Ljudmila Vsevolodovna Keldysh (To Her Centenary, Part I)

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Introduction

Ljudmila Vsevolodovna Keldysh (1904–1976) was a remarkable mathematician. She attacked and solved problems of great complexity with supreme creative effort. But she was also a remarkable woman, a mother of five in a family of scientists with the inner strength to overcome the specific hardships facing Russian intellectuals during the first two thirds of the 20th century.

These days, there is nothing unusual about a woman choosing mathematics as her career and becoming a professional of high quality. However, before the second half of the 20th century this was a rare event, exemplified chiefly by Sophia Kovalevsky and later Emmy Nöther. So the choice of mathematics as a profession by a young woman from a family rather far-removed from science, even though belonging to the Russian intelligentsia, was an act that bespoke an exceptional strength of mind. And Ljudmila Keldysh's life has proved that this choice was not a juvenile whim but a genuine calling.

The research work of L.V. Keldysh splits distinctly into three periods. The first (before World War II) was exclusively devoted to studying the structure of Borel sets and culminated in her doctorate; successfully defended in 1941 (the complete text appeared in Trudy MIAN in 1945).

After the war, she investigated continuous mappings of compact sets in the context of dimension-raising mechanisms. This led her to certain structure theorems as well as to the construction of remarkable examples, which answered basic questions that previously seemed completely inaccessible.

At the end of the 1950s, L.V. turned to geometric topology, a subject that was, at that time, beginning to make progress on basic problems concerning manifolds: combinatorial and topological equivalence, triangulability, classification of embeddings, etc. To this subject matter, L.V. brought the techniques that she was successfully using in her work on continuous mappings, especially the notion of pseudoisotopy which turned out to be very helpful. She organized a seminar on geometric topology, initially within the framework of the Moscow topology seminar of P.S. Alexandrov. In the final decade and a half of L.V.'s life, her seminar played an important role, and in many instances a decisive one, in attaining now classical solutions of these problems.

Family History

Ljudmila Keldysh was raised in the family of a prominent engineer; she was the oldest among seven children. The name 'Keldysh' is of Turkic origin, possibly originating

Vsevolod Mikhailovich Keldysh Ljudmila Keldysh as a young girl

from the word "kel di" = "he has come" (the ending "sh" is a diminutive of endearment), despite no record of Turkic ancestors in the family. Originally, the name could have been given to a long-awaited boy, born after several girls.

Foma Simonovich Keldysh, the great-grandfather of L.V., was a psalm-reader at an orthodox church in Warsaw. His son Mikhail, born in 1839, studied medicine at military academies in Warsaw and St.Petersburg. After graduation, he saw military service in Caucasus and in the war with Turkey. Later, he was engaged in both practical and scientific epidemiology. In particular, he composed a medico-topographic description of the Caspian region. His service was rewarded with the rank of general and a title of nobility. He died in 1920.

His son, Mikhail Keldysh, Ljudmila's grandfather, was married to the first cousin of General Brusilov (known for the famous "Brusilov's breakthrough" in World War I).

Ljudmila's father, Vsevolod Mikhailovich Keldysh, was born in 1878 in Vladikavkaz during Mikhail's endless wanderings around the south of Russia. He graduated from the Polytechnical Institute in Riga as an engineer in structural mechanics. Also in Riga, he married Maria Alexandrovna Skvortsova. For professional reasons, like his father, he had to move from one town to another. Ljudmila Keldysh was born in Orenburg (in the region of Saratov, to the east of the Volga) on March 12th 1904 (the other six children were born in Riga, Helsinki, St.Petersbourg, and Moscow).

Ljudmila's mother Maria (born in 1879) came from a family of Russian nobility. Maria's grandfather, Nikolai Skvortsov, was a general. In the Caucasian wars (1817–1864) he was badly wounded and two Georgian women, mother and daughter, nursed him back to health. He helped the daughter enter the famous Smol'ny Institute in St. Petersburg (a high-level educational institution for aristocratic young ladies) and later married her. Maria's father was also a general and his wife, Sofia Iosifovna Covzan, had Polish roots.

According to family legend, Maria's great-grandfather, a physician, was once visiting friends in the Ukraine and came upon a sick girl abandoned by gypsies. He cured the girl, left her at the country-seat of these friends and married her when she grew up. One may judge the truth of this legend by the gypsy-like features of some family members, especially Ljudmila's brother Mstislav and, to a lesser extent, L.V. herself.

During World War I, trials and troubles befell the 11-year-old Ljudmila, hitherto a happy girl. When the German troops approached Riga, the Polytechnical Institute was transferred to Moscow. The family had six children, the youngest girl only a year old. They lived in a suburb, the boys commuting to Moscow daily by train to continue school, sometimes on the roof of an overfilled wagon. Food was a problem; on certain days the rations consisted only of roast onion. Gradually life became easier, partly because of help from the ARA, an American welfare organization supporting post-war Russia. A seventh child, a girl, was born to the family in 1920. Vsevolod Keldysh felt that one must have faith in a better future and the family called the last-born daughter Vera, meaning faith.

