Operator local quenches in massive scalar field theory

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OUTLINE

- Introduction
- CFT_2 methods
- Simplest free field theories
  - Two-dimensional flat space
  - Living on a cylinder
- Massless theory in $d > 2$ dimensions
- Adding mass
- Applications and future prospects

Introduction

Definitions

Quantum quench is a sudden change in the system that produces a time-dependent excited state.

There are two classes of quenches: **global** and **local**.

The global quenches are triggered by changing parameters of the Hamiltonian **homogeneously** [Calabrese and Cardy, '06; Das, Galante, Myers, '15].

The local quenches are triggered by adding an interaction **locally** to the Hamiltonian or changing parameters **locally** in the Hamiltonian [Calabrese and Cardy, '07; Nozaki, Numasawa, Takayanagi, '13; Asplund, Bernamonti et al., '15].

Introduction

Refer to V. Pushkarev's talk

Motivation:

- General probe of non-equilibrium QFTs [Das, Galante, Myers, ‘15]
- Applications in:
  - condensed matter [Ganahl et al., ‘12]
  - high energy physics [Zhang et al., ‘22]
  - information theory [e, Numasawa, et al., ‘14; He, Shu, ‘20]
  - physics of the Early Universe [Carrilho, Ribeiro, ‘17]
- Duals in AdS/CFT, entanglement and thermalization [Balasubramanian, ‘11; Nozaki, Numasawa, Takayanagi, ‘13; Hartman, Maldacena, ‘13; Ageev and Aref’eva, ‘16, ‘18, ‘19; Aref’eva, Khramtsov, Tikhanovskaya, ‘17; Ageev, ‘19 and many other]

Introduction

Setup

The local quench introduced in [Calabrese and Cardy, '07] is called **the geometric quench**, which has been partially generalized in [Doyon, Lucas et al., '14] to a higher-dimensional case. This quench protocol involves joining two different theories with a boundary at some time moment.

Figure. A sketch of the geometric local quench setup. Joining two separated semi-infinite systems produces an instantaneous local interaction between two endpoints (the picture is taken from [Nozaki, Numasawa, Takayanagi, '13]). From the viewpoint of the new Hamiltonian, a locally excited state is generated just after this local quench. Therefore, we can generally characterize the local quenches by local excitations. These excitations will propagate to other regions under the time-evolution.

INTRODUCTION

SETUP

Another setup is called **the operator local quench**, where a localized excited state is prepared by inserting some local operator into the path-integral that prepares the state [Nozaki, Numasawa, Takayanagi, '13; Asplund, Bernamonti et al., '15].

The quenched state in the Schrödinger picture $|\Psi(t)\rangle$ is created by the action of the local operator O at the space-time point (t_0, x_0) on the ground state $|0\rangle$

$$\boxed{|\Psi(t)\rangle = \mathcal{N} \cdot e^{-iH(t-t_0)} \cdot e^{-\varepsilon H} O(t_0, x_0)|0\rangle.} \tag{1}$$

Here, $\mathcal{N}$ provides the unit norm of the state; ε is a damping factor which preserves a finite norm of the state through the regularization of the UV degrees of freedom.

Technically, the local quench is given in the limit $\varepsilon \to 0$, but we take it finite in calculations.

Introduction

Setup

We choose the spacetime point of the quenching operator O as $(t_0, x_0) = (0, 0)$. The presence of the damping factor ε effectively shifts the time as $t_0 \to t_0 - i\varepsilon$. Physically, this defines a "local" excitation with the characteristic length 2ε.

After the operator local quench, the evolution of an observable defined by a local operator $\mathcal{O}$ on the state $|\Psi(t)\rangle$ is given by

$$\boxed{\langle \mathcal{O}(t,x)\rangle_O \equiv \frac{\langle\Psi|\mathcal{O}(t,x)|\Psi\rangle}{\langle\Psi|\Psi\rangle} = \frac{\langle 0|O^\dagger(i\varepsilon,0)\mathcal{O}(t,x)O(-i\varepsilon,0)|0\rangle}{\langle 0|O^\dagger(i\varepsilon,0)O(-i\varepsilon,0)|0\rangle}.} \tag{2}$$

