = \frac{1}{2}\tanh^2\varphi$.



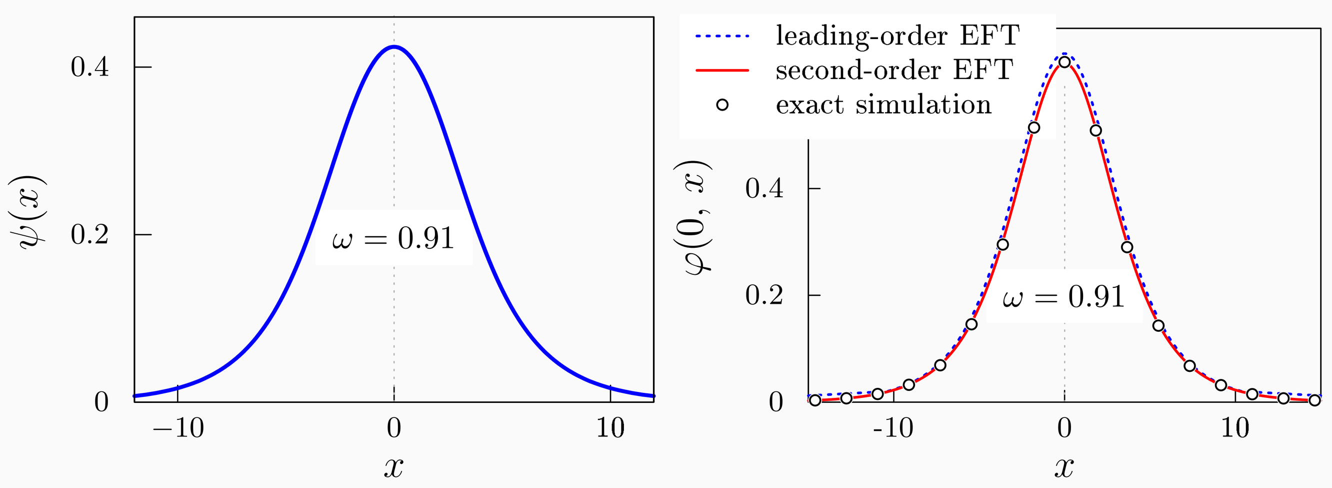
Emergence of an oscillon in numerical simulation from Gaussian initial data $\varphi = \varphi_0 \exp(-r^2/\sigma^2)$ with $\varphi_0 = 0$ and $\sigma = 20$ in 3 spatial dimensions. It goes on to live $\gtrsim 10^5$ periods.

Oscillons are widely met in cosmology:

- nucleate[1,2] during generation of axion or ultra-light dark matter
- accompany[3] cosmological phase transitions
- formed[4] by inflaton field during preheating

Problem: Why are oscillons so long-lived? How to describe them?

1D oscillons example: $V(\varphi) = \frac{1}{2}\tanh^2\varphi$



One-dimensional oscillon profiles for $\omega = 0.91$. Left: Leading order EFT oscillon profile $\psi(x)$. Right: Comparison between exact oscillon profile $\varphi(0, x)$, obtained from numerical simulations (shown in circles), and the EFT oscillon profiles in the leading and second orders (shown in dashed and solid lines, respectively), obtained by canonical transformation from $\psi(x)$ at $t = 0$.

Existence, longevity, stability

Existence of an oscillon with $I(r = 0) = I_0$: $\quad (\Omega = \partial h/\partial I)$

$$\Omega(I_0) < m, \qquad h(I_0)/I_0 < m$$

Longevity: profile equations lead to $\omega - \Omega \sim (mR)^{-2} \ll 1$, thus

$$\left|\frac{d\Omega}{dI}\right| \ll \frac{\Omega}{I}$$

– conditions for EFT
– longevity of oscillon

Such potential $V(\varphi)$ is *almost quadratic*!

Linear stability : $dN(\omega)/d\omega < 0$ (Vakhitov-Kolokolov[6] criterion)

Higher-order corrections

Goal: Develop asymptotic expansion in R^{-2}:

$$\mathcal{S}_{\text{eff}} = \underbrace{\mathcal{S}^{(1)}_{\text{eff}}}_{R^0 + R^{-2}} + \overbrace{\underbrace{\mathcal{S}^{(2)}_{\text{eff}}}_{R^{-4}} + \underbrace{\mathcal{S}^{(3)}_{\text{eff}}}_{R^{-6}} + \ldots}^{\text{corrections}}$$

Field corrections: $\quad I = \underbrace{\bar{I}}_{\text{slow}} + \underbrace{\delta I}_{\textit{fast}}, \qquad \theta = \underbrace{\bar{\theta}}_{\text{slow}} + \underbrace{\delta\theta}_{\textit{fast}}$

$$\langle \delta I \rangle = \langle \delta\theta \rangle = 0, \quad \delta I \ll I, \quad \delta\theta \ll \theta$$

