SIMC Youth Race

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Steklov Mathematical Institute of RAS,
conference hall, 9 floor

Holographic Anisotropic Model for Heavy Quarks in Anisotropic Hot Dense QGP: Magnetic Catalysis

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My collaborators:

Holographic Anisotropic Model for Heavy Quarks
in Anisotropic Hot Dense QGP: Magnetic Catalysis

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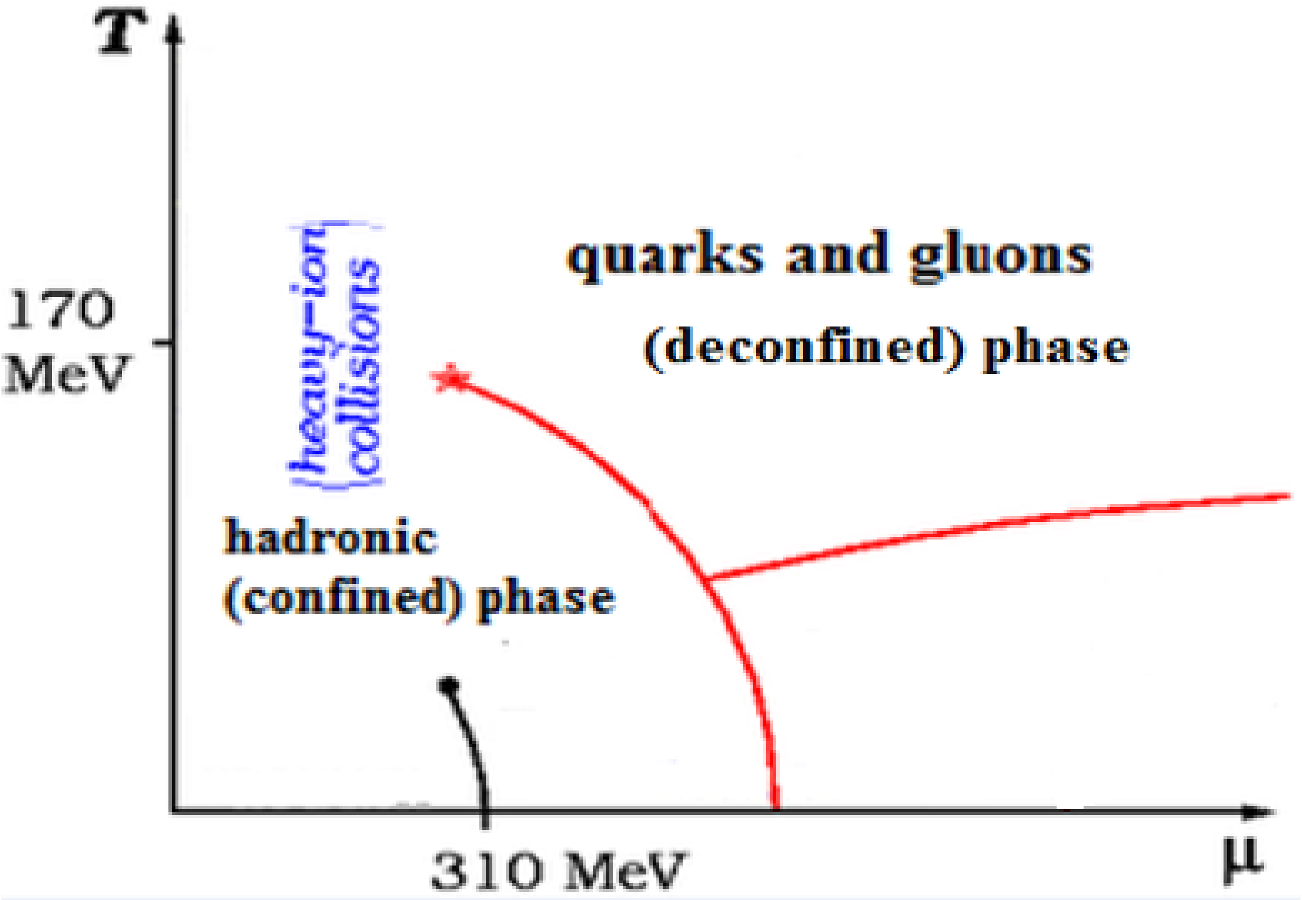
Work is in progress.

Outline:

- Introduction
- Set up a Question?
- Approach: AdS/CFT or Gauge/Gravity Duality
- Results
- Summary

Introduction: (QCD phase diagram)

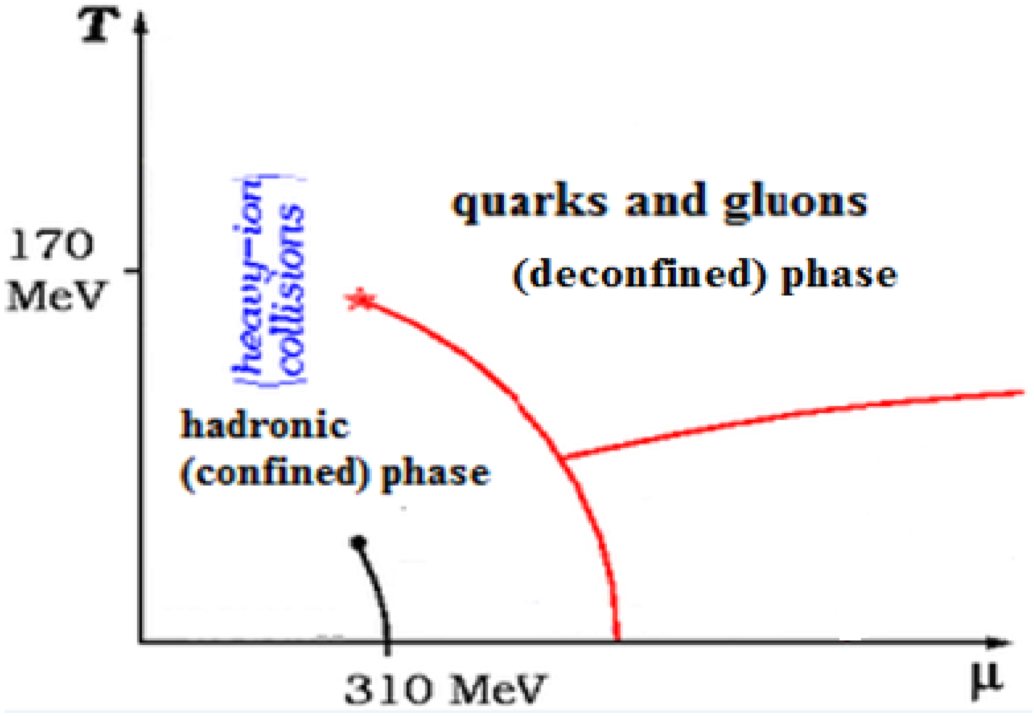


One of the most important open problems in the study of QCD at finite temperature and density is the determination of the phase diagram of the theory.

Introduction: (Heavy Ion collision)

RHIC
Au-Au collision
2Tc

LHC
Pb-Pb collision
5Tc



$T_c \sim 170$ MeV

Experimental results show that:
QGP is strongly coupled

$$\frac{\eta}{s} \sim 1/4\pi$$

Nucl. Phys. A 750, 64 (2005)

Introduction:

Heavy ion collisions (HIC):

Can teach us about properties of the high temperature phase of QCD.

Noncentral relativistic HIC → Anisotropic Plasma

There is a very strong magnetic field, in the early stages of relativistic heavy ion collision.

Set up a Question:

What is the effect of the magnetic field on the phase transition temperature? (confinement/deconfinement)

Two phenomenon:

1- Inverse Magnetic Catalysis (IMC) (Mamo. ‘15, Aref’eva et al. ‘20 , Dudal et al. ‘19, ….)

2- Magnetic Catalysis (MC) …….??? (Gusynin et al. ‘94, He et al. ‘20, …)

2nd Question:

What is the effect of the anisotropy on the phase (confinement/deconfinement) transition temperature?

Noncentral relativistic HIC $\Rightarrow$ Anisotropic quark gluon plasma

Approach:

QGP as a phase of QCD is strongly coupled.

So, we use Non-Perturbative approach:

AdS/CFT duality (That is an example of holographic principle)

Adv. Theor. Math. Phys. 2 (1998)

AdS/CFT correspondence: (Maldacena)

IIB String Theory on $\mathrm{AdS}_5 \times S^5 \Leftrightarrow \mathcal{N} = 4\,,\mathrm{d} = 4$ SYM theory

Interesting limit:

Classical gravity ⟺ Strongly coupled QFT

$$Gravity \Leftrightarrow \mathrm{Gauge}$$

Adv. Theor. Math. Phys. 2 (1998)

Methods:

Top-down models: Directly constructed from string theory:

D3-D7 model

J. Babington, J. Erdmenger, N. J. Evans, Z. Guralnik , I. Kirsch,
M. Kruczenski, D. Mateos, R. C. Myers and D. J. Winters,…

D4-D8 model

T. Sakai and S. Sugimoto

Bottom-up models: (phenomenological)

Introduce a dilaton field

J. Erlich, E. Katz, D. T. Son and M. A. Stephanov,
A. Karch, B. Batell and T. Gherghetta, U. Gursoy, E. Kiritsis,…

Duality:

Gravity	⟺	Gauge
Anti-de Sitter Space (AdS)	⟷	Vacuum state
Black hole	⟷	Thermal state
Scalar field	⟷	Scalar operator

JHEP 02 (2000) 039 and JHEP 07, 086 (2014)

Our 1st Question:

What is the effect of the magnetic field on the phase transition temperature? (confinement/deconfinement)

How can we set up a holographic approach?

