Erratum to *Topology of Homology Manifolds* by Bryant, Ferry, Mio, and Weinberger

On page 457 of [BFMW] there is a mistake between the first and second lines of the second paragraph. As a result, some of the results in the paper are not correctly proved in the generality asserted. In

more detail, our construction of homology manifolds is as a limit of an infinite sequence of polyhedral approximations, each of which is produced via controlled surgery as a union of two manifolds with boundary via a controlled homotopy equivalence of their boundaries. In order to get started, we needed to have a degree one normal map from a manifold to X (see e.g. [Br, Wa]), the underlying homotopy type of the homology manifold to be constructed.

We asserted that such a normal map exists whenever the nonconnective total surgery obstruction for X vanishes. However, this only follows from the vanishing of the connective total surgery obstruction [Ra, KMM]. At the prime 2, the connective theory is a factor of the nonconnective theory and there is no problem; however at odd primes, the issue is equivalent to whether X has a KO[1/2]-orientation (or equivalently an $\mathbb{L}^*(\mathbb{Z})$-orientation) according to Sullivan [Su] ([Ra]). Whenever the orientation class of X is Steenrod orientable, i.e. there exist a manifold M and a degree one map $M \to X$ (even without normal data), the push forward of the controlled symmetric signature class of M gives such a class for X and there is no issue.

In particular, in dimension 6, René Thom [Th] shows that all homology classes are the images of closed manifolds, and thus, our results are true as stated. Further, all of the examples in the paper (e.g. the simply connected case and the homology manifold not homotopy equivalent to a manifold) are unchanged. The classification of homology manifolds homotopy equivalent to a manifold, and the periodicity of the structure sets is also unchanged.

We do not know whether all homology manifolds possess degree one (normal) maps from manifolds. This was a theorem in [FP], but it suffers from the same difficulty as [BFMW].

The changes to the results of the paper are two-fold: the calculation of bordism of homology manifolds is now out of reach, and the main theorem requires the orientability hypothesis (e.g. the Steenrod representability of the fundamental class of X). The proofs of the theorems are unchanged, except that one starts from the given normal invariant which is lifted to $H_x(X; \mathbb{L})$ (as the total surgery obstruction vanishes) and one constantly modifies patch structures produced from the normal invariant by using Wall realizations of this element of the controlled L-group $L_c(X \downarrow X) \cong H_x(X; \mathbb{L})$ rather than expressing this process in terms of maps into G/Top.

We remark that in the important special case of $X = K(\pi, 1) = Z/\pi$ with Z contractible, where the fundamental group π is a word hyperbolic group whose boundary is a sphere (the high-dimensional Cannon conjecture, studied in [BLW, FLW]), X does have a normal invariant, given explicitly by considering $Z \times_\pi \partial Z \to X$ as a reduction of the negative of the Spivak fibration over X.

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