Holographic RG flows in 3d supergravity

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Introduction

The objects of our study are holographic renormalization group (RG) flows in a 3d supergravity model with the scalar field potential depending on the dimensionless finite parameter a^2. These flows are investigated from the side of the dynamical system theory. For that purpose we reduce the gravitational equations of motion to an autonomous dynamical system.

Holographic RG flows are those which have a qualitative correspondence with RG flows from QFT: they start at one UV fixed point, end at an adjacent IR fixed point and the coupling is a monotonic function of the energy. It is important to investigate such fixed points for their asymptotic and stability to explore the energetic limits of the dual theories, since it can provide information about transitions in the dual CFT. In this regard, we find equilibrium points of the system, analyze them for stability and restore asymptotic solutions near the critical points. We also find two types of solutions: with asymptotically AdS metrics and with hyperscaling violation in the metric. We write down possible RG flows between an unstable (saddle) UV fixed point and a stable (stable node) IR fixed point. This poster does not mention our analysis of the bifurcations in the model [1].

The goal of the work *is the description of the holographic RG flows:*

1. *To obtain the holographic RG flows for 3d supergravity model*
2. *To consider properties of the observables at the equilibrium points on these flows*

Holographic model and the holographic β-function

We consider action functional of the 3-dimensional gravitational system with negative cosmological constant Λ_{uv} [2–3]:

$$S = \frac{1}{16\pi G_3}\int d^3x\sqrt{|g|}\left(R - \frac{1}{a^2}\partial(\phi)^2 - V(\phi)\right) + G.H.Y., \tag{1}$$

where $G.H.Y.$ is the Gibbons–Hawking–York boundary term, and the potential of the scalar field

$$V(\phi) = 2\Lambda_{uv}\cosh^2\phi\left[(1-2a^2)\cosh^2\phi + 2a^2\right] \tag{2}$$

has the different behavior for $0 < a^2 \leq 1/2, \quad 1/2 < a^2 < 1 \quad$ and $\quad a^2 \geq 1$ in principal.

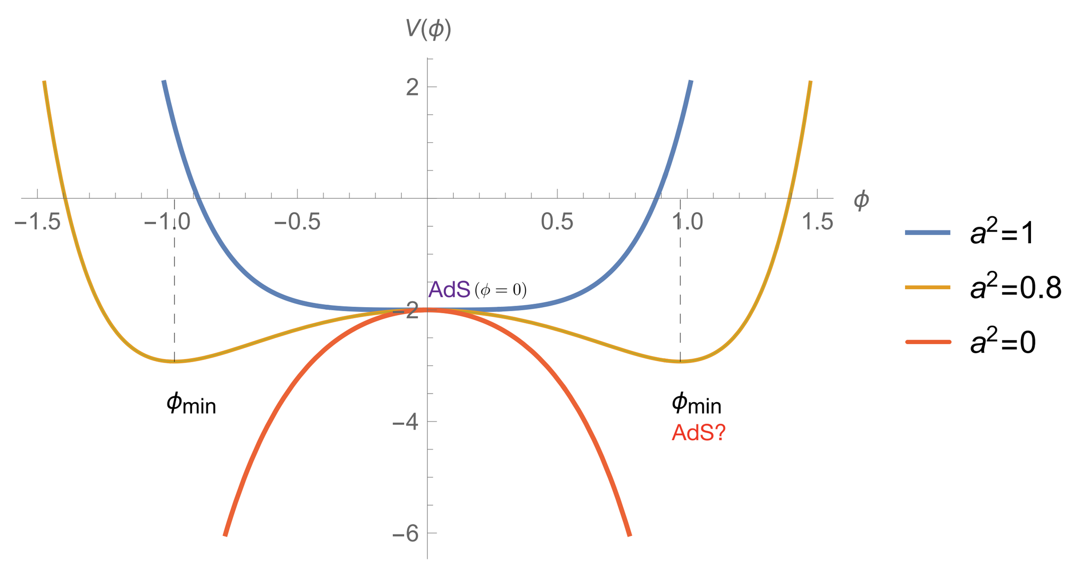


Figure 1. View of the potential $V(\phi)$ according to the parameter a^2.

To take into account the holographic approach, we look for asymptotic solutions for the metric in domain wall coordinates [4–6]:

$$ds^2 = e^{2A(w)}\left(-dt^2 + dx^2\right) + dw^2, \quad w \in (w_0, +\infty), \quad \phi = \phi(w), \tag{3}$$

that can be interpreted as a holographic renormalization group flow, where w is a holographic coordinate, A is a scale factor such that e^{2A} corresponds to the energy scale E of the dual theory, and $\lambda = e^{\phi(w)}$ must be identified as its running coupling. Connection of β-functions in QFT and AdS/CFT:

$$\beta = \frac{d\lambda}{d\log E}\bigg|_{QFT} = \frac{d\phi}{dA}\bigg|_{Holo} \tag{4}$$

At the UV and IR limits there are theories for which the β-function is zero.