Our Model:

$$\mathcal{L} = \sqrt{-g}\left[R - \frac{f_0(\phi)}{4}F_0^2 - \frac{f_1(\phi)}{4}F_1^2 - \frac{f_3(\phi)}{4}F_3^2 - \frac{1}{2}\partial_\mu\phi\,\partial^\mu\phi - V(\phi)\right]$$

$$\phi = \phi(z),$$

$$\text{Electric anzats } F_0: \quad A_0 = A_t(z), \quad A_{i=1,2,3,4} = 0,$$

$$\text{Magnetic anzats } F_k: \quad F_1 = q_1\, dx^2 \wedge dx^3, \quad F_3 = q_3\, dx^1 \wedge dx^2.$$

F_0 $\longleftrightarrow$ Chemical potential

F_1 $\longleftrightarrow$ Primary anisotropy

F_3 $\longleftrightarrow$ Magnetic field

JHEP 07, 161 (2021)

Our ansatz for the metric:

$$ds^2 = \frac{L^2}{z^2}\mathfrak{b}(z)\left[-g(z)\,dt^2 + dx_1^2 + \left(\frac{z}{L}\right)^{2-\frac{2}{\nu}} dx_2^2 + e^{c_B z^2}\left(\frac{z}{L}\right)^{2-\frac{2}{\nu}} dx_3^2 + \frac{dz^2}{g(z)}\right]$$

$$\mathfrak{b}(z) = e^{2A(z)}$$

Warp factor

isotropic $\nu = 1$

Anisotropic $\nu = 4.5$

Varying the Lagrangian with the ansatz of metric:

Equations of motion:

$$A_t'' + A_t' \left(\frac{\mathfrak{b}'}{2\mathfrak{b}} + \frac{f_0'}{f_0} + \frac{\nu - 2}{\nu z} + c_B z \right) = 0$$

$$\mathrm{g}'' + \mathrm{g}' \left(\frac{3\mathfrak{b}'}{2\mathfrak{b}} - \frac{\nu + 2}{\nu \mathrm{z}} + \mathrm{c_B z} \right) - \left(\frac{\mathrm{z}}{\mathrm{L}} \right)^2 \frac{\mathrm{f_0} \left(\mathrm{A_t'} \right)^2}{\mathfrak{b}} - \left(\frac{\mathrm{z}}{\mathrm{L}} \right)^{\frac{2}{\nu}} \frac{\mathrm{q_3^2 f_3}}{\mathfrak{b}} = 0$$

$$\mathfrak{b}'' - \frac{3(\mathfrak{b}')^2}{2\mathfrak{b}} + \frac{2\mathfrak{b}'}{\mathrm{z}} - \frac{4\mathfrak{b}}{3\nu \mathrm{z}^2} \left(1 - \frac{1}{\nu} + \left(1 - \frac{3\nu}{2} \right) \mathrm{c_B z^2} - \frac{\nu \mathrm{c_B^2 z^4}}{2} \right) + \frac{\mathfrak{b} \left(\phi' \right)^2}{3} = 0$$

$$2g' \left(1 - \frac{1}{\nu} \right) + 3g \left(1 - \frac{1}{\nu} \right) \left(\frac{\mathfrak{b}'}{\mathfrak{b}} - \frac{4\left(\nu + 1\right)}{3\nu z} + \frac{2 c_B z}{3} \right) + \left(\frac{L}{z} \right)^{1 - \frac{4}{\nu}} \frac{L\, e^{-c_B z^2} q_1^2\, f_1}{\mathfrak{b}} = 0$$

Solving EOMs:

1st gauge field:

$$A_t'' + A_t'\left(\frac{\mathfrak{b}'}{2\mathfrak{b}} + \frac{f_0'}{f_0} + \frac{\nu-2}{\nu z} + c_B z\right) = 0$$

Blackening function:

$$g'' + g'\left(\frac{3\mathfrak{b}'}{2\mathfrak{b}} - \frac{\nu+2}{\nu z} + c_B z\right) - \left(\frac{z}{L}\right)^2 \frac{f_0 (A_t')^2}{\mathfrak{b}} - \left(\frac{z}{L}\right)^{\frac{2}{\nu}} \frac{q_3^2 f_3}{\mathfrak{b}} = 0$$

$$f_3 = 2\left(\frac{L}{z}\right)^{\frac{2}{\nu}} \mathfrak{b} g \, \frac{c_B z}{q_3^2}\left(\frac{g'}{g} + \frac{3\mathfrak{b}'}{2\mathfrak{b}} - \frac{2}{\nu z} + c_B z\right)$$

$$g'' + g'\left(\frac{3\mathfrak{b}'}{2\mathfrak{b}} - \frac{\nu+2}{\nu z} - c_B z\right) - 2g\left(\frac{3\mathfrak{b}'}{2\mathfrak{b}} - \frac{2}{\nu z} + c_B z\right) c_B z - \left(\frac{z}{L}\right)^2 \frac{f_0 (A_t')^2}{\mathfrak{b}} = 0$$

Magnetic catalysis for heavy quarks:

Warp factor:

$$\mathfrak{b}(z) = e^{2\mathcal{A}(z)} = e^{-cz^2/2 - 2pz^4}$$

Gauge coupling function:

$$f_0 = e^{-(R_{gg} + \frac{c_B q_3}{2})z^2} \frac{z^{-2+\frac{2}{\nu}}}{\sqrt{\mathfrak{b}}}$$

2004.01965 [hep-th]]

Boundary conditions:

1st gauge field: $A_t(0) = \mu, \quad A_t(z_h) = 0$

Blackening function: $g(0) = 1, \quad g(z_h) = 0$

Dilaton field: $\phi(z_0) = 0$

In search of Magnetic Catalysis: (MC)

We need to find: g(z)

Temperature and Entropy:

$$T = \frac{\sqrt{g_{tt}{}' \, g^{zz}{}'}}{4\pi}\Big|_{z=z_h} = \frac{\sqrt{g_{00}{}' \, g^{55}{}'}}{4\pi}\Big|_{z=z_h} = \frac{|g'|}{4\pi}\Big|_{z=z_h}$$

$$s = \frac{\sqrt{g_{xx}\, g_{y_1y_1}\, g_{y_2y_2}}}{4}\Big|_{z=z_h} = \frac{\sqrt{g_{11}\, g_{22}\, g_{33}}}{4}\Big|_{z=z_h}$$

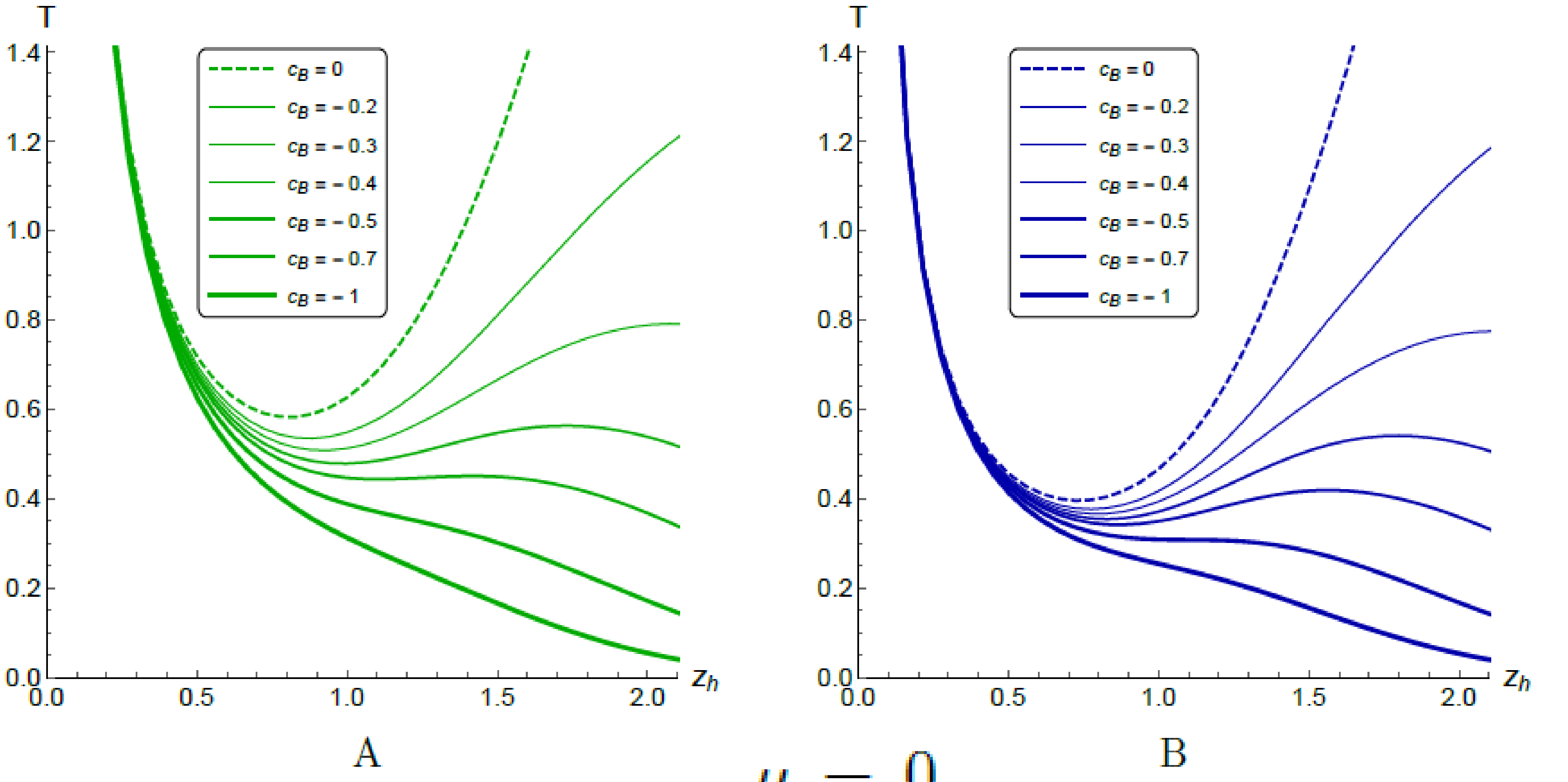
$$s = \frac{1}{4}\left(\frac{L}{z_h}\right)^{1+\frac{2}{\nu}} e^{-(2R_{gg}-c_B)\frac{z_h^2}{2}-3pz_h^4}$$