At the beginning of the 1920s the family lived in Ivanovo (a city north-east of Moscow) where Ljudmila finished her secondary school education. A lecture by N.N. Lusin, then teaching in Ivanovo, determined her choice of profession. She entered the Physics and Mathematics Department of Moscow University where Lusin eventually became her scientific advisor.

N.N. Lusin

N.N. Lusin (1883–1950) was the head of the Moscow mathematical school known by the name of 'Lusitania'. He came to Moscow University in 1901. At that time mathematical life there was very dynamic and he actively participated in it (he was the secretary of a mathematical circle headed by N.E. Zhukovsky). But in 1905, in a period of revolutionary uprisings, the political life in the country and especially in Moscow became extremely insecure and it affected the university. There were student strikes and at one time Lusin had a cache of bombs under his bed! Fortunately his teacher D.F. Egorov, the leading Moscow mathematician of the time, succeeded in arranging for Lusin's departure to France.

The opportunity to hear the glorious French scientists: Poincaré, Hadamard and Darboux, and very intensive work at the Sorbonne library had a profound influence on him. Lusin's work in Hadamard's seminar in 1912 was especially important. There he had personal contact with eminent mathematicians, notably Lebesgue and Borel, whose ideas exerted a strong influence on his thoughts on the foundations of mathematics.

He returned to Moscow in 1914 and began to teach at the mathematics department of the university. He had an exceptional teaching style and was particularly good

Nikolai Nikolaevich Lusin (1883 – 1950)

at attracting disciples to scientific creativity. The work of his seminars never ended when the bell rang and continued up to the door of his apartment and frequently inside it. His students, forming the Lusitania[1] group (including M.Y. Suslin, D.E. Men'shov, A.Y. Khinchin, P.S. Alexandrov, P.S. Uryson, L.A.Lusternik, A.N. Kolmogorov, N.K. Bari, P.S. Novikov and L.V. Keldysh), later became leading researchers in fundamental and applied mathematics and some founded their own schools in many different branches of mathematics.

In the harsh times of the revolutionary crisis of 1917, some Moscow professors, Lusin among them, moved to Ivanovo to teach at the new Polytechnical Institute (without breaking their relationship with Moscow University). He was in Ivanovo from 1918 to 1922 and there Ljudmila Keldysh, still a schoolgirl, met the man who was to orient her future scientific life.[2]

In her Parents' Family

In the 1920s, Vsevolod Keldysh, Ljudmila's father, taught at a military-engineering academy where he was head of the Chair of Ferroconcrete and later head of the Chair of Structural Mechanics. At that time the construction industry was rapidly developing in the Soviet Union and he became one of the leading experts in such important constructions as the Dniepr power plants, the Moscow-Volga Canal and the Moscow metro (underground). He also achieved the rank of general. When the youngsters around him would begin to criticize things in an insolent manner, he would cool them down with the words, "Now you are not satisfied with modern life, but you should know I could not find a job to my liking in the tsarist days and had to go into teaching. And now I have a fascinating professional activity."

[1]It was the name of a grand ocean liner torpedoed by a German submarine in 1915, a famous event of the First World War. One may learn about it at http://www.lusitania.net/. Incidentally, Lusitania was an antique Roman province on the site of what is now Portugal.

[2]Readers may make more detailed acquaintance with Lusin's mathematics, philosophy and religion in a remarkable article, "A comparison of two cultural approaches to mathematics: France and Russia, 1890–1930" by L. Graham and J.-M. Kantor (http://www.math.jussieu.fr/~kantor/).

Ljudmila Keldysh on a hike

The family was big; alongside the seven children, it included Vsevolod's mother and mother-in-law, neither of whom received a pension since they were widows of tsarist generals (not to mention their noble origins!) But the family had sufficient money to live on because the head of the family belonged to a well-paid category. Vsevolod Keldysh retired at the age of 80 and he died suddenly at 87 in 1965. His wife Maria Alexandrovna had died before him in 1953.

The family had strict ethical principles, which was rather typical for the intellectual world of Russia at that time. The mother devoted her whole life to the family, tutoring her children in foreign languages (German and French) and trying to teach them good taste and a love of music. The parents played piano-for-four-hands together, and regularly led family outings to the opera and to concerts of classic music. They did not deprive themselves of such pleasure even in the harsh post-revolutionary times, when they had to return home from Moscow on foot. But only one of the children, Yuri, who spent hours sitting deeply engrossed at the piano, made music his profession, eventually becoming a musicologist (rather than a concert pianist as he had originally hoped). Ljudmila also learned to play the piano and still loved to play later in her life.

The family was not religious. Only grandmother Sophia Iosifovna would take the youngest daughter, Vera, to church; the parents would react by explaining that the daughter "will understand what's what when she grows up." Nevertheless, the family observed the main religious feasts in the traditional orthodox spirit with *paskha*,[3] *kulichi*,[4] and painted eggs for Easter, attending the Church of Christ the Redeemer where the best vocalists from the Bolshoy Theatre sang in the Easter services. L.V. Keldysh once told one of us, her students, that at difficult moments of her life she would enter an empty church and stand there in silence for a short while.

