HOLLENBECK-VERBITSKY CONJECTURE ON BEST CONSTANT INEQUALITIES FOR ANALYTIC AND CO-ANALYTIC PROJECTIONS

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Abstract. We address the problem of finding the best constants in inequalities of the form:

$$\|\left(|P_+f|^s + |P_-f|^s\right)^{\frac{1}{s}}\|_{L^p(\mathbb{T})} \leq A_{p,s}\|f\|_{L^p(\mathbb{T})},$$

where P_+f and P_-f denote analytic and co-analytic projection of a complex-valued function $f \in L^p(\mathbb{T})$, for $p \geq 2$ and all $s > 0$, thus proving Hollenbeck-Verbitsky conjecture from [1]. We also prove the same inequalities for $1 < p \leq \frac{4}{3}$ and $s \leq \sec^2 \frac{\pi}{2p}$. This serves as an example of plurisubharmonic obstacle problem.

We will show how this result implies best constants inequalities for the projections on the real-line and half-space multipliers on $\mathbb{R}^n$ and an analog for analytic martingales. A remark on an isoperimetric inequality for harmonic functions in the unit disk will also be given.

The talk is based on recent result of the author and an earlier result with Marković. Some preceding results and possible extensions will be discussed.

References

[1] B. Hollenbeck and I.E. Verbitsky, *Best constant inequalities involving the analytic and co-analytic projection*, Oper. Theory Adv. Appl. **202** (2010), 285-295.