Free energy:

$$F = -\int s\, dT = \int_{z_h}^{\infty} s\, T' dz.$$

Temperature:

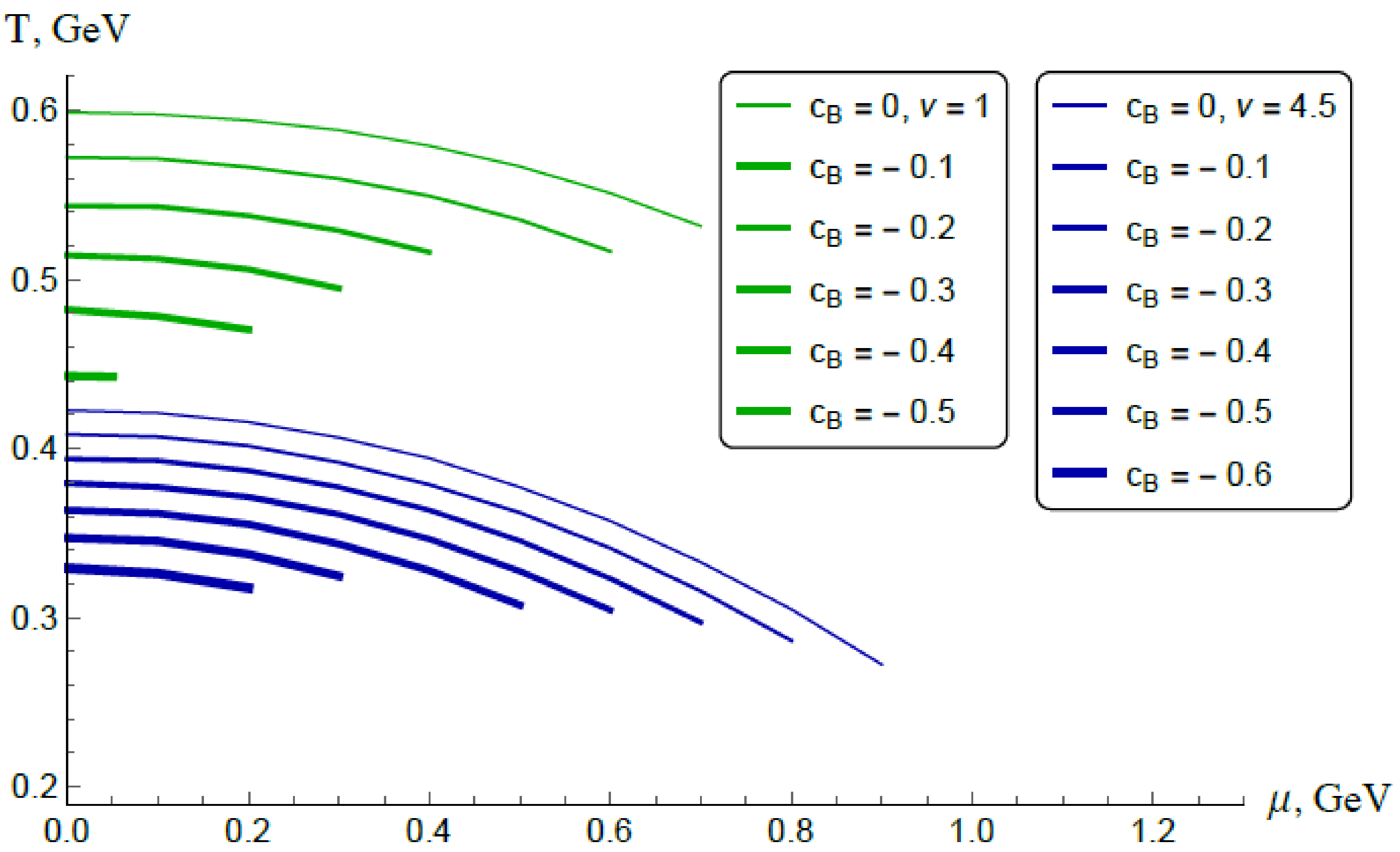
Warp factor: $\mathfrak{b}(z) = e^{2\mathcal{A}(z)} = e^{-cz^2/2 - 2pz^4}$



$\mu = 0$

IMC:

Warp factor: $\mathfrak{b}(z) = e^{2\mathcal{A}(z)} = e^{-cz^2/2 - 2pz^4}$



Warp factor: $\mathrm{b}(z) = e^{2A(z)} = e^{-cz^2/2-2pz^4}$

NO MC phenomenon was observed!!!!

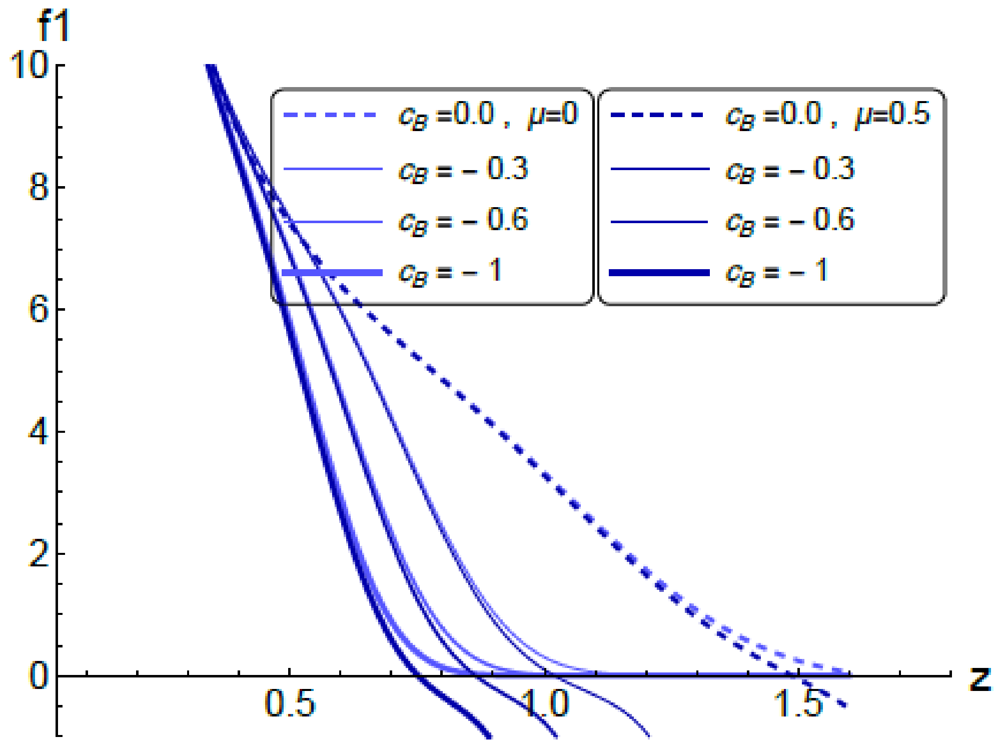
New Warp factor: $\mathfrak{b}(z) = e^{2A(z)} = e^{-cz^2/2 - 2(p - c_B q_3)z^4}$

$$ds^2 = \frac{L^2}{z^2}\mathfrak{b}(z)\left[-g(z)\,dt^2 + dx_1^2 + \left(\frac{z}{L}\right)^{2-\frac{2}{\nu}} dx_2^2 + e^{c_B z^2}\left(\frac{z}{L}\right)^{2-\frac{2}{\nu}} dx_3^2 + \frac{dz^2}{g(z)}\right]$$

Gauge coupling function: $f_0 = e^{-(R_{gg}+\frac{c_B q_3}{2})z^2}\,\frac{z^{-2+\frac{2}{\nu}}}{\sqrt{\mathfrak{b}}}$

Our model can possess the Linear Regge trajectory for meson mass spectra.