There were frequent parties of young people, mainly friends of the eldest daughter Ljudmila – mathematics students and postgraduates, among them P.S. Novikov, A.N. Kolmogorov and P.S. Alexandrov, all of whom were future full members of the USSR Academy of Sciences, I.V. Arnold, a well-known mathematician, later the father of Vladimir Arnold (another academician to be), and musician friends of Yuri. There were games, laughter and serious conversations.

The determined character of the eldest daughter had a great influence on the other children. Following her example, Mstislav (the future President of the USSR Academy of Science) joined the Mathematics Department of Moscow University against the wishes of his father, who wanted Mstislav to follow in his footsteps and become a construction engineer.

Their father's profession was inherited only by Lyubov, his favourite child. She eventually became the keeper of the family archives. The youngest daughter, Vera, now works at the well-known Aerohydrodynamics Institute. She has written a short history of the family in a book devoted to Mstislav Keldysh, from which I have borrowed some of the family information [1].

In 1934, L.V. Keldysh and P.S. Novikov were married. From the very first days of their life together, L.V. not only became P.S. Novikov's wife and mother of numerous children, but, as one may put it, the guardian angel of her husband's mathematical creativity. With full understanding, she always gave precedence to her husband's work over her own. Among the participants of Lusitania, the two of them were the closest to Lusin himself both in temperament and scientific interest. Together they wrote a remarkable commentary to his book on analytic sets, later analyzed his works in a special article; their first works were aimed at solving problems within Lusin's fields of interest. From 1934, they held positions at the Steklov Mathematical Institute. Later their scientific interests separated but L.V. still tried to keep an eye on the work of P.S., participating in seminars whenever he spoke about his results. She closely followed his proof of the undecidability of the word problem in group presentations. Indeed, with S.I. Adyan, she edited the manuscript written by P.S. Later, she persuaded Adyan to rejoin P.S. and finalize the latter's research on the negative solution of the Burnside problem.

Pre-war Research: Borel Sets

The name of L.V. Keldysh became well-known after her very first result, which Lusin included in the beginning of his famous lecture course in 1930 (see [2]). She constructed a beautiful arithmetical example (using continued fractions) of a set belonging to the fourth Borel class. It was the first serious advance in the Borel classification of sets since 1905 when Baire had given the first non-trivial example of a set of the third class. She loved to remember that when she was introduced to Heinz Hopf, who was visiting Moscow, he exclaimed, "Ah! The fourth class example!"

An important motivation behind descriptive set theory, that is the description of processes to build sets of different Borel classes, is the measurability problem, one of the basic questions in analysis. Borel and Lebesgue gave two definitions of measurability. That of Borel is minimal in a sense; it studies those sets that may be obtained from intervals by applying the union and intersection operations to countable ensembles of sets. (And the measure may be extended by the transfinite process to all these B-sets.) There arises a transfinite classification of such sets. The Lebesgue definition covers a much larger class of sets, although one builds nonmeasurable sets in his sense. These ideas relate

[3] Sweetened curds with butter and various other ingredients such as raisins, shaped as pyramids with crosses imprinted upon them.

[4] A kind of cake made from fancy pastry with raisins and spices and formed into a cylinder.

to the famous Continuum Hypothesis and especially to questions about effectiveness of constructions used when Zermelo's Axiom of Choice is rejected. The construction of a Lebesgue nonmeasurable set was ineffective in this sense but Lebesgue had also given an effective example of a set nonmeasurable in Borel's sense. These problems demanded detailed investigation of the continuum and analysis of the effectiveness of construction processes for different classes of subsets. All this became the object of investigations by Lusin and his school.

For Lusin, philosophy of science came before strictly mathematical problems. This broader interest led to a profound involvement in questions about the foundation of mathematics. L.V. Keldysh recalled that Lusin repeatedly stated that the essence of the study of the continuum consists of a groping towards the limits of human set-theoretical knowledge and he predicted the existence of non-provable assertions (for example, the impossibility of proving measurability of his projective sets which was ascertained in 1951 by P.S. Novikov).

The natural beginning to this activity was a detailed study of Borel sets. It is well-known that the first success here was a solution by P.S. Alexandrov and F. Hausdorf of the problem about existence of a perfect core in every uncountable B-set. A new operation invented by Alexandrov gave sets beyond Borel classification, in addition to Borel sets. They were discovered by M.Y. Suslin who gave them the name 'A-sets' but they are also known as 'Suslin sets'.

