$\mathcal{N}=2$ cubic higher spin vertices and rigid symmetries of hypermultiplet

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Abstract

We present off-shell unconstrained formulation of $\mathcal{N}=2$ cubic $(\frac{1}{2},\frac{1}{2},\mathrm{s})$ couplings of higher spins [1] to hypermultiplet using $\mathcal{N}=2$ harmonic superspace approach [2]. The constructed vertices are associated with the gauging of the global symmetries of the hypermultiplet. We also describe the corresponding algebra of these symmetry transformations and the conservation laws of $\mathcal{N}=2$ Noether current superfields.

Harmonic superspace

4D $\mathcal{N}=2$ superspace defined as coset:

$$\mathbb{R}^{4|8}=\frac{\{M_{ab},P_a,Q^i_\alpha,\bar{Q}^i_{\dot\alpha},su(2)\}}{\{M_{ab}\},su(2)}=(x^a,\theta^i_\alpha,\bar\theta^i_{\dot\alpha})\,.$$

4D $\mathcal{N}=2$ harmonic superspace can also be defined as coset:

$$\mathbb{HR}^{4+2|8}=\frac{\{M_{ab},P_a,Q^i_\alpha,\bar{Q}^i_{\dot\alpha},su(2)\}}{\{M_{ab},u(1)\}}=\mathbb{R}^{4|8}\times S^2=(x^a,\theta^i_\alpha,\bar\theta^i_{\dot\alpha},u^{\pm i})\,.$$

- Harmonic superfields $Q^{(n)}(x^a,\theta^i_\alpha,\bar\theta^i_{\dot\alpha},u^{\pm i})$ have infinitely many components.
- Harmonic superspace have new invariant subspace containing only half of the original Grassmann variables. Analytic superspace (analog of chiral superspace in $\mathcal{N}=1,d=4$):

$$\mathbb{HA}^{4+2|4}=\frac{\{M_{ab},P_a,Q^i_\alpha,\bar{Q}^i_{\dot\alpha},su(2)\}}{\{M_{ab},Q^+_\alpha,\bar{Q}^+_{\dot\alpha},u(1)\}}=(x^a,\theta^+_\alpha,\theta^+_{\dot\alpha},u^{\pm i})=\zeta_A\,.$$

- Harmonic superspace allow off-shell hypermultiplet:

$$S_{hyper}=-\frac{1}{2}\int d^4xd^4\theta^+du\;q^{+a}\mathcal{D}^{++}q^+_a=-\int d^4xd^4\theta^+du\;\tilde{q}^+\mathcal{D}^{++}q^+\,. \qquad (1)$$

- Here $q^+_a(x,\theta^+,u)=(q^+,-\tilde{q}^+)$, $q^{+a}=\epsilon^{ab}q^+_b$. q^+ is unconstrained analytic superfield with component expansion:

$$q^+(x^a,\theta^+,u)=f^i(x)u^+_i+\theta^{+\alpha}\psi_\alpha+\bar\theta_{\dot\alpha}\bar\chi^{\dot\alpha}+\dots\,.$$

- Harmonic derivative:

$$\mathcal{D}^{++}=\partial^{++}-2i\theta^{+\rho}\bar\theta^{+\dot\rho}\partial_{\rho\dot\rho}+\theta^{+\hat\mu}\partial^+_{\hat\mu}+i(\hat{\theta}^+)^2\partial_5\,,\quad \partial^{++}=u^{+i}\frac{\partial}{\partial u^{-i}}\,,\quad \hat\mu=(\mu\,,\dot\mu)\,.$$

  Here x^5 is auxiliary coordinate and is used for description of massive hypermultiplet in the presence of central charge, $\partial_5q^+:=imq^+$.
- Hypermultiplet equations of motion after exclusion of auxiliary fields gives:

$$\mathcal{D}^{++}q^+=0\quad\Rightarrow\quad(\Box+m^2)f^i=0\,,\quad i\partial_{\alpha\dot\alpha}\bar\chi^{\dot\alpha}+m\psi_\alpha=0\,,\quad i\partial_{\alpha\dot\alpha}\psi^\alpha-m\bar\chi_{\dot\alpha}=0\,.$$