2^{nd} coupling function:



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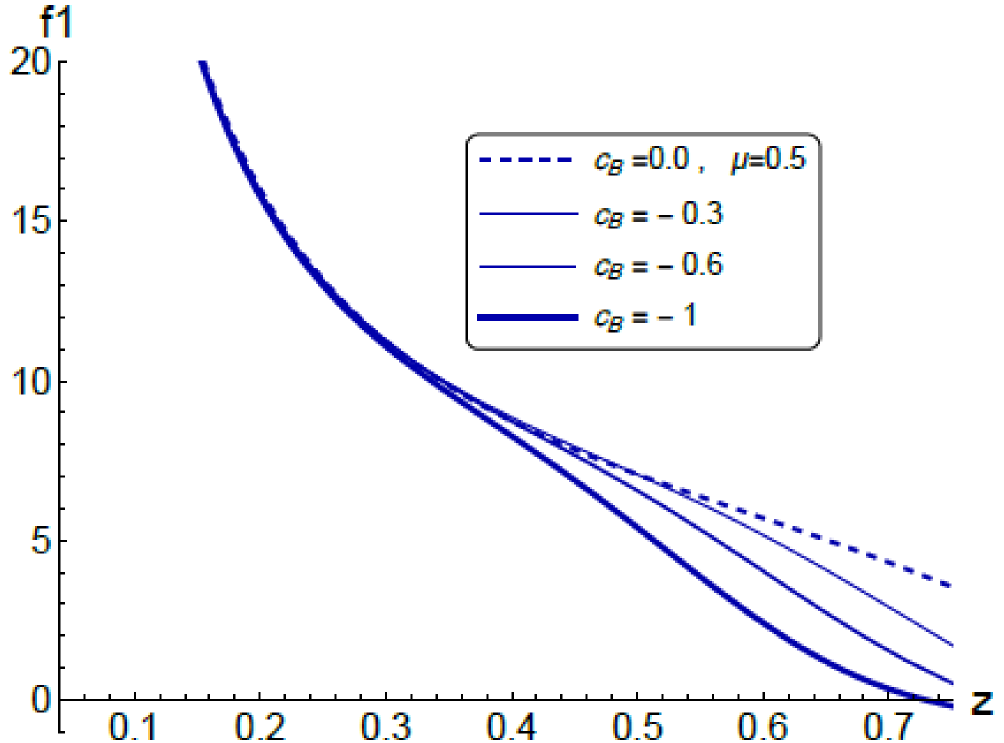
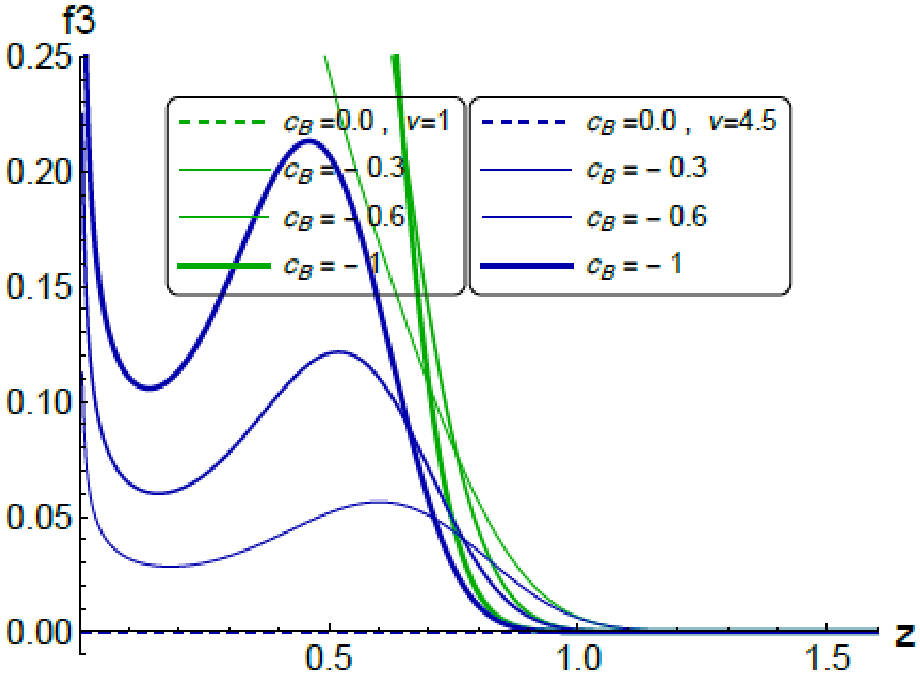


B

The Null Energy Condition (NEC) should be satisfied by choosing the appropriate 2^{nd} horizon.

3rd coupling function:



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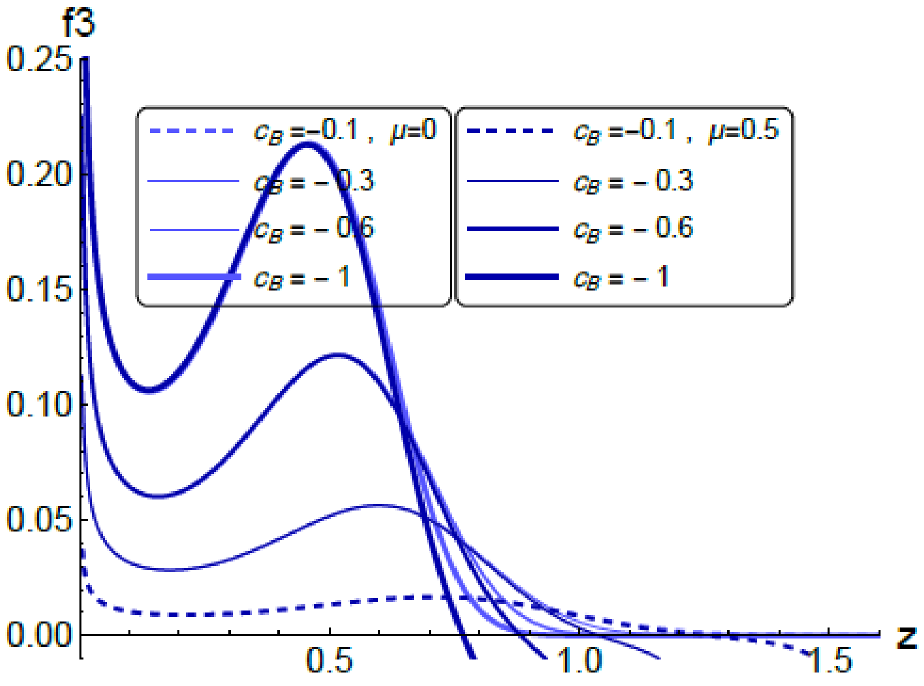
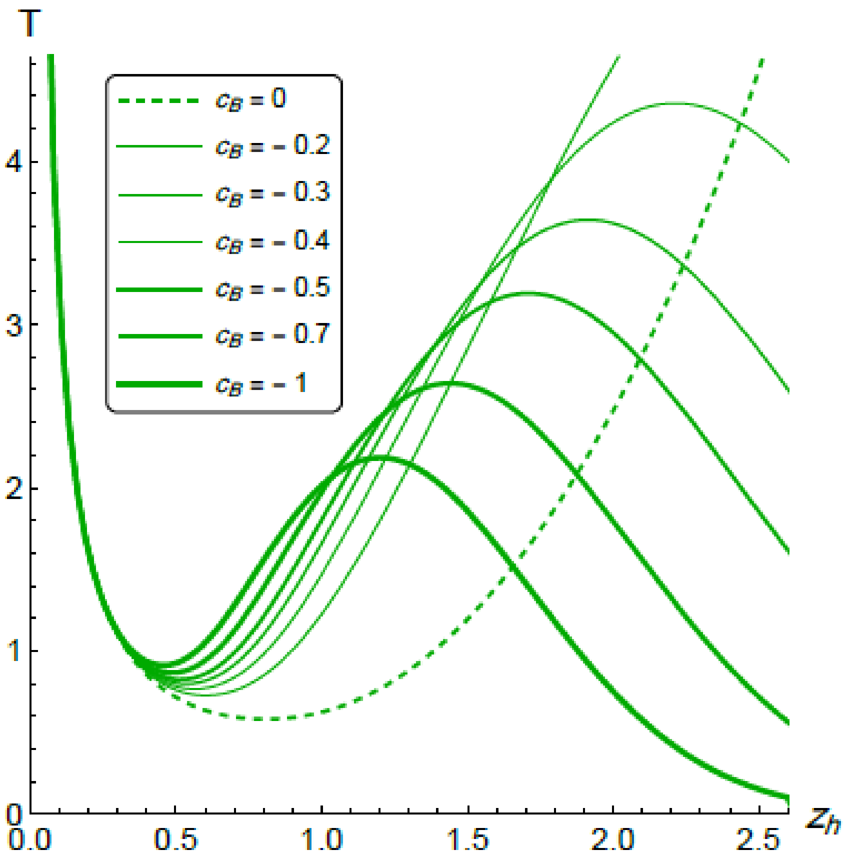


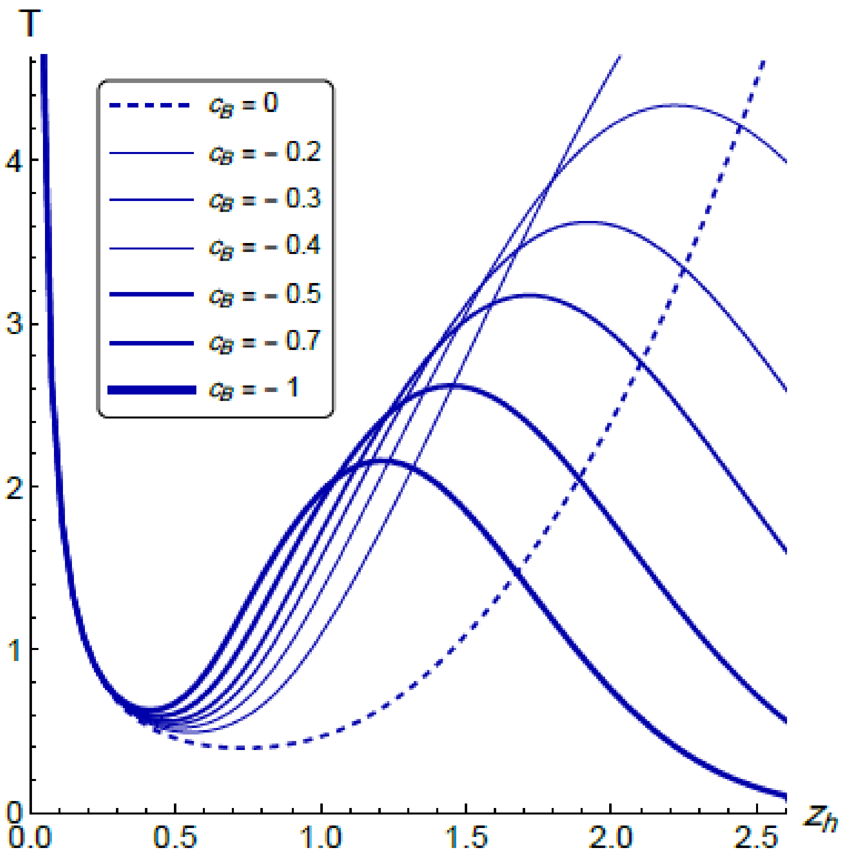
B

The Null Energy Condition (NEC) should be satisfied by choosing the appropriate 2nd horizon.