Lusin specified the Vallée Poussin classification of Borel sets, or B-sets, which is based on the set-theory notion of limits. He has introduced, in particular, a notion of an *element* of the given transfinite class α. Elements are countable intersections that are not countable unions of sets belonging to lower classes. Their countable unions give all sets of the class α. Based on an analogy with uncountable compacts that are homeomorphic to unions of the Cantor set with countable sets of points, he posed in general terms the problem of finding *canonical* elements such that every Borel set be a countable union of canonical elements. But he gave no precise definition of this notion: "We call *canonical* those elements of the given class K_α *that has a sufficiently simple property* and such that one obtains every set of the class K_α taking the union of a countable set of the canonical elements… It would be desirable to have in every class a finite number of kinds of canonical elements such that the elements of the same kind be homeomorphic between them." [2]

L.V. Keldysh succeeded ultimately in building arithmetic examples for *all* countable classes α in this Vallée Poussin classification, and this construction is effective if a is effectively given (that is with a recursive set of lesser transfinite numbers). This construction defines such a set of the given class (in the Baire space of irrational points) with series of conditions on sequences of partial quotients in continued fraction developments of its points. For example, the condition for her initial example of fourth class demands that there be an infinite set of different partial quotients, each of which repeats infinitely many times. (Baire's set of 1905 consists of irrational points whose partial quotients increase unboundedly.) [3, 4, 5]

Together with this construction she formulated a strict notion of canonical elements and with its help gave a rather complete description of any B-set [5]. She called an element A of the given class *canonical* if every open-closed subset is *universal* in this class (that is, it is possible to embed in *A* any other set *B* of this, or lesser, class as a closed subset) and if it has first Baire category on itself[5] in every open-closed subset of its closure. She showed that these elements are homeomorphic for every class (according to a theorem by M.A. Lavrentijev [6], elements of different classes are not homeomorphic, beginning with the third class) and that every element of the given class consists of a canonical element and a countable union of sets belonging to lesser classes. She showed that Baire's construction for the third class and her construction for other classes give precisely the canonical elements in every class. Keldysh's definition of canonical elements, properly speaking, did not exactly answer Lusin's question because countable unions of sets of the first category have themselves the same first category. So this definition and questions that she posed here, and her investigation of elements with second category, eventually engendered a large amount of investigation (see [7] and the detailed paper by A. Ostrovski [8], which contains more references).

Several other important results followed. Her works of that period on descriptive set theory fully displayed her personal style: discovery of the underlying concepts of the chosen subject through an infallible intuition and painstaking construction of the fundamental examples. The geometric viewpoint and descriptive set theory techniques that she used in these studies later facilitated her transition to research in topology.

Stalinism

This first period of L.V.'s research took place in the ghastly period of Stalinism. The first victim of it in the family was Nikolay Alexandrovich, brother of her mother and a former officer in the tsarist army. His past prevented him from reorganizing his life and the family assisted him. He was arrested at the very beginning of that period and became a political prisoner, working during the early 1930s on the construction of the White Sea-Baltic Canal. Later, in wartime, he was arrested again and perished in one of the Stalin camps.

In 1935 L.V.'s mother, Maria Alexandrovna, was arrested and Vsevolod Mikhailovich had to appear before a specially appointed commission before they were allowed to go free. The arrest was within the framework of a campaign for appropriating gold and jewellery from the wealthy before the revolution but they were granted clemency and their property was returned, jokingly declared to be mere trinkets.

The next victim was L.V.'s brother Mikhail. He was part of a large group of about a hundred postgraduates and professors from the History Department of Moscow

[5]That is, it is a countable sum of closed subset nowhere dense in it.

University who were arrested in 1936. Only in the 1990s did the family learn that he was executed in 1937.

Another brother, Alexandr, was arrested in 1938 as a 'French spy'. Fortunately, a change of leadership occurred in the *organs* (as the secret police liked to call themselves) due to which many people, Alexandr among them, regained their liberty. Then the espionage accusation was changed to that of antisemitism but his first wife (an ethnic Jew), and others, testified in his defence. Later he fought in the war and after it, although he had dreamed of becoming a dramatic actor, became an administrator in the state concert foundation and organised theatrical tours all over the USSR.

World War II

The period of the war in Russia was extremely hard for the family. The critical day in Moscow was October 16th 1941. The Germans were on the outskirts of Moscow. L.V. was to leave Moscow with her three sons; the first, Leonid (from her short-lived first marriage) was only ten, while the youngest, Sergey, was three. That day was the last possible chance to board a train going east from Moscow but no one could give the exact destination. Only with the help of A.N. Kolmogorov, who presented them as members of his family, was the family able to move out of Moscow. They first reached Gor'ky by train and then Kazan by a Volga riverboat several days later. Arrangements were made there for personnel evacuated from Academy of Sciences institutes. However, this family did not have the status of *evacuated* personnel but that of *refugees* and were therefore without any provisions or warm clothes, without guarantee of any housing, essentially without means of support. However, they did get a roof over their heads, living with several hundred similar refugees on mattresses in the Kazan University indoor sports facility.

A month later, Petr Sergeevich reached Kazan, gravely ill. He was awaiting serious surgery and physicians had doubts about its outcome. The family obtained a room in the corridor of the university student dormitory. During the next months of the cold and hungry winter and the subsequent spring of 1942, L.V. passed between the dormitory hall where the children waited for her and the hospital ward where she nursed her husband. The distance was ten kilometres and there was no transportation in that part of Kazan.

