Suppression exponent for multiparticle production in ϕ^4 theory (based on arXiv:2212.03268 [hep-ph])

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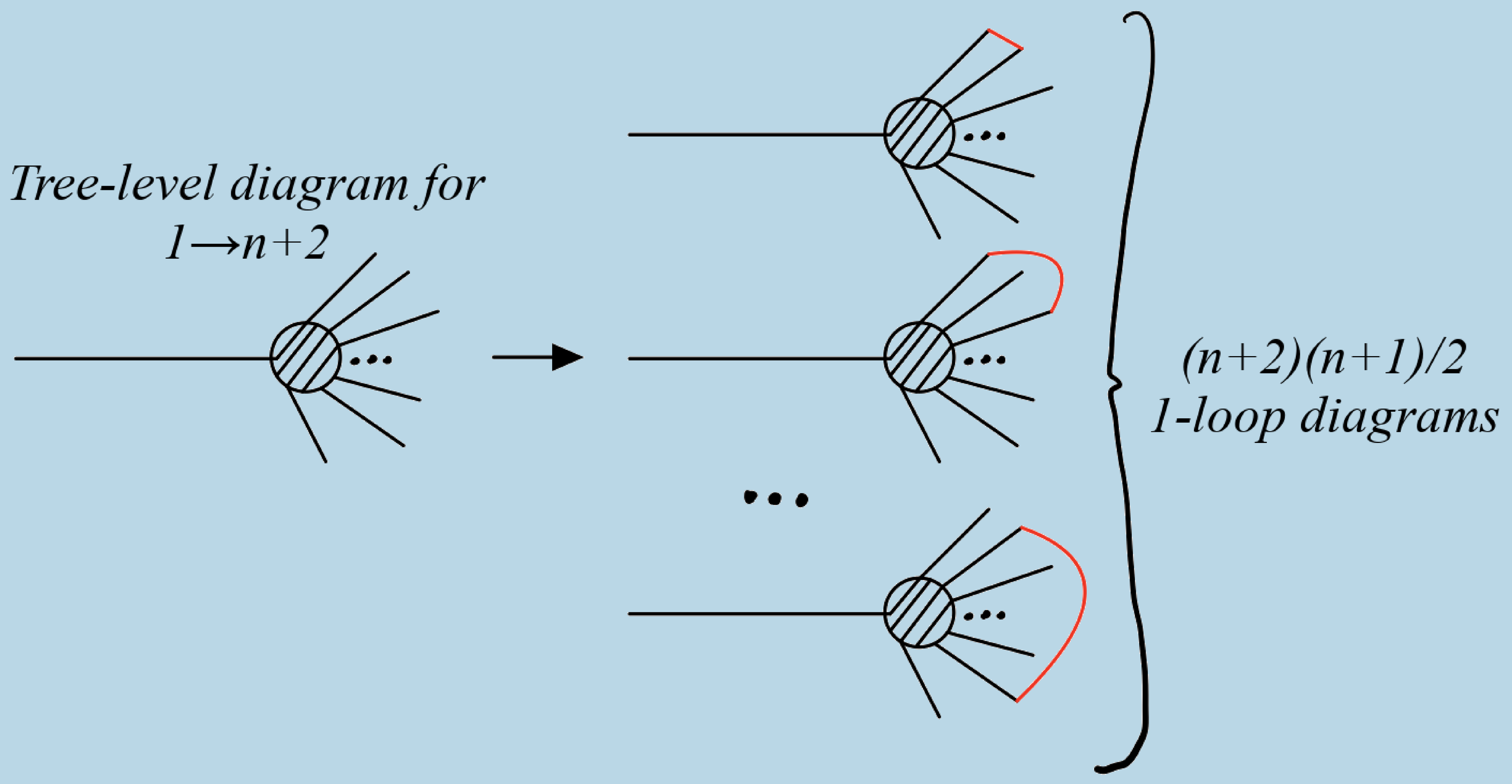
Problem

Perturbation method is efficient for computing few-to-few scattering amplitudes in weakly coupled field theories. But an increasing amount of contributing diagrams can make it unreliable if the number n of particles in the final state exceeds the inverse coupling constant λ^{-1} of the theory. In theory with the action

$$S = (2\lambda)^{-1} \int d^4x \left[(\partial\phi)^2 - m^2\phi^2 - \phi^4/2\right] , \qquad (1)$$

tree-level amplitudes are proportional to $n!$, and loops give additional powers of n. This effect can be traced by counting the number of contributing diagrams which is $n!$ for the tree-level and has additional powers of n for loops. Number of 1-loop diagrams for process $1 \to n$ can be estimated from tree-level diagrams for $1 \to n+2$ as schematically shown below.