Temperature:

$$\mathfrak{b}(z) = e^{2A(z)} = e^{-cz^2/2 - 2(p - c_B\, q_3)z^4}$$

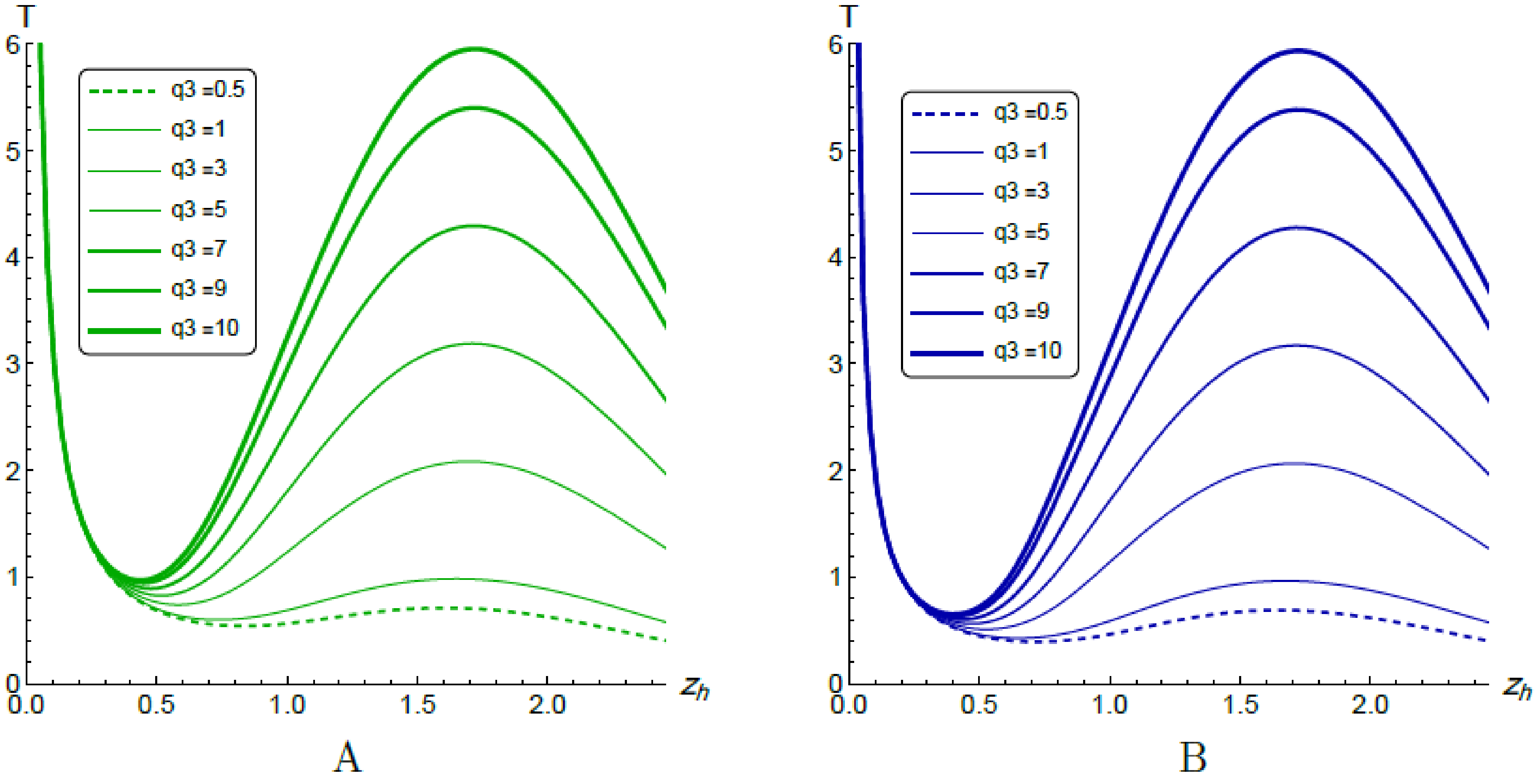




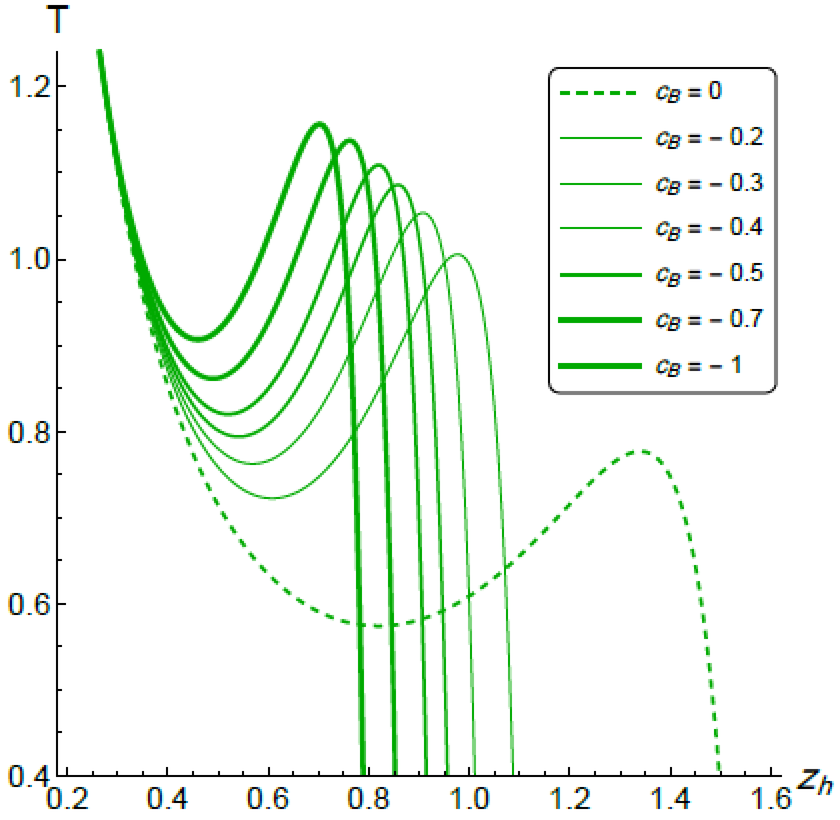
$$q_3 = 5$$

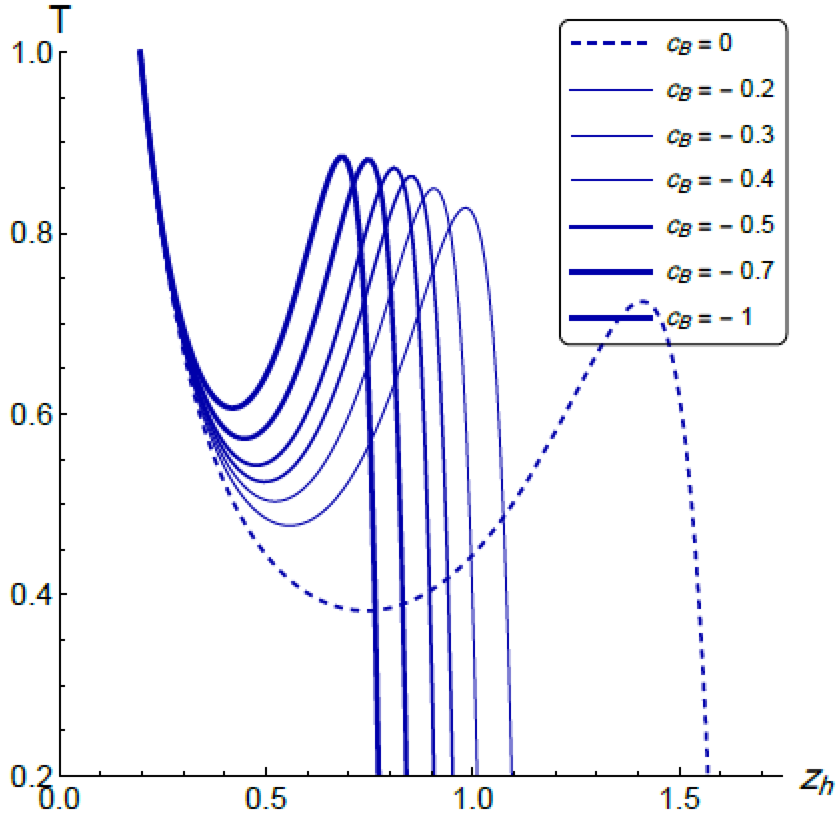
$$\mu = 0$$