Besides the daily ration of 300 grams of bread per person and one teaspoonful of granulated sugar, there was just enough money to purchase a 50-kilogram bag of coarsely ground rye flour a month. This flour, boiled in water, became the basic food for the family over the next two years.

After the return to Moscow (in 1943), the family's life returned to normal little by little. The two daughters rejoined their three brothers and once again L.V. Keldysh had to call upon all her energy and force of character in order to efficiently play her family role and continue active research work, without which she could not imagine her life.

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The final second part of this article is to appear in the next issue of the Newsletter. The author is very grateful to members of L.V. Keldysh's family for acquaintance with their family history and the permission to use it in this outline. Also used with thanks are recollections of L.V. Keldysh's students: L.V. Sandrakova, A.B. Sossinsky and M.A. Stanko. Many thanks go moreover to A.B. Sossinsky and L.C. Siebenmann for their efforts to adapt this text, written by a Russian author, for the understanding of European readers.

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Ljudmila Vsevolodovna Keldysh (To Her Centenary, Part II)

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The Keldysh–Novikov family

The children's education was the most important part of the family life of L. V. Keldysh and P. S. Novikov, just as it had been in L. V.'s paternal family; colleagues at the Steklov Institute once asked her to give a talk on education and later recalled that it was a memorable lecture. The children were constantly being educated, not through moralizing speeches but by their parents' personal example and the whole family life-style. The dominance of spiritual values over material ones was implicit. She insisted that the children should be neatly clothed and have good food but without excess. ("Cheap chic!" was her characteristic observation when she encountered pretentiousness.)

Another important trait of their lives was a love of the outdoors, of fresh air and of spending as much time as possible out of town. Even during the war years, when all her drive was directed to the survival of the family, she would always find and rent a room in some village a few dozen kilometres from Moscow and take the children there for the whole summer. She loved the beauty of nature and country houses. Even in later life, she did not care for the comforts of vacation resorts but preferred a tent in a forest on some river-bank. She considered her husband's smoking to be a serious flaw but she was unable to overcome it.

There were no rigid schemes in the intellectual education of the children. The only things that L. V. Keldysh regarded as an obligatory supplement to school education were learning foreign languages and reading. Books were ordinarily, but not exclusively, given as presents and she tried to diversify their choice of reading material as much as possible. Their reading gave occasions for conversation at the table about history, art, science and life. This was precisely what constituted the children's family education. As they grew up, the role of father in these conversations became more significant. If some ethical question was touched upon, one of the parents would speak about "what is good and what is bad", usually rather categorically, but only in passing, without discussing details, as if assuming that there was nothing to discuss. But in other cases, the conversation would run as a discussion or even as a contest, never as a homily from the parents to the children.

The parents would frequently mention that choice of profession should be based only on one's vocation and capabilities. In that sense, they were themselves excellent examples, since both came from families comprising no mathematicians, in fact no scientists at all. Nevertheless, all three sons chose the physical and mathematical sciences as their professions. The eldest, Leonid, is a physicist and a full member of the Academy of Sciences. For a certain period he was Academician-Secretary in the Section of General Physics and Astronomy of the Russian Academy of Sciences. The youngest, Sergey, is an academician and a Fields medalist whose name is known to the entire international mathematics community. The third son, Andrey, was also a very gifted mathematician in the opinion of I. R. Shafarevich, his research advisor, but he perished tragically before realizing his potential. Only the youngest sister, Elena, is an exception to the scientific trend; she is a philologist and translator, who teaches French and Portuguese at the Institute of International Relations. The elder sister, Nina, died after a long illness.

Ljudmila Keldysh and Petr S. Novikov at the ICM, Moscow 1966

The choice made by the three sons was possibly due to the fact that discussions of scientific problems between their parents (who had related scientific interests) were often emotional and this was a part of home life. Moreover, many of the closest friends of the parents from their student years, such as the physicists A. A. Andronov and M. A. Leontovich, and the astronomer N. N. Pariisky, were not only outstanding scientists but also had wide ranges of interests and very impressive personalities. Another reason was that in the period in which they were growing up, the physical and mathematical sciences were flourishing and had become popular throughout the world.

The range of interests of the family extended far beyond the limits of science itself and touched upon practically the entire cultural sphere; one may say that culture was a lifeform for this family. In the post-war years, the two artists O. A. and V. V. Domogatski became close family friends. She was a sculptor and he was a graphic artist with extensive knowledge of the arts, sculpture, art history and history in general and a large circle of acquaintances in the Moscow intelligentsia. Other people from the art world came to the house and original works of such remarkable Russian painters as Fonvisin, Falk, Kuznetsov, Meshcherin, and Krymov adorned the walls of the house. Another family friend was the well-known actor and stage director I. I. Solov'ev, who would provide information about new shows and the theatre world in general.