- Unconstrained analytic prepotential of $\mathcal{N}=$ **2 Maxwell multiplet** appear as connection in harmonic derivative:

$$\mathcal{D}^{++}\quad\Rightarrow\quad\mathcal{D}^{++}+i\kappa_1V^{++}J,\qquad Jq^{+a}=i(\tau^3)^a_{\;b}q^{+b}\,,\qquad(\tau_3)^a_{\;b}=\left(\begin{smallmatrix}1&0\\0&-1\end{smallmatrix}\right)\,. \qquad (2)$$

- Unconstrained analytic prepotentials of $\mathcal{N}=$ **2 supergravity** appear as vielbeins in harmonic derivative:

$$\mathcal{D}^{++}\quad\Rightarrow\quad\mathcal{D}^{++}+\kappa_2h^{++\mu\dot\mu}\partial_{\mu\dot\mu}+\kappa_2h^{++\hat\mu+}\partial^-_{\hat\mu}+\kappa_2h^{++5}\partial_5\,. \qquad (3)$$

Rigid symmetries of hypermultiplet

The **rigid symmetries** of free hypermultiplet (1) considered in [2] are realized by the following transformations:

$$\delta^{(s)}_{rig}q^{+a}=-\hat\Lambda^{\alpha(s-2)\dot\alpha(s-2)}_{rig}\partial^{s-2}_{\alpha(s-2)\dot\alpha(s-2)}(J)^{P(s)}q^{+a},\qquad P(s)=\frac{1+(-1)^{s+1}}{2}, \qquad (4)$$

where the differential operator $\hat\Lambda^{\alpha(s-2)\dot\alpha(s-2)}_{rig}$ is defined as

$$\hat\Lambda^{\alpha(s-2)\dot\alpha(s-2)}_{rig}:=\Lambda^{\alpha(s-2)\dot\alpha(s-2)M}\partial_M,\qquad M=(\alpha\dot\alpha,\alpha,\dot\alpha,5), \qquad (5)$$

with the parameters of the transformations as coefficients:

$$\Lambda^{(\alpha(s-2)\alpha)(\dot\alpha(s-2)\dot\alpha)}=\lambda^{\alpha(s-1)\dot\alpha(s-1)}-2i\lambda^{\alpha(s-1)(\dot\alpha(s-2)-}\bar\theta^{+\dot\alpha)}-2i\theta^{+(\alpha}\lambda^{\alpha(s-2))\dot\alpha(s-1)-}, \qquad (6a)$$

$$\Lambda^{\alpha(s-2)\dot\alpha(s-2)5}=\lambda^{\alpha(s-2)\dot\alpha(s-2)5}+2i\lambda^{(\alpha(s-2)\beta)\dot\alpha(s-2)-}\theta^+_\beta+2i\lambda^{\alpha(s-2)(\dot\alpha(s-2)\dot\beta)-}\bar\theta^+_{\dot\beta}\,, \qquad (6b)$$

$$\Lambda^{\alpha(s-1)\dot\alpha(s-2)+}=\lambda^{\alpha(s-1)\dot\alpha(s-2)+},\qquad\lambda^{\alpha(s-1)\dot\alpha(s-2)\pm}=\lambda^{\alpha(s-1)\dot\alpha(s-2)i}u^\pm_i\,, \qquad (6c)$$

$$\Lambda^{\alpha(s-2)\dot\alpha(s-1)+}=\lambda^{\alpha(s-2)\dot\alpha(s-1)+},\qquad\lambda^{\alpha(s-2)\dot\alpha(s-1)\pm}=\lambda^{\alpha(s-2)\dot\alpha(s-1)i}u^\pm_i\,. \qquad (6d)$$

All the parameters $\lambda^{\alpha(s-1)\dot\alpha(s-1)}$, $\lambda^{\alpha(s-2)\dot\alpha(s-2)5}$, $\lambda^{\alpha(s-1)\dot\alpha(s-2)i}$, $\lambda^{\alpha(s-2)\dot\alpha(s-1)i}$ in the operator $\hat\Lambda^{\alpha(s-2)\dot\alpha(s-2)}_{rig}$ are coordinate-independent. The analytic operator (5) satisfies the condition:

$$[\mathcal{D}^{++},\hat\Lambda^{\alpha(s-2)\dot\alpha(s-2)}_{rig}]\partial^{s-2}_{\alpha(s-2)\dot\alpha(s-2)}=0\,.$$

This property ensure the invariance of the free hypermultiplet action (1) under (4).