Introduction

Setup

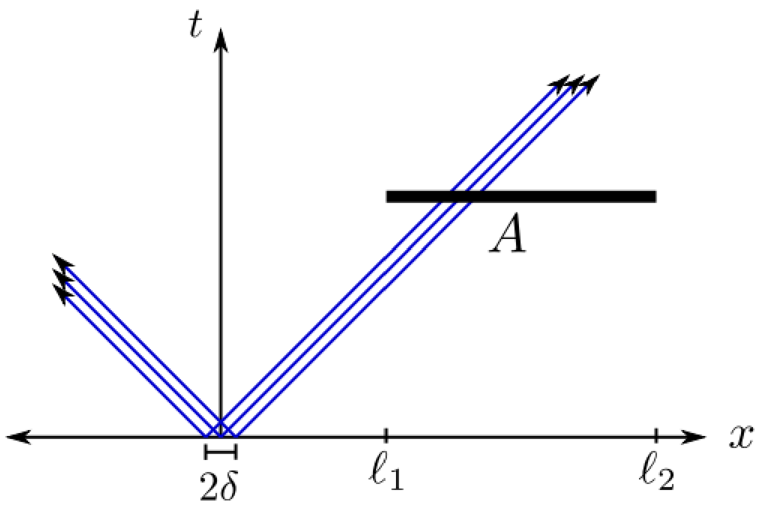


Figure. Setup for the operator local quench. The initial state has a localized excitation near $x = 0$, which then propagates outward. The picture is taken from [Asplund, Bernamonti et al., '15]; $\delta \equiv \varepsilon$.

CFT_2 METHODS

In CFT_2, this setup is exactly solvable due to conformal symmetry.

We introduce the **holomorphic (or lightcone) coordinates** $(z, \bar{z})$ related to Euclidean coordinates (τ, x) as $z = x + i\tau$, $\bar{z} = x - i\tau$.

First, let us review the derivation of the energy density dynamics following the local quench, based on **Ward identities**. Ward identity for a primary operator $O_{(h,\bar{h})}$ and the holomorphic part of the stress-energy tensor $T(z)$ is

$$\langle T(z) O_0 \dots O_N \rangle = \sum_{k=0}^{N} \left(\frac{h}{(z - z_k)^2} + \frac{1}{z - z_k} \partial_{w_k} \right) \langle O_0 \dots O_N \rangle \equiv D\left[\langle O_0 \dots O_N \rangle \right], \tag{3}$$

with the analogous identity holding for $\overline{T}(\bar{z})$; $O_i \equiv O(z_i, \bar{z}_i)$.

CFT$_2$ METHODS

2-point function of a primary operator $O_{(h,\bar{h})}$ is fixed by conformal symmetries

$$\langle O(z_0,\bar{z}_0)O(z_1,\bar{z}_1)\rangle = \frac{1}{(z_1-z_0)^{2h}(\bar{z}_1-\bar{z}_0)^{2\bar{h}}}. \tag{4}$$

3-point function necessary to calculate the energy density evolution accordint to the operator local quench protocol is

$$\begin{aligned}\langle T(z)+\overline{T}(\bar{z})\rangle_{O_{(h,\bar{h})}} &\equiv \frac{\langle O(z_0,\bar{z}_0)\left(T(z)+\overline{T}(\bar{z})\right)O(z_1,\bar{z}_1)\rangle}{\langle O(z_0,\bar{z}_0)O(z_1,\bar{z}_1)\rangle} = \frac{D\left[\langle O_0 O_1\rangle\right]}{\langle O_0 O_1\rangle} \\ &= \frac{h(z_1-z_0)^2}{(z_1-z)^2(z_0-z)^2} + \frac{\bar{h}(\bar{z}_1-\bar{z}_0)^2}{(\bar{z}_1-\bar{z})^2(\bar{z}_0-\bar{z})^2}.\end{aligned} \tag{5}$$

CFT_2 methods

Transforming back to Euclidean and performing the analytic continuation $\tau \to it$ with the substitution of points $t_0 = -i\varepsilon$, $t_1 = i\varepsilon$, the energy density evolution after the local quench by a primary operator $O_{(1,0)}$ is given by

$$\boxed{\langle \mathcal{E}(t,x)\rangle_{O_{(1,0)}} = -\langle T(t,x) + \overline{T}(t,x)\rangle_{O_{(1,0)}} = \frac{4\varepsilon^2}{\left((x-t)^2+\varepsilon^2\right)^2}.} \tag{6}$$

Note that the energy density is chiral for this specific choice of the quenching operator, and generically, the perturbation propagates along both parts of the lightcone.