Dynamical system for the holographic model

According to the anzatz equations of motion in terms of w variable look like

$$\begin{cases} 2\dot{A}^2 + V - \frac{\dot{\phi}^2}{a^2} = 0, \\ \ddot{A} + \frac{\dot{\phi}^2}{a^2} = 0, \\ \ddot{\phi} + 2\dot{A}\dot{\phi} - \frac{a^2}{2}V_\phi = 0. \end{cases} \tag{5}$$

To obtain an autonomous system of first-order differential equations describing the RG flow, we introduce new variables [7–8]:

$$X = \frac{\dot{\phi}}{\dot{A}}, \quad Z = e^{-\phi}, \quad 0 < Z < +\infty \quad \forall\phi. \tag{6}$$

In this case we obtain the dynamical system:

$$f(Z, X) = \frac{dZ}{dA} = -ZX, \tag{7}$$

$$g(Z, X) = \frac{dX}{dA} = \left(\frac{X^2}{a^2} - 2\right)\left(X + 2a^2\cdot\frac{\left(2a^2(Z^8-1) - (Z^2-1)(Z^2+1)^3\right)}{(Z^2+1)^4 - 2a^2(Z^4-1)^2}\right). \tag{8}$$

Lets note that the analytical function f represents the change of V due to the change of the energy scale. And the function g represents the change of β-function due to the change ofthe energy scale, since X is introduced as overstretched β-function. The equilibrium points of this system, corresponding to the fixed points of the field theory, may be found as the result of equations:

$$\begin{cases} f(Z,X)\Big|_{Z_c,X_c} = 0, \\ g(Z,X)\Big|_{Z_c,X_c} = 0. \end{cases} \Rightarrow \begin{cases} 1. \quad Z_c = 0, \quad X_c = -a\sqrt{2}, \\ 2. \quad Z_c = 0, \quad X_c = a\sqrt{2}, \\ 3. \quad Z_c = 0, \quad X_c = -2a^2, \\ 4. \quad Z_c = 1, \quad X_c = 0, \\ 5-6. \quad Z_c = \sqrt{\frac{1\pm2|a|\sqrt{1-a^2}}{2a^2-1}}, \quad X_c = 0. \end{cases}$$

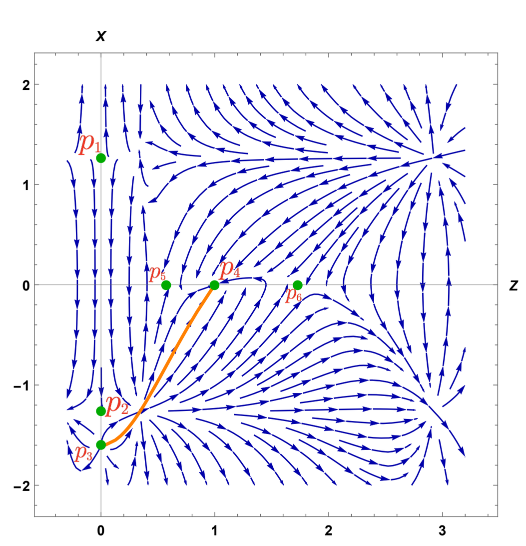


Figure 2. The RG flow for the paramter value $a^2 = 0.8$ and representation of its equilibrium points.

Classification of the equilibrium points

Stability analysis of the equilibrium points is provided by linear perturbations of the dynamical system in the vicinity of these points. With respect to the eigenvalues λ in every equilibrium point and a^2 we obtain the following classification:

point	$0 < a^2 < \frac{1}{2}$	$a^2 = \frac{1}{2}$	$\frac{1}{2} < a^2 < 1$
1	$a < 0$: unst.node	$a = -\frac{1}{\sqrt{2}}$: none	$a < 0$: saddle
	$a > 0$: saddle	$a = \frac{1}{\sqrt{2}}$: saddle	$a > 0$: saddle
2	$a < 0$: saddle	$a = -\frac{1}{\sqrt{2}}$: saddle	$a < 0$: saddle
	$a > 0$: unst.node	$a = \frac{1}{\sqrt{2}}$: none	$a > 0$: saddle
3	saddle	none	unst.node
4	stab.node	stab.node	stab.node
5-6	saddle	saddle	saddle

At some points a stability type is different according to the parameter a. It is an indicator of bifurcations of the theory. It also can be observed that at the value $a^2 + \pm1/\sqrt{2}$ for the first three points there is a regime when stability cannot be defined. In this connection, the system should be analyzed via an extra approach [1].

Asymptotic behavior of the theory near the critical points

Plugging the points coordinates into EOM we estimated the behavior of the potential according to the corresponding asymptotic for the scalar field. And recostruction of the form of the metric can help to investigate if it is AdS or the hyperscaling spacetime at considering point.