L. V. Keldysh was a "natural object" of curiosity for journalists, especially around March 8th (the so-called International Woman's Day, an important day in Soviet Russia). She would react to this with respectful annoyance. In particular, talk about "feminine mathematics" made her smile. At the end of the 1940's, a journalist was appointed to write about this exceptional woman who was a professor of mathematics and a mother of five, about her work, her family and her social activity. He spent much time and effort, conversed with L. V. herself and with many people around her, all this only to telephone and say with chagrin, "our editor has said that my write-up is not fit to print because education in your family is too individualistic and, as a result, your eldest son even is not in the Young Communist's League!"

In day-to-day life, L. V. followed strict ethical rules, quite rigorous but without any element of hypocrisy or bigotry. Her demands were particularly strict when applied to herself. As to others, she was much more indulgent; she was well-disposed and always ready to help. However, decency and culture were to her the highest merits of a human being and these two qualities were inseparable in her eyes.

In the same vein, she invariably sought to help those who were in trouble or victims of injustice, although she was never an active fighter against abstract evil because of the innate absence of aggressiveness in her character and her constant preoccupation with family and science.

Here is a characteristic example: in the postwar excesses of Stalinism, commissions for purging workers' collectives were organized and such a commission appeared at the Steklov Institute. At the time, V. A. Rokhlin (still a young mathematician and not yet as famous as he would become) was employed by the institute as an assistant to L. S. Pontryagin. When Rokhlin obtained his doctorate degree, knowing that as a former war prisoner of the Germans he had no chance of getting a research position at the institute, he handed in his resignation before the commission began functioning. However, the commission destroyed his letter of resignation and organized a meeting of the institute's staff, which decided that Rokhlin should be fired as an "anti-Soviet element". Only seven people voted against this motion, P. S. Novikov and L. V. Keldysh among them. (The other five were B. N. Delonay, A. O. Gelfond, A. A. Markov, L. S. Pontryagin and I. R. Shafarevich).

Another stand of L. V. Keldysh against injustice was her signature of the "Letter of the 99", written in defense of the mathematical logician A. S. Essenin-Volpin, who, in 1968, was forcibly placed in a mental institution by the KGB.

It is hard to imagine today how much courage was needed then to display such high moral conduct. Throughout her life, she never wavered from it.

Scientific work of L. V. after the war

The work of L. V. in descriptive set theory was completed just before the war. A report on these results, which composed her thesis, was published in a note "On the structure of B-sets" in Doklady (1941) [3] but the full exposition appeared (for an obvious reason) only toward the end of the war. It was a paper in French, with the same title, in Mathematical Sbornik [4]. In a 1945 volume of Trudy MIAN, she summed up a decade of her work in this domain [5] and then began to work on topological themes.

The note in 1945 Doklady [7], dedicated to open mappings of A-sets, lies between her work on descriptive set theory and topology. Its basic result states that every A-set in Baire space is the image under an open mapping of a set that is an intersection of a F_σ-set and a G_δ-set. But it had been shown by Hausdorff that the class G_δ is invariant relative to open mappings.[1] L. V. obtained her result, unexpected in view of this classical theorem of Hausdorff, by an original geometric construction.

Raising of dimension. Light mappings.

Toward the end of this period, the topological theme in L. V.'s work came to the fore and she turned naturally to the investigation of continuous mappings of compact spaces. This work occupied the second decade of her mathematical activity (1947–1957). The distinctive feature of her topological work was again its concrete geometric character and, as in B-set theory, her aim to structure the clarification of mappings of a general kind and construct decisive examples. She was especially interested in problems posed by P. S. Alexandrov, her elder comrade in Lusitania and chief of the Moscow topological school. These problems concerned the mechanism of dimension raising. Here, three classes of mappings play the main role: light (or null-dimensional, having null-dimensional preimages of points), monotone (with connected preimages of points) and open. Any mapping is a composition of monotone and light mappings, which suggests the importance of studying them. As for open mappings, one long-term expectation was that they are sufficiently close to topological properties of spaces to conserve dimension. It is easy to show that an open mapping cannot raise dimension of the line segment and a monotone open mapping cannot raise dimension of the square. It is somewhat more difficult to show that one cannot raise dimension of surfaces by monotone and light open mappings (see [8]).

Almost ten years of study of the raising dimension mechanism led L. V. Keldysh to results that remain significant achievements. She has written complicated geometric constructions of examples, which have shown, in the domain of elemental-geometric objects, that the condition of openness doesn't prevent the possibility of dimension raising in both the light case and in the monotone (even for manifolds) case. Alexandrov's conjecture, stating that for a compact space any continuous mapping that finitely raises dimension is a composition of a finite-to-one mapping and a mapping that does not raise dimension, was rejected in the general case rather quickly. L. V. Keldysh showed in [9] how one must change the conjecture. She began with a study of the structure of was a sculptor and he was a graphic artist with extensive knowledge of the arts, sculpture, art history and history in general and a large circle of acquaintances in the Moscow intelligentsia. Other people from the art world came to the house and original works of such remarkable Russian painters as Fonvisin, Falk, Kuznetsov, Meshcherin, and Krymov adorned the walls of the house. Another family friend was the well-known actor and stage director I. I. Solov'ev, who would provide information about new shows and the theatre world in general.