Solve equations for $\delta I(\bar{I}, \bar{\theta})$, $\delta\theta(\bar{I}, \bar{\theta})$, plug the result into (averaged) action, and use the stationary ansatz $\psi \equiv \sqrt{\bar{I}}\, \mathrm{e}^{-i\bar{\theta}} = \psi(\mathbf{x})\, \mathrm{e}^{-i\omega t}$ to get

$$\mathcal{S}_{\text{eff}} = \mathcal{S}^{(1)}_{\text{eff}} + \mathcal{S}^{(2)}_{\text{eff}},$$

$$\mathcal{S}^{(2)}_{\text{eff}} = \int dt\, d^d\boldsymbol{x} \Big[\underbrace{d_1}_{d_i = d_i(|\psi|^2),} (\partial_i \psi)^4 + \underbrace{d_2\, \psi \Delta \psi (\partial_i \psi)^2 + d_3\, (\Delta\psi)^2}_{\textit{four } \mathbf{x}\text{-derivatives } \sim\, O(R^{-4})} \Big]$$

At all orders of the EFT **global symmetry persists** due to averaging, thus the EFT oscillons are eternal.

Action-angle coordinates — smooth variables for the EFT

We are considering **wide oscillons**: $\quad R \gg m^{-1}, \quad |\partial_i \varphi| \ll m\varphi.$

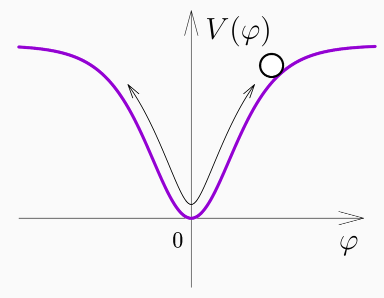
Roughest approximation: $\boxed{\partial_t^2 \varphi - \cancel{\Delta\varphi} = -V'(\varphi)} \implies$ Nonlinear oscillator, exactly solvable.

Single integral of motion: $\quad h \equiv (\partial_t \varphi)^2/2 + V(\varphi) = \text{const}\,.$

Action-angle variables:

$$I(\varphi, \dot{\varphi}) = \frac{1}{2\pi} \oint \sqrt{2(h - V)}\, d\varphi,$$

$$\theta(\varphi, \dot{\varphi}) = \frac{\partial}{\partial I} \int^{\varphi} \sqrt{2(h - V(\varphi'))}\, d\varphi'$$



Canonical transformation: $\varphi = \Phi(I, \theta)$, $\dot{\varphi} = \Pi(I, \theta)$, so that $h = h(I)$.

Mechanical solution: $I = \text{const}, \quad \theta = \Omega(I)\, t, \quad \Omega(I) = \partial h/\partial I.$

In field theory:
oscillon is $\mathbf{x}$-dependent $\implies$ fields $I(\mathbf{x})$ and $\theta(t, \mathbf{x})$ slowly depend on $\mathbf{x}$.

Example: $V(\varphi) = \frac{1}{2}\tanh^2\varphi$

$$\varphi = \operatorname{arcsinh}\left(\sqrt{\frac{I(2-I)}{1-I}}\, \cos\theta\right)$$

$$h(I) = I - I^2/2.$$

Effective Field Theory: leading-order effective action

We start with the exact action:

$$\mathcal{S} = \int dt\, d^d\boldsymbol{x} \Bigg(\underbrace{\frac{1}{2}\dot{\varphi}^2 - V(\varphi)}_{I\partial_t\theta - h} - \underbrace{\frac{1}{2}(\partial_i\varphi)^2}_{\text{subleading}} \Bigg)$$

Averaging over period:

$$(\partial_i \varphi)^2 \longrightarrow \langle (\partial_i \varphi)^2 \rangle = \frac{1}{2\pi} \int_0^{2\pi} (\partial_i \Phi(I, \theta))^2\, d\theta \approx \frac{(\partial_i I)^2}{\mu_I(I)} + \frac{(\partial_i \theta)^2}{\mu_\theta(I)},$$

where **slow-varying** $\partial_i I$, $\partial_i \theta$ are moved ***out*** of the average, $\quad \mu_I \equiv \left\langle (\partial_I \Phi)^2 \right\rangle^{-1}, \quad \mu_\theta \equiv \left\langle (\partial_\theta \Phi)^2 \right\rangle^{-1}$

Effective action in the leading order

$$\mathcal{S}_{\text{eff}} = \int dt\, d^d\boldsymbol{x} \left(I\partial_t\theta - h(I) - \frac{(\partial_i I)^2}{2\mu_I(I)} - \frac{(\partial_i \theta)^2}{2\mu_\theta(I)} \right)$$