Example of an asymptotic solution at the point 4:

- $Z = 1$: $\boxed{\phi = 0 \quad \Rightarrow \quad V(\phi) = 2\Lambda_{uv}}$, and a conclusion from EOM is:

$$A \sim \sqrt{-\Lambda_{uv}}\,(w - w_0)\,, \tag{9}$$

By the metric reconstruction it was obtained:

$$ds^2 \cong e^{\sqrt{-\Lambda_{uv}}(w-w_0)}\left(-dt^2 + dx^2\right) + dw^2 \quad \Rightarrow \quad \text{EOM is valid } \forall \quad a \tag{10}$$

	$V(\phi)$	**Type according to energy scale**	UV/IR
p_3	$V \to -\infty$, $a^2 \in (0; \frac{1}{2})$ $V \to +\infty$, $a^2 \in (\frac{1}{2}; 1]$	unstable (saddle, $a^2 \in (0; \frac{1}{2})$) stable (stable node, $a^2 \in (\frac{1}{2}; 1]$)	IR
p_4	const	unstable (unstable node for all a)	UV
p_5	const	unstable (saddle for all a)	IR/UV, $a^2 \in (\frac{1}{2}; 1)$
p_6	const	unstable (saddle for all a)	IR/UV, $a^2 \in (\frac{1}{2}; 1)$

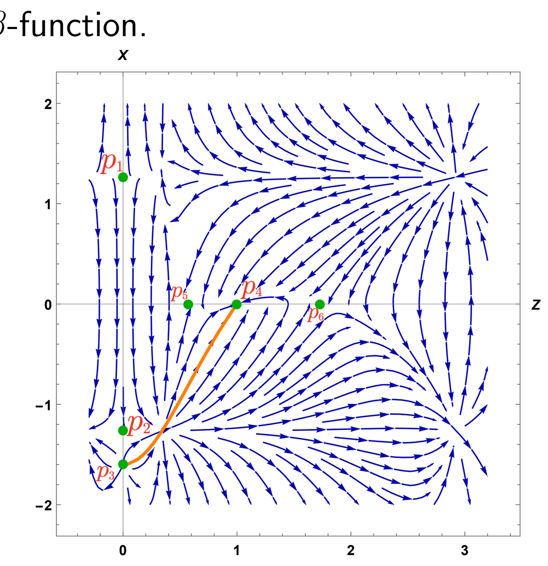
Examples of the phase portraits and RG flows of the dynamical system

Note that axis Z is related with V, and X corresponds to the β-function.

Possible RG flows for $\frac{1}{2} < a^2 < 1$:

1. p_4 (UV, AdS_3, $\phi = 0$) to p_3 (IR, $\phi \to +\infty$)
2. p_4 (UV, AdS_3, $\phi = 0$) to p_5, p_6 (IR, $\phi \to +\infty$)
3. p_5, p_6 (UV, AdS_3, $\phi = 0$) to p_3 (IR, $\phi \to +\infty$)

At this phase portrait: $a^2 = 0.8$. The orange line represents the exact solution of (7)-(8), [3].

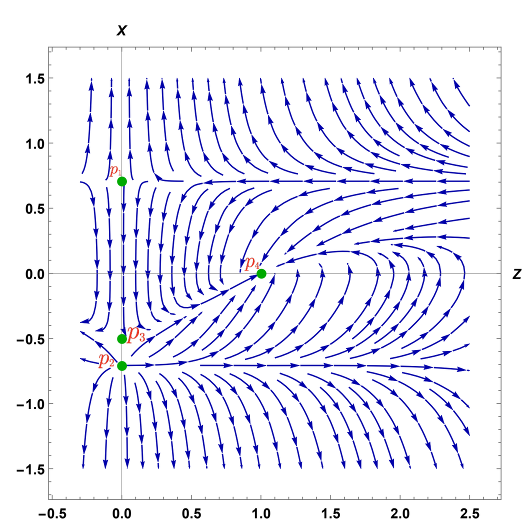


Possible RG flows for $0 < a^2 < \frac{1}{2}$:

1. p_4 (UV, AdS_3, $\phi = 0$) to p_3 (IR, $\phi \to +\infty$)

For such a type of IR points we found that the stability of the points changes if the parameter a belongs to different regions, that means here is a bifurcation when $a^2 = 1/2$. And the potential changes its asymptotic behavior at this value of a.

At this phase portrait: $a^2 = 0.25$



Conclusions & outlook

1. The metrics near the critical points, corresponding to a constant scalar field, are asymptotically AdS spacetimes
2. Near the critical points with the scalar field tending to infinity we have obtained metrics with hyperscaling violation
3. Solutions near two critical points (with $\phi \to +\infty$) don't fulfill the equations of motion
4. For three asymptotically AdS solutions we have found the constraint on the parameter: $a^2 > \frac{1}{2}$, so these solutions correspond to local minima of the potential

Open questions:

1. Systematic description of the model for the case with non-zero finite T (for another potential in [9])
2. Investigation of a connection with phase transitions in a dual $N = 2$ $d = 2$ conformal field theory

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