It was observed [1] that the parts of perturbative series going in powers of λn can be resummed into an exponent. Inclusive probability $\mathcal{P}_{\text{few}\to n}(E)$ of producing $n \sim \lambda^{-1} \gg 1$ scalar quanta with total energy E from the few-particle initial state $\hat{\mathcal{O}}|0\rangle$ is expected to have the form [1],

$$\mathcal{P}_{\text{few}\to n}(E) \equiv \sum_f |\langle f; E, n|\hat{\mathcal{S}}\,\hat{\mathcal{O}}|0\rangle|^2 \sim \mathrm{e}^{F(\lambda n,\, \varepsilon)/\lambda} , \qquad (2)$$

where the sum covers all final states with given n and E, $\hat{\mathcal{S}}$ is the S-matrix. The exponent F is conjectured to be *universal* [2], i.e. independent of the operator $\hat{\mathcal{O}}$ as long as the latter creates $\ll \lambda^{-1}$ particles from the vacuum. This makes F a function of two variables: λn and $\varepsilon \equiv E/n - m$.

In our study we calculate the exponent F at $n \gtrsim \lambda^{-1}$ semiclassically.

Semiclassical method

We use the semiclassical technique of D.T. Son [3]. It applies at $n \gg 1$ and $\lambda \ll 1$ and is based on the universality of the exponent in (2).

- We take $\hat{\mathcal{O}} = \exp\left\{-\lambda^{-1} \int d^3\boldsymbol{x}\, J(\boldsymbol{x})\, \hat{\phi}(0, \boldsymbol{x})\right\}$, that describes a classical source $J(\boldsymbol{x})$ producing few particles at $J \ll O(\lambda^0)$.
- At finite J, we represent the probability (2) in the form of a path integral and evaluate the latter in the saddle-point approximation.
- We search for saddle-point configurations ϕ_{cl} that satisfy

$$\Box\phi_{\text{cl}} + m^2\phi_{\text{cl}} + \phi_{\text{cl}}^3 = iJ(\boldsymbol{x})\,\delta(t) \qquad (3)$$

  and the boundary conditions at $t \to \pm\infty$ depending on ε and λn.
- We numerically compute $F(\lambda n, \varepsilon) = \lim_{J\to 0} F_J(\lambda n,\, \varepsilon)$, where F_J in the right-hand side is a functional calculated on ϕ_{cl}.

References

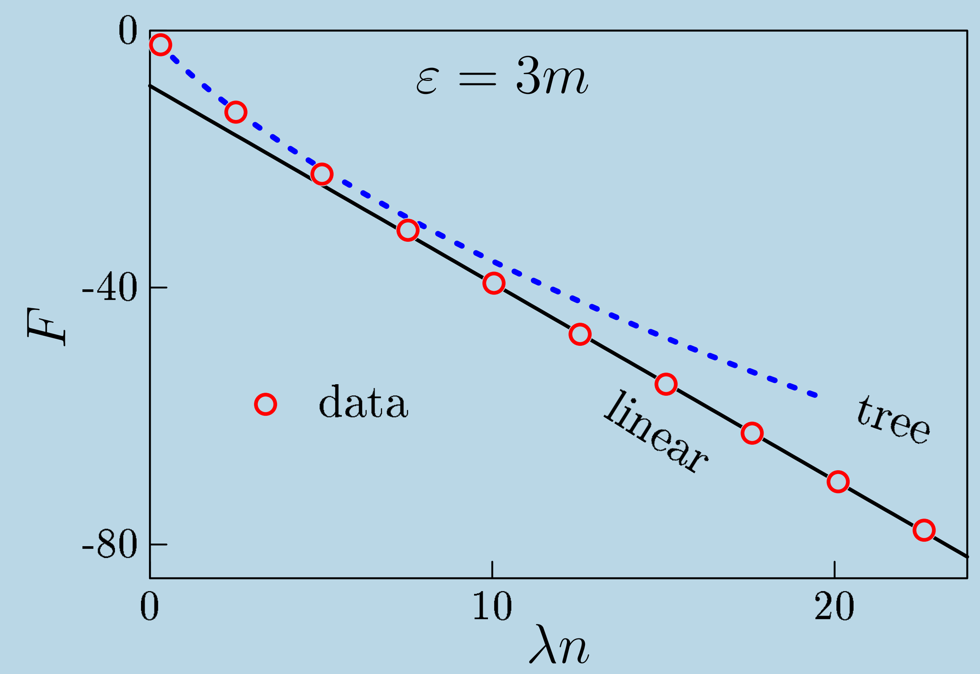
[1] M. V. Libanov, V. A. Rubakov, D. T. Son and S. V. Troitsky, Phys. Rev. D **50** (1994), 7553-7569 doi:10.1103/PhysRevD.50.7553 [arXiv:hep-ph/9407381 [hep-ph]].

[2] M. V. Libanov, D. T. Son and S. V. Troitsky, Phys. Rev. D **52** (1995), 3679-3687 doi:10.1103/PhysRevD.52.3679 [arXiv:hep-ph/9503412 [hep-ph]].

[3] D. T. Son, Nucl. Phys. B **477** (1996), 378-406 doi:10.1016/0550-3213(96)00386-0 [arXiv:hep-ph/9505338 [hep-ph]].

Numerical results

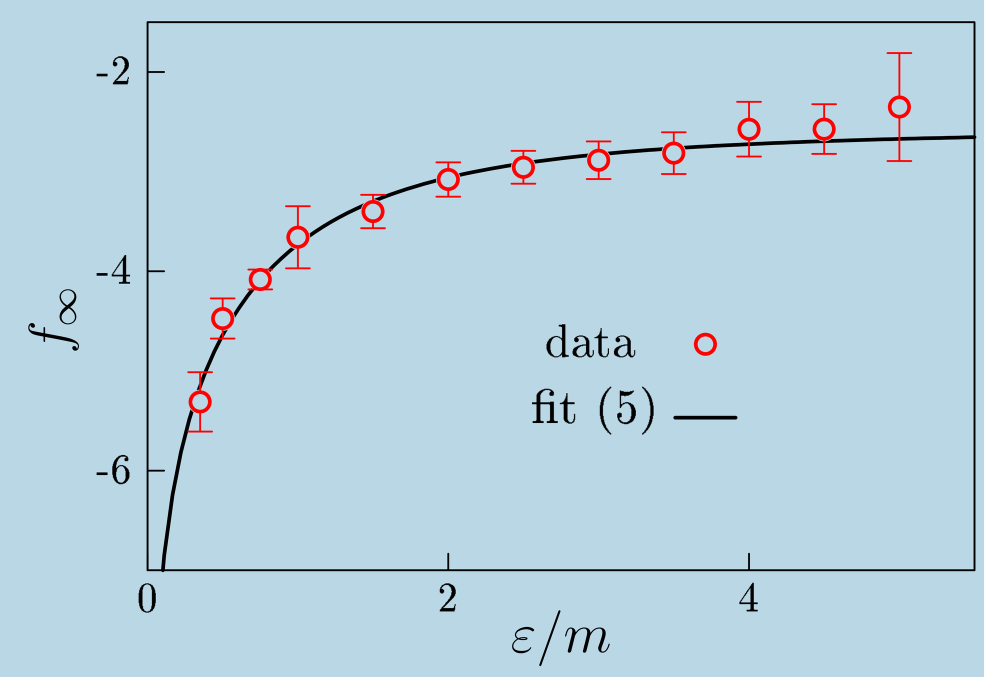
Our result for $F(\lambda n, \varepsilon)$ is demonstrated in the figure below (circles) at the exemplary value $\varepsilon = 3m$.



F monotonically decreases with λn. At $\lambda n \ll 1$ it coincides with the contribution of tree-level diagrams (dashed line). In the limit $\lambda n \gg 1$ our numerical data are well fitted by the linear function (solid line):

$$F \to \lambda n f_\infty(\varepsilon) + g_\infty(\varepsilon) \qquad \text{at} \qquad \lambda n \to +\infty , \qquad (4)$$

where f_∞ and g_∞ are negative. Results at other ε have similar qualitative behavior, although f_∞ and g_∞ depend on ε. Figure below



demonstrates the slope $f_\infty(\varepsilon)$. It can be approximated by the expression (solid line)

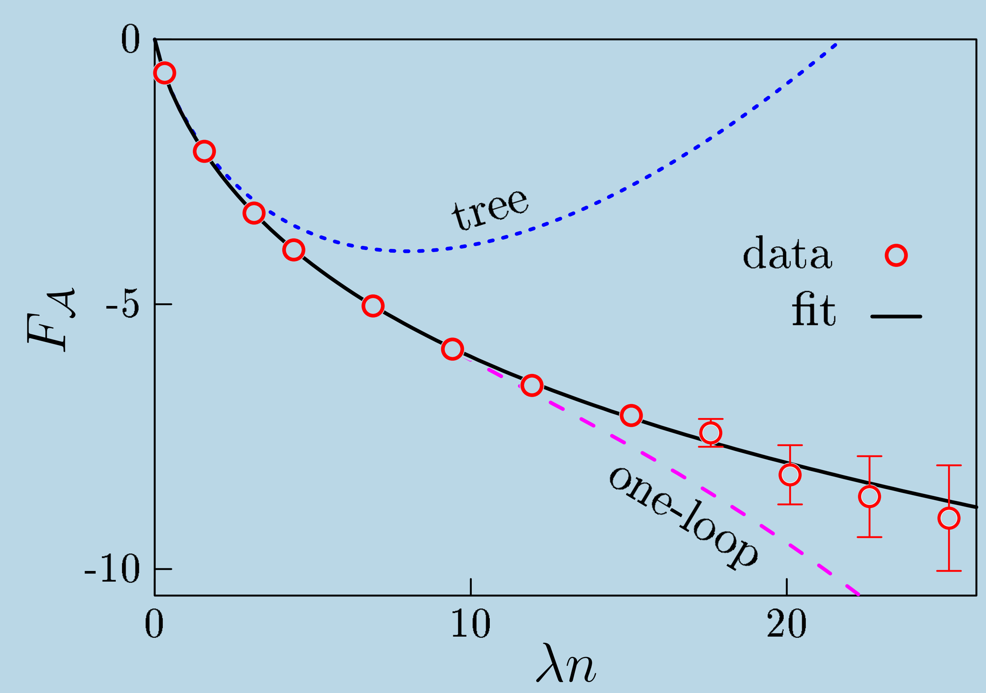
$$f_\infty(\varepsilon) \approx -\frac{3}{4} \ln\left[(d_1 m/\varepsilon)^2 + d_2\right] , \qquad (5)$$

Minimal slope $f_\infty \to -2.57 \pm 0.06$ is achieved in the limit $\varepsilon \to +\infty$.

The probability (2) can also be used to calculate the amplitude $\mathcal{A}_n$ of producing n particles at the mass threshold. It is determined by the ratio of the probability to the n-particle phase volume $\mathcal{V}_n(\varepsilon)/n!$ at $\varepsilon \to 0$:

$$|\mathcal{A}_n|^2 \sim \lim_{\varepsilon\to 0} \frac{n!}{\mathcal{V}_n}\, \mathrm{e}^{F/\lambda} \sim n!\, \mathrm{e}^{2F_{\mathcal{A}}(\lambda n)/\lambda} . \qquad (6)$$

Extrapolating numerical results to $\varepsilon = 0$, we get the exponent $F_{\mathcal{A}}(\lambda n)$ which is displayed by circles with errorbars in the figure below.



At small λn these data are close to tree-level (dotted line) and one-loop (dashed line) exponents. The fitting function (solid line) interpolates between tree-level exponent at small λn and linear asymptotic at large λn.

Overall, our results prove exponential suppression of the multiparticle production probability at $n \gg 1$ and arbitrary ε in the unbroken $\lambda\phi^4$ theory. This semiclassical method can also be used for description of multiparticle production in other theories.