Transformations (4) have an interesting algebraic structure:

$$\left[\delta^{(s_1)}_{rig|F},\delta^{(s_2)}_{rig|F}\right]\sim\delta^{(s_1+s_2-2)}_{rig|B}+\delta^{(s_1+s_2-4)}_{rig|B}\Box. \qquad (7)$$

Here labels B and F means bosonic or fermionic part of corresponding symmetry transformation. In case $s_1=s_2=2$ this is $\mathcal{N}=2$ SUSY algebra. For the case $s_1=s_2=s>2$, this gives some generalizations of the $\mathcal{N}=2$ supersymmetry algebra.

$\mathcal{N}=2$ Noether current superfields of hypermultiplet

According to Noether's theorem, rigid symmetries (4) imply conservation laws of current superfield:

$$\mathcal{D}^{++}\left(q^{+a}\partial^{s-1}_{\alpha(s-1)\dot\alpha(s-1)}(J)^{P(s)}q^+_a\right)=0\,,$$

$$\mathcal{D}^{++}\left(q^{+a}\partial^{s-2}_{\alpha(s-2)\dot\alpha(s-2)}\partial_5(J)^{P(s)}q^+_a\right)=0\,,$$

$$\mathcal{D}^{++}\left(q^{+a}\partial^-_\alpha\partial^{s-2}_{\alpha(s-2)\dot\alpha(s-2)}(J)^{P(s)}q^+_a\right)=q^{+a}[\mathcal{D}^{++},\partial^-_\alpha]\partial^{s-2}_{\alpha(s-2)\dot\alpha(s-2)}(J)^{P(s)}q^+_a\,,$$

$$\mathcal{D}^{++}\left(q^{+a}\partial^-_{\dot\alpha}\partial^{s-2}_{\alpha(s-2)\dot\alpha(s-2)}(J)^{P(s)}q^+_a\right)=q^{+a}[\mathcal{D}^{++},\partial^-_{\dot\alpha}]\partial^{s-2}_{\alpha(s-2)\dot\alpha(s-2)}(J)^{P(s)}q^+_a\,.$$

At the component level, each of the superfield laws leads to the conservation of a bosonic or fermionic current. As an example, we give the corresponding component conserved currents for the case of spin **3**:

$$Y_{(\alpha\beta)\rho(\dot\alpha\dot\beta)\dot\rho}=\frac{i}{4}\Big(f^i\partial_{\alpha\dot\alpha}\partial_{\beta\dot\beta}\partial_{\rho\dot\rho}\bar{f}_i-\partial_{\alpha\dot\alpha}\partial_{\beta\dot\beta}\partial_{\rho\dot\rho}f^i\bar{f}_i-\partial_{\rho\dot\rho}f^i\partial_{\alpha\dot\alpha}\partial_{\beta\dot\beta}\bar{f}_i+\partial_{\alpha\dot\alpha}\partial_{\beta\dot\beta}f^i\partial_{\rho\dot\rho}\bar{f}_i\Big),$$

$$Y_{\alpha\rho\dot\alpha\dot\rho}=\frac{m}{4}\left(\partial_{\alpha\dot\alpha}\partial_{\rho\dot\rho}f^i\bar{f}_i-\partial_{\alpha\dot\alpha}f^i\partial_{\rho\dot\rho}\bar{f}_i-\partial_{\rho\dot\rho}f^i\partial_{\alpha\dot\alpha}\bar{f}_i+f^i\partial_{\alpha\dot\alpha}\partial_{\rho\dot\rho}\bar{f}_i\right),$$

$$Y^i_{(\alpha\beta)(\dot\alpha\dot\rho)\rho}=-\frac{i}{4}\chi_\rho\partial_{(\alpha\dot\alpha}\partial_{\beta)\dot\rho}\bar{f}^i-\frac{i}{4}\psi_\rho\partial_{(\alpha\dot\alpha}\partial_{\beta)\dot\rho}f^i,$$

$$\bar{Y}^i_{(\alpha\beta)(\dot\alpha\dot\beta)\dot\rho}=-\frac{i}{4}\bar\chi_{\dot\rho}\partial_{\alpha(\dot\alpha}\partial_{\beta\dot\beta)}f^i-\frac{i}{4}\bar\psi_{\dot\rho}\partial_{\alpha(\dot\alpha}\partial_{\beta\dot\beta)}\bar{f}^i.$$

Conservation laws have form:

$$\partial^{\rho\dot\rho}Y_{\dots\rho\dot\rho}=0.$$

Hypermultiplet cubic couplings

Analytic-superspace localization of parameters (6) produce the spin s, spin $s-1$ and spin $s-\frac{1}{2}$ gauge transformations, correspondingly. Introduction of appropriate gauge superfields allows to compensate these gauge transformations and leads to cubic vertices.