L. V. Keldysh was a "natural object" of curiosity for journalists, especially around March 8th (the so-called International Woman's Day, an important day in Soviet Russia). She would react to this with respectful annoyance. In particular, talk about "feminine mathematics" made her smile. At the end of the 1940's, a journalist was appointed to write about this exceptional woman who was a professor of mathematics and a mother of five, about her work, her family and her social activity. He spent much time and effort, conversed with L. V. herself and with many people around her, all this only to telephone and say with chagrin, "our editor has said that my write-up is not fit to print because education in your family is too individualistic and, as a result, your eldest son even is not in the Young Communist's League!"

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$$f = \varphi_{k-1}\psi_{k-1}\dots\varphi_1\psi_1 \quad (1)$$

where all ψ_p are light mappings that don't raise dimension and all φ_p are two-to-one mappings, each of which raises dimension by one. As a two-to-one mapping cannot raise dimension more than one, L. V. Keldysh's result is conclusive and gives the complete clarification to the picture of irreducible Peano mappings.

She also indicated property of a mapping that allows it to be represented in the form (1).

The technique that she worked out in this study allowed her to understand the structure of light mappings of any finite dimensional compact space, especially light open mappings. Amongst open mappings, only homeomorphisms are irreducible. But L. V. Keldysh indicated a curious class of mappings that contains both irreducible and open mappings, i.e. *piecewise* mappings that send pieces onto pieces, where one calls the closure of an open set a *piece*. (She asked if any light mapping of a compact space satisfies this property; it seems the answer is not known so far.) She showed that any light piecewise mapping is a composition of an irreducible and an open mapping, where the second uniformly (that is on open subsets) does not raise dimension. Then she showed that in the case of an open light mapping, one can choose the irreducible component so that it has the property mentioned above providing it can be presented in the form (1). In conclusion, a light open mapping of a compact space that raises dimension by k can be written as a composition of $2k+1$ mappings, where the odd mappings do not uniformly raise dimension and the even mappings are two-to-one and each raises dimension by one. So, the dimension increase is due to two-to-one irreducible gluings [10, 11].

The rather complicated technique of proofs of these results are evidently rooted in various techniques of descriptive set theory, especially the sieve operation. This is definitely true for the result that has crowned this series of works by L. V. Keldysh, her example of a light open mapping of the Menger curve onto the square [12].

It was A. N. Kolmogorov who constructed the first unexpected example of dimension raising by light open mappings [13]: the 2-adic group acts on the Menger curve and the orbit space of the action is the Pontrjagin two-dimensional surface (which breaks the sum theorem in dimension theory). The dimensional insufficiency of this continuum left doubts but there were new examples of open mappings of a one-dimensional continuum, whose range was the square. In these examples, mappings had the form of compositions of a monotone but not light open mapping and a light open mapping that does not raise dimension. So, the dimension raising was not due to a light open mapping.

Keldysh's example from [12], one of the classics of geometric topology, shows visually the dimension raising mechanism accompanied by openness in the concrete case of the mapping of a one-dimensional continuum (identified with the Menger curve by R. D. Anderson's criterium [17]) onto the square. Once, A. N. Kolmogorov said to A. B. Sossinsky that he had given up on the topic when he realized that he was unable to compete with L. V. Keldysh. Later B. A. Pasynkov developed a nice technique, with the help of which it became possible, using Keldysh's example as a starting point, to build a light open mapping of the Menger curve even onto the Hilbert cube [14].

Other people later built simpler examples, however these examples seem to reveal to a lesser degree the dimension raising mechanism [15, 16]. D. J. Wilson shows in [19] that for every locally connected continuum one can build an open mapping of the Menger curve onto it in such a way that every point preimage is homeomorphic to the Cantor set.

The study of light open mappings has significance, from the point of view of an open classical Hilbert–Smith problem, to the possibility of an effective action of a p-adic group on manifolds. Such an action would give a light open mapping onto orbit space that necessarily raises the dimension by two or more, as was shown by Yang [18]. It would be rather exotic; for example, if this action is free and its factor-space is locally simply connected the mapping is a locally trivial fibering and its space certainly cannot be any manifold. Light open mappings of *manifolds* with dimension three and more that are not finite-to-one were built by D. Wilson [19] and J. Walsh [21] in 1972–76. The technique used by them is very similar to the technique of the next series of Keldysh's works, which concern monotone open mappings.

Dimension raising. Monotone mappings.

It was well-known that monotone mappings do not raise dimension of surfaces. R. D. Anderson in 1952 gave an example of a monotone-open mapping of a one-dimensional continuum onto the Hilbert cube.

It was much more difficult to build a monotone mapping of *a cube onto a cube of higher dimension*. L. V. Keldysh constructed in 1955 [22] such a mapping of the 3-cube onto the 4-cube that easily generalises to a monotone mapping of any cube of dimension more than 2 onto any other one, including the Hilbert cube. It was essential in her examples that the mapping was irreducible (in this case, the set E of one-point preimages of points is an everywhere dense G_δ and the mapping is homeomorphic on it).

The next year, she showed [23] that if the set E just mentioned is arcwise connected for a given mapping $f\colon K \to M$ of continuum K onto manifold M of dimension more than 2, then one can transform f into a *monotone open* mapping $K \to M$ with the help of an arbitrary small shift of M onto itself (by a pseudoisotopy, see below). So, she obtained the basic result: it is possible to map openly and monotonically the 3-cube onto a cube of any dimension.

She made the following remark, which emphasizes the geometric core in her construction of a monotone mapping of 3-cubes onto 4-cubes: it turns out that it may be obtained as a limit of ε-dense embeddings with the property that any path in the 4-cube may be δ-shifted into the image of the 3-cube (and $\delta \to 0$ as $\varepsilon \to 0$).

Departing from this remark, A. V. Chernavsky built mappings of cubes onto cubes of higher dimension [24], which generalizes Keldysh's example in the following way. For every k and for p such that $2p+3 \leq k$, there exists a mapping of the k-dimensional cube onto cubes of any higher dimension including the Hilbert cube, with point preimages acyclic up to dimension p. This mapping is the limit of embeddings of the k-cube with conditions that generalize the conditions of Keldysh's example. Keldysh's method transforms this mapping by a pseudoisotopy into a monotone open one. This result is exact because of R. L. Frum-Ketkov's theorem that when a mapping of a k-cube onto polyhedrons of higher dimension has preimages of points acyclic up to dimension p then $k \geq 2p+3$ [25].

Geometric topology of manifolds. L. V. Keldysh's seminar

This seminar began in 1956–57 as a spin-off of the seminar at MSU headed by P. S. Alexandrov. The assiduous participants of it were (in order of participation) A. V. Chernavsky, M. A. Stan'ko, A. B. Sossinsky and E. V. Sandrakova. But there were also doctoral students from other cities (M. S. Farber from Bacu, V. P. Kompaniec from Kiev, S. V. Matveev from Cheljabinsk, Van ny Kyong from Viet-Nam, etc.) and distinguished visitors from abroad (M. H.A. Newman, E. C. Zeeman, O. G. Harrold, J. Kister, D. Henderson, etc.). Especially influential was a visit by R. H. Bing.

The term "geometric topology", coined by M. H.A. Newman at the 1954 Congress, refers to topology in which algebraic or analytic tools play a secondary role or in which topological properties of manifolds are the central issue. In the 1963 list of the most important problems of geometric topology, J. W. Milnor included: the Poincaré conjecture in dimensions three and four, triangulability and Hauptvermutung for manifolds, the topological invariance of Pontrjagin classes and of simple homotopy type, the annulus conjecture and, finally, the double suspension conjecture. Today they are all to a large extent solved, but at the time they held centre-stage both in L. V. Keldysh's seminar and worldwide. For some of these conjectures, ideas introduced by participants of L. V. Keldysh's seminar exerted an essential influence. She gave to this seminar all her energy, meticulously checking the results of her disciples and lending them support of any kind. [30]

The work of L. V. from this period is connected, in one way or another, with the notion *pseudoisotopy*, that is a one-parameter family of mappings $g_t : \mathbb{R}^n \to \mathbb{R}^n$ with g_t homeomorphisms for $t < 1$. In her last articles, she developed an approach to study topological embeddings through constructions of pseudoisotopies which transform standard embeddings into the given ones (see [29]). M. A. Stan'ko happily inverted this idea; he introduced the notion *embedding dimension* as the minimal dimension of a polyhedron onto which one may shift the given compact space by a small pseudoisotopy. This notion turned out to be central in the future development of geometric topology.

The seminar of L. V. Keldysh ceased in 1974. At the close of the 60's, events in Russian political life had impinged on normal scientific life; as already mentioned, P. S. Novikov and L. V. Keldysh had supported dissidents such as Essenin-Volpin. This led to conflicts, which considerably shortened the lives of these two remarkable people. In 1973, L. V. took

Disciples of L. V. Keldysh, from left to right: V. P. Kompaniets, M. A. Stan'ko, A. V. Chernavsky, E. V. Sandrakova, A. B. Sossinsky

her retirement in protest against the dismissal of her collaborator, which she took to be unjust. Then Petr Sergeevich Novikov died in 1974. In February 1976, Ljudmila Vsevolodovna Keldysh also passed away. Just one week before her death, freshly returned from hospital after major surgery, she came to the Steklov Institute for the traditional meeting with her students.

The author is very grateful to members of L. V. Keldysh's family for acquaintance with their family history and permission to use it in this outline. Also used with thanks are recollections of L. V. Keldysh's students L. V. Sandrakova, A. B. Sossinsky and M. A. Stanko. Many thanks again go to A. B. Sossinsky and L. C. Siebenmann for their efforts to adapt this text, written by a Russian author, for the understanding of European readers.

Notes

1. An open mapping is a mapping that sends open subsets onto open subsets
2. An irreducible mapping takes no proper closed subspace onto the whole image (or, in another way: the subset of unicity points $\{x = f^{-1}fx\}$ is everywhere dense).

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