Temperature: $\mu = 0$ $c_B = -0.5$

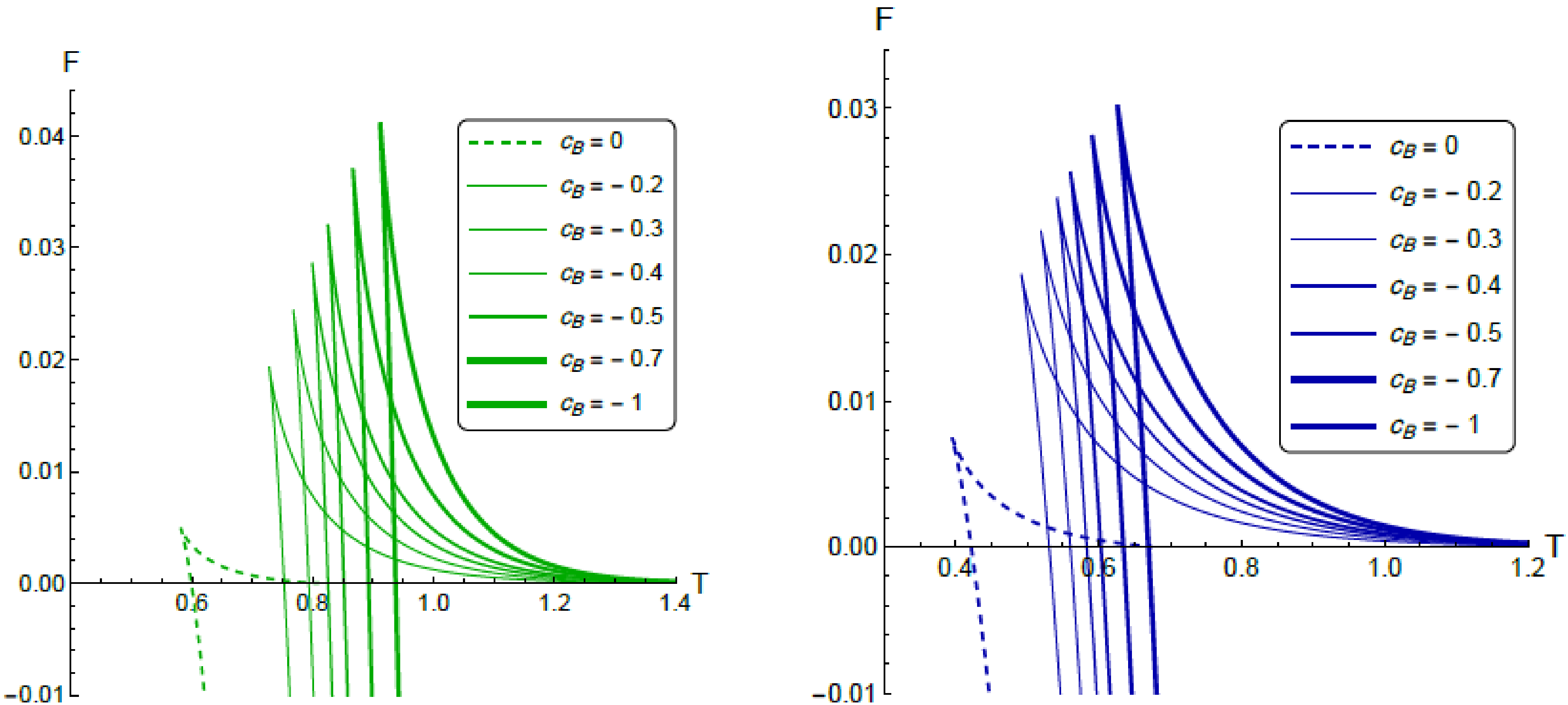


Temperature: $\mu = 0.3$ $q_3 = 5$

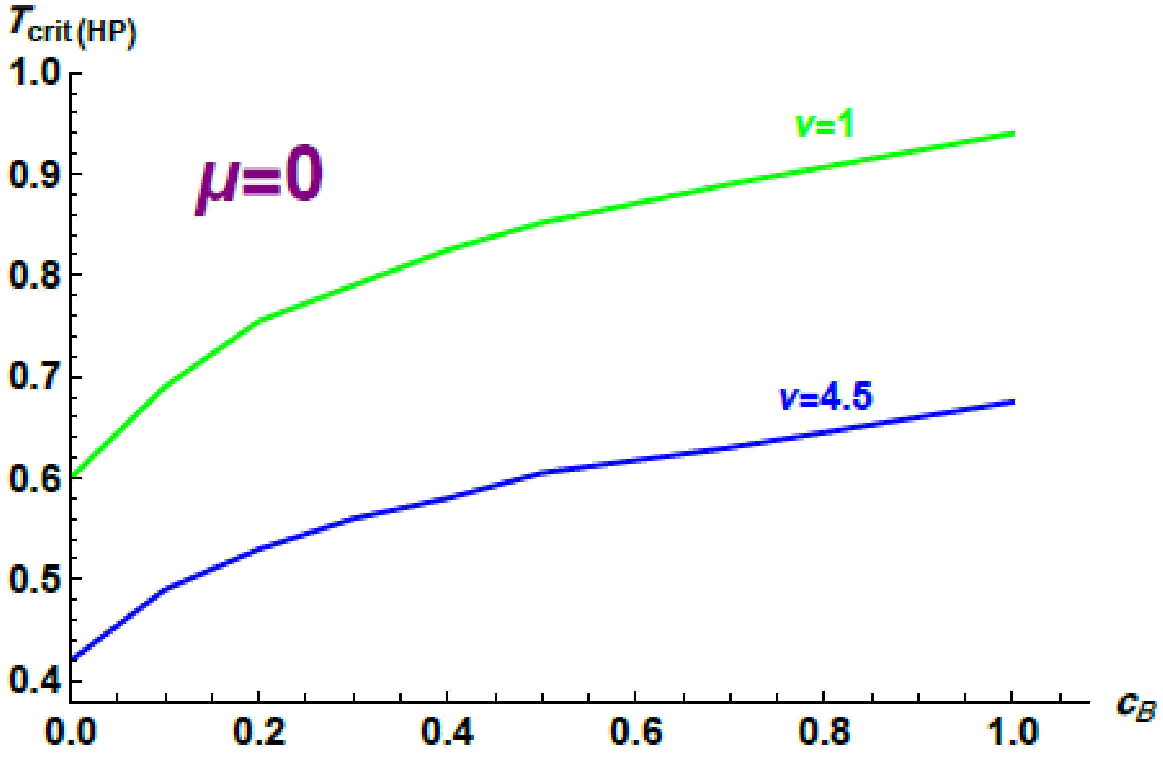




Free energy: $\mu = 0 \quad q_3 = 5$



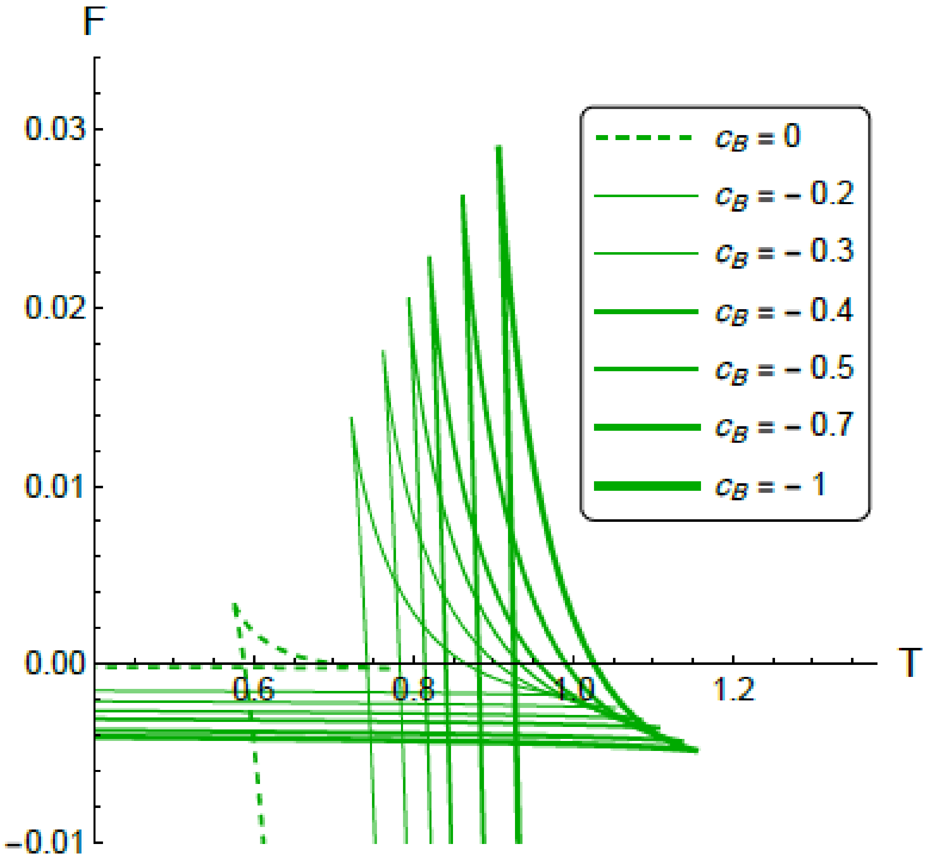
Phase diagram:

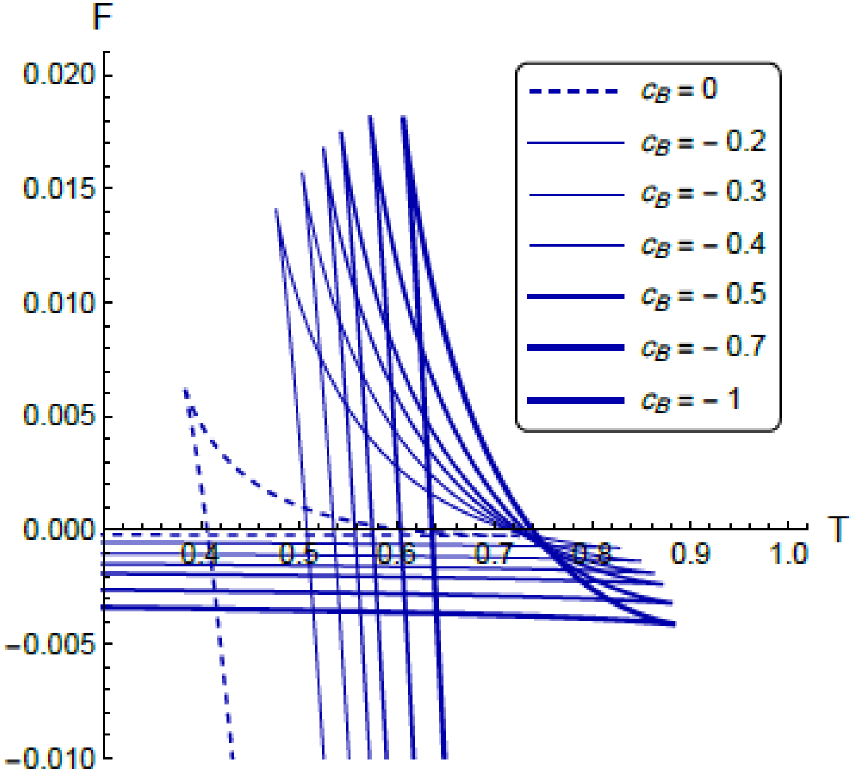


$$q_3 = 5$$

MC phenomenon is obtained.

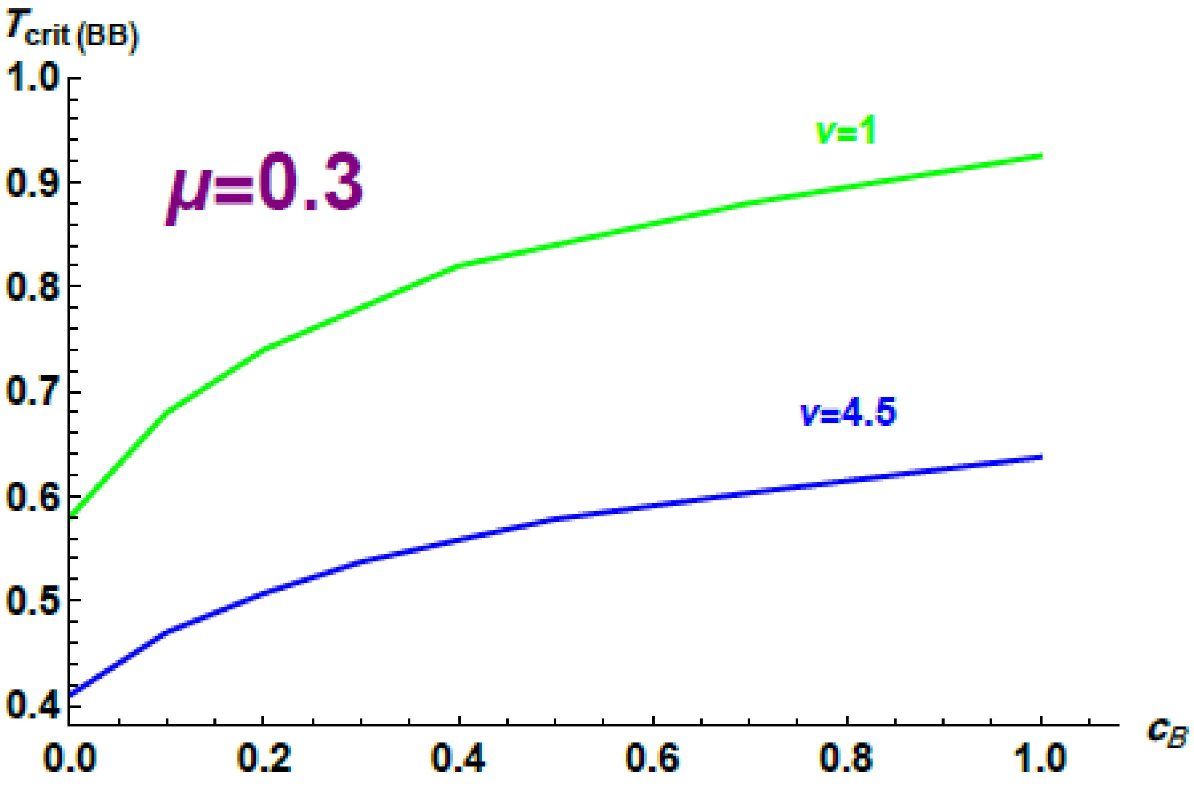
Free energy:





$q_3 = 5$

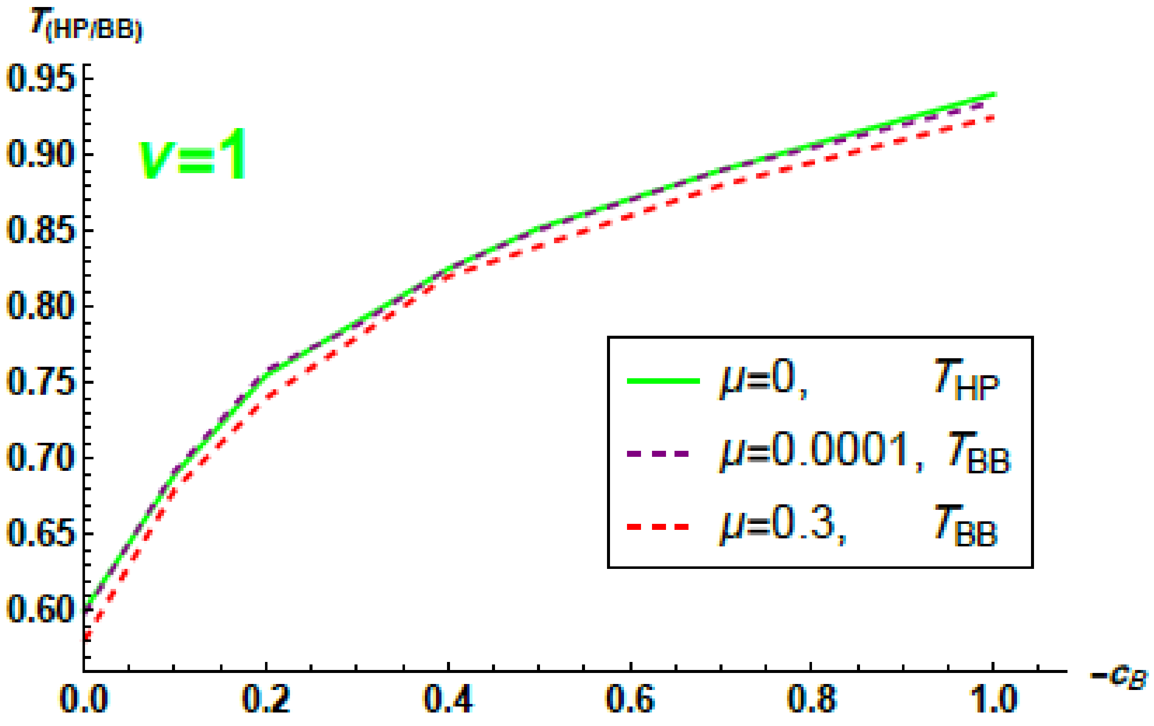
Phase diagram:



$$q_3 = 5$$

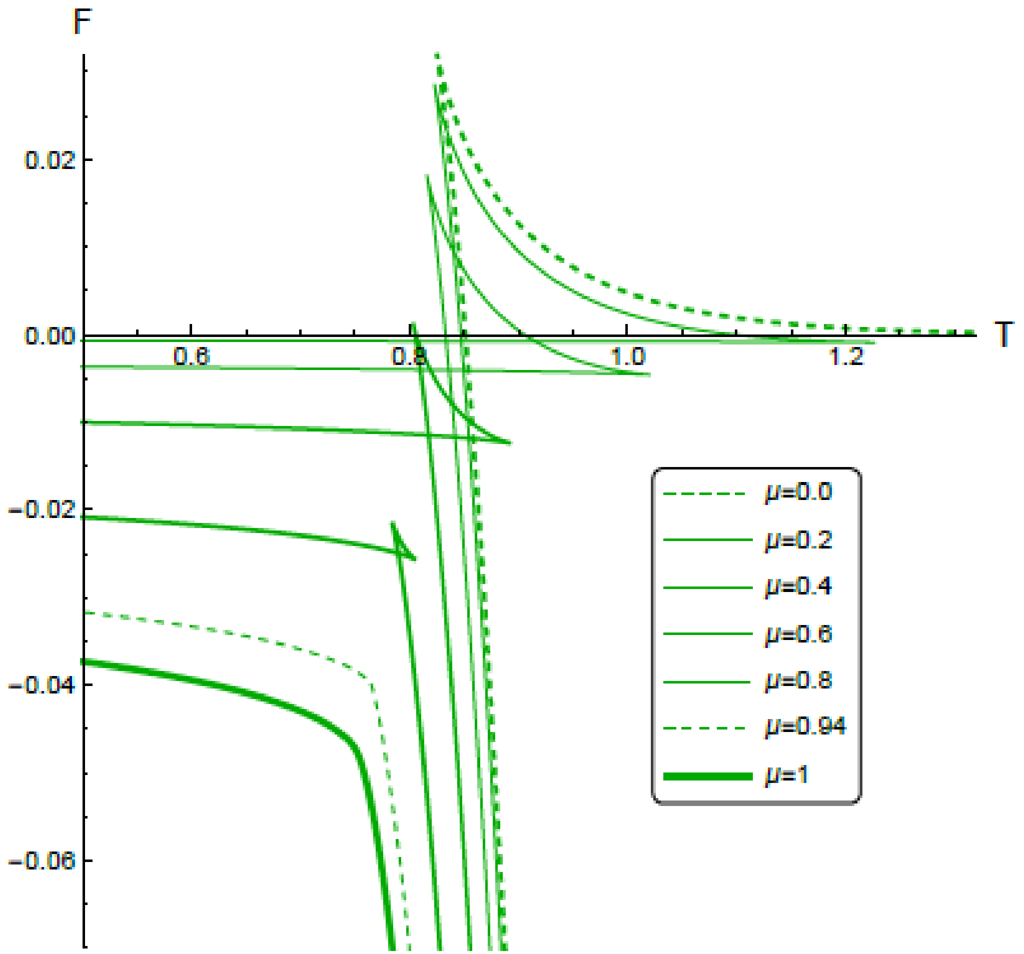
MC phenomenon is obtained.