Here we present cubic $(\frac{1}{2},\frac{1}{2},\mathrm{s})$ vertices. Their form is **fully fixed** by supergauge transformations and $\mathcal{N}=2$ rigid supersymmetry.

- Spin **1** hypermultiplet coupling:

$$S_{spin\;1}=-\frac{1}{2}\int d^4xd^4\theta^+du\;q^{+a}\left(\mathcal{D}^{++}+\kappa_1V^{++}J\right)q^+_a\,. \qquad (9)$$

- Spin **2** hypermultiplet coupling:

$$\hat{\mathcal{H}}^{++}_{(2)}:=h^{++\alpha\dot\alpha}\partial_{\alpha\dot\alpha}+h^{++\hat\mu+}\partial^-_{\hat\mu}+h^{++5}\partial_5\,,$$

$$S_{spin\;2}=-\frac{1}{2}\int d^4xd^4\theta^+du\;q^{+a}\left(\mathcal{D}^{++}+\kappa_2\hat{\mathcal{H}}^{++}_{(2)}\right)q^+_a\,. \qquad (10)$$

- General spin s hypermultiplet couplings:

$$\hat{\mathcal{H}}^{++}_{(s)}:=\Big(h^{++\alpha(s-1)\dot\alpha(s-1)}\partial_{\alpha\dot\alpha}+h^{++\alpha(s-1)\dot\alpha(s-2)+}\partial^-_\alpha$$
$$+h^{++\alpha(s-2)\dot\alpha(s-1)+}\partial^-_{\dot\alpha}+h^{++\alpha(s-2)\dot\alpha(s-2)}\partial_5\Big)\partial^{(s-2)}_{\alpha(s-2)\dot\alpha(s-2)},$$

$$S_{spin\;s}=-\frac{1}{2}\int d^4xd^4\theta^+du\;q^{+a}\left(\mathcal{D}^{++}+\kappa_s\hat{\mathcal{H}}^{++}_{(s)}J^{P(s)}\right)q^+_a. \qquad (11)$$

- Corresponding **gauge transformations** have universal form:

$$\delta^{(s)}_\lambda q^{+a}=-\frac{\kappa_s}{2}\left\{\hat\Lambda^{\alpha(s-2)\dot\alpha(s-2)},\partial^{s-2}_{\alpha(s-2)\dot\alpha(s-2)}\right\}(J)^{P(s)}q^{+a}$$
$$-\frac{\kappa_s}{2}\partial^{s-2}_{\alpha(s-2)\dot\alpha(s-2)}\Omega^{\alpha(s-2)\dot\alpha(s-2)}(J)^{P(s)}q^{+a},$$

$$\delta_\lambda V^{++}=[\mathcal{D}^{++},\lambda],\quad\delta_\lambda\hat{\mathcal{H}}^{++}_{(2)}=[\mathfrak{D}^{++},\hat\Lambda_{(2)}],\qquad\delta_\lambda\hat{\mathcal{H}}^{++}_{(s)}=[\mathfrak{D}^{++},\hat\Lambda_{(s)}],$$

  where

$$\hat\Lambda_{(s)}=\Lambda^{\alpha(s-2)\dot\alpha(s-2)M}\partial_M\partial^{s-2}_{\alpha(s-2)\dot\alpha(s-2)} \qquad (12)$$

  and all λ-parameters are arbitrary analytic functions.

So form of this couplings is universal and cubic $(\frac{1}{2},\frac{1}{2},\mathrm{s})$ vertices can be constructed by gauging of global "higher spin" symmetries of hypermultiplet action (1). For the future details see [2].

References

[1] I. Buchbinder, E. Ivanov and N. Zaigraev, *Unconstrained off-shell superfield formulation of 4D, $\mathcal{N}=2$ supersymmetric higher spins*, JHEP **12** (2021), 016 [arXiv:2109.07639 [hep-th]].

[2] I. Buchbinder, E. Ivanov and N. Zaigraev, *Off-shell cubic hypermultiplet couplings to $\mathcal{N}=2$ higher spin gauge superfields*, JHEP **05** (2022), 104 [arXiv:2202.08196 [hep-th]].