Example: $V(\varphi) = \frac{1}{2}\tanh^2\varphi$

$$\mu_I = I(2 - I)^2(1 - I)^2, \qquad \mu_\theta = I^{-1} - 1$$

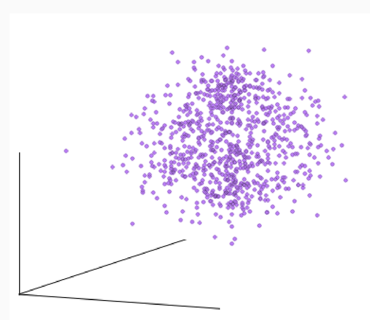
Introducing one complex field $\boxed{\psi(t, \boldsymbol{x}) = \sqrt{I} \cdot \mathrm{e}^{-i\theta}}$ gives

$$\mathcal{S}_{\text{eff}} = \int dt\, d^d\boldsymbol{x} \Big(\underbrace{i\psi^* \partial_t \psi - h - \frac{|\partial_i \psi|^2}{2\mu_1}}_{\text{nonlinear Schrödinger model}} - \frac{1}{2\mu_2} \left[\psi^{*2} (\partial_i \psi)^2 + \text{h.c.} \right] \Big),$$

Form factors:
$\mu_i = \mu_i(|\psi|^2)$,
$h = h(|\psi|^2)$

Global symmetry: $\quad \theta \to \theta + \alpha \iff \psi \to \mathrm{e}^{-i\alpha}\psi \implies$ Conserved charge: $\quad N = \int d^d\mathbf{x}\, I = \int d^d\mathbf{x}\, |\psi|^2$

Charge conservation allows the appearance[5] of *stable non-topological solitons* i.e. **oscillons**.



Attraction + charge conservation = solitons!

Stationary Ansatz $\quad \theta = \omega t \iff \psi = \psi(\mathbf{x}) \cdot \mathrm{e}^{-i\omega t}$ gives the **oscillon profile equation**:

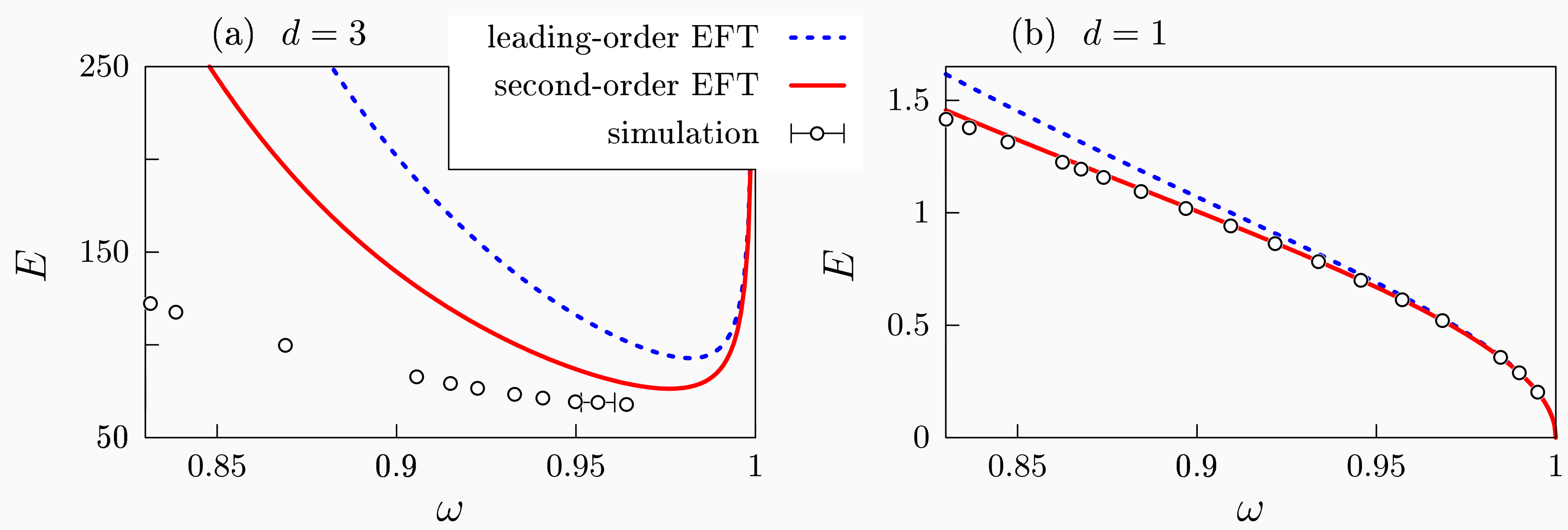
$$-\frac{2\psi^2}{\mu_I} \Delta\psi - (\partial_i \psi)^2 \frac{d}{d\psi} \left(\psi^2/\mu_I \right) + \Omega\psi = \omega\psi$$

Alternatively: minimize the energy E at fixed charge N.

Leading-order oscillon profile: $\varphi(t, \mathbf{x}) = \Phi(\psi^2(\mathbf{x}), \omega t)$

N — "number of particles"
ω — "energy of a particle"
$dE/dN = \omega$

$E(\omega)$ comparison for $V(\varphi) = \frac{1}{2}\tanh^2\varphi$



The energy $E(\omega)$ of d-dimensional oscillons in the leading-order EFT (dotted lines) and second-order EFT (solid lines). Circles with errorbars show results of exact numerical simulations. EFT gives qualitatively correct predictions at $d = 3$, but becomes precise at $d = 1$ and exact at $\omega \approx m$.

References

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