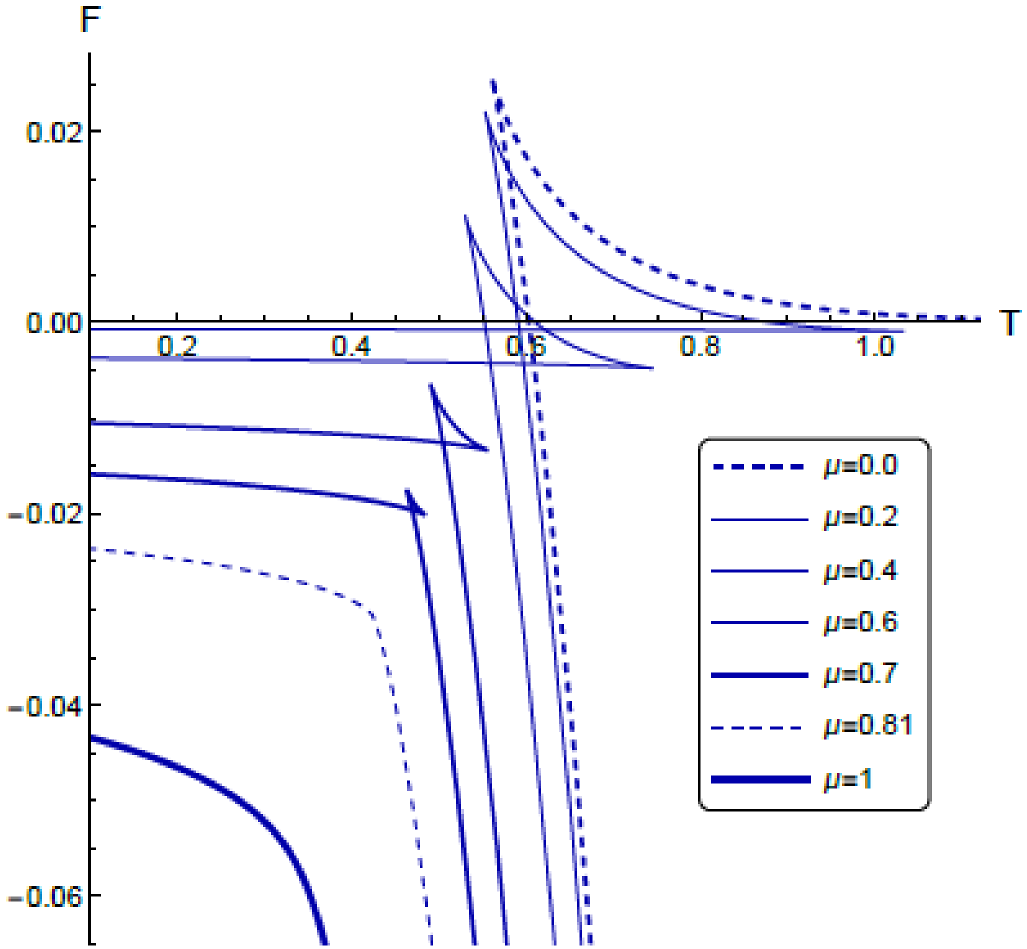
Phase diagram when: $\mu \to 0$



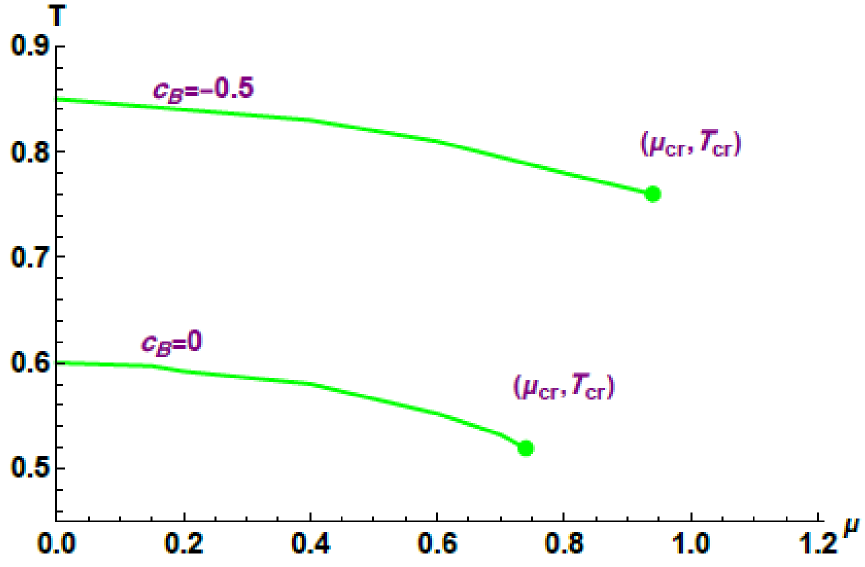
$q_3 = 5$

Free energy: $c_B = -0.5$ $q_3 = 5$





Phase diagram for different cases of anisotropy:



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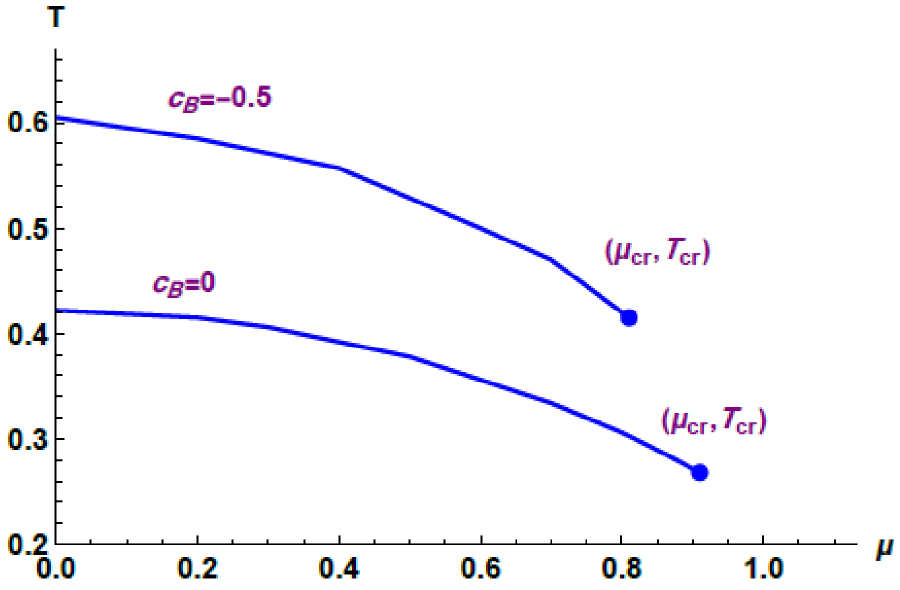

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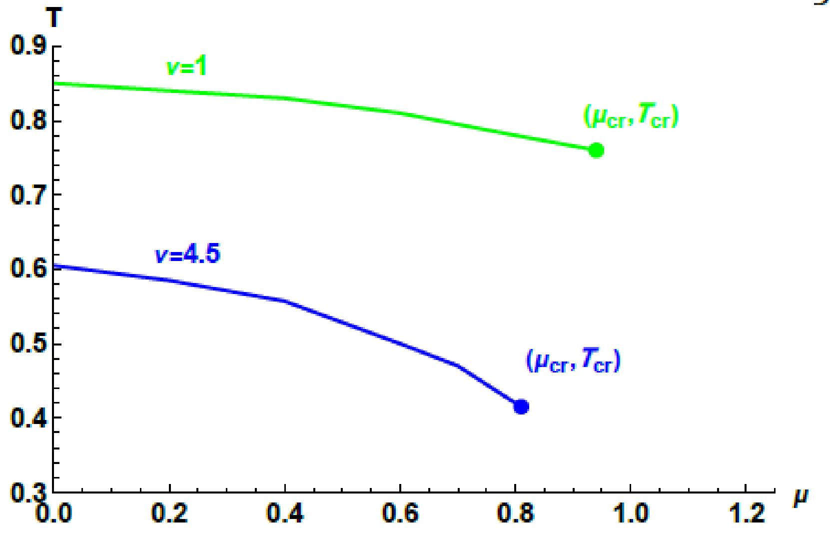

C

Summary:

- We developed the model to possess MC phenomenon.

- Increasing the primary anisotropy decreases the confinement/deconfinement and black hole black hole phase transition.

- Although, increasing anisotropy via magnetic field increases the critical transition temperatures.

Works in progress:

- Renormalization group flow….. (MIAN)

- Unquenched set up for quarks ……(KU Leuven and NIT India)

Thanks…