SIMPLEST FREE FIELD THEORIES

TWO-DIMENSIONAL FLAT SPACE

Now, let us derive the same result using Euclidean coordinate-space propagator and Wick's theorem.

The action for **a free massless scalar field theory** in 2d Euclidean spacetime reads

$$S = \frac{1}{8\pi} \int d\tau \, dx \left((\partial_\tau \phi)^2 + (\partial_x \phi)^2 \right) . \tag{7}$$

Simplest free field theories

Two-dimensional flat space

Let us perform calculations in holomorphic coordinates.

For technical purposes, we define a formal expression for the "2-point function" of the operator ϕ

$$\langle \phi(z_1, \bar{z}_1)\phi(z_0, \bar{z}_0)\rangle = -\ln\left[(z_1 - z_0)(\bar{z}_1 - \bar{z}_0)\right] + \text{IR divergence}. \tag{8}$$

2-point function of the operator $\partial\phi$ do not possess IR divergences

$$\langle \partial\phi(z_1, \bar{z}_1)\partial\phi(z_0, \bar{z}_0)\rangle = \partial_{z_1}\partial_{z_0}\langle \phi(z_1, \bar{z}_1)\phi(z_0, \bar{z}_0)\rangle = -\frac{1}{(z_1 - z_0)^2}. \tag{9}$$

This result coincides with the CFT 2-point function with $h = 1$, $\bar{h} = 0$.

Simplest free field theories

Two-dimensional flat space

The evolution of the energy density after the local $\partial\phi$-quench is described by

$$\langle \mathcal{E}(z,\bar{z})\rangle_{\partial\phi} = \frac{\langle \partial\phi(z_1,\bar{z}_1)|\mathcal{E}(z,\bar{z})|\partial\phi(z_0,\bar{z}_0)\rangle}{\langle \partial\phi(z_1,\bar{z}_1)\partial\phi(z_0,\bar{z}_0)\rangle}. \quad (10)$$

The energy density $\mathcal{E}$ in terms of the holomorphic and anti-holomorphic parts of the CFT_2 stress-tensor is given by

$$\mathcal{E}(z,\bar{z}) = -(T(z)+\overline{T}(\bar{z})), \quad (11)$$

where

$$T(z) \equiv -\frac{1}{2}\,\partial\phi(z,\bar{z})\,\partial\phi(z,\bar{z}), \quad (12)$$

$$\overline{T}(\bar{z}) \equiv -\frac{1}{2}\,\bar{\partial}\phi(z,\bar{z})\,\bar{\partial}\phi(z,\bar{z}). \quad (13)$$

Simplest free field theories

Two-dimensional flat space

Here we first meet **a composite operator** — $(\partial\phi(z))^2$, i.e. an operator consisting of two merged at one spacetime point.

Composite field operators contain **UV divergences** as we merge the points (small distances $\implies$ UV modes).

We exploit **the point-splitting regularization** $w - z = \overline{w} - \bar{z} \equiv \delta$, and consider the limit $\delta \to 0$.

To obtain a finite answer for expressions of the form $\langle\Psi(x_1)|\mathcal{O}(x)^2|\Psi(x_2)\rangle$, we exploit the subtraction scheme defined as

$$
\begin{aligned}
&\langle\Psi(x_1)|\mathcal{O}(x)^2|\Psi(x_2)\rangle\Big|_{\text{finite}} \\
&= \lim_{y\to x}\Big[\langle\Psi(x_1)|\mathcal{O}(x)\mathcal{O}(y)|\Psi(x_2)\rangle - \langle\Psi(x_1)|\Psi(x_2)\rangle\langle 0|\mathcal{O}(x)\mathcal{O}(y)|0\rangle\Big].
\end{aligned}
\tag{14}
$$

Simplest free field theories

Two-dimensional flat space

Therefore, we have

$$\left.\frac{\langle\partial_{z_1}\phi(z_1,\bar z_1)|\mathcal{E}(z,\bar z)|\partial_{z_0}\phi(z_0,\bar z_0)\rangle}{\langle\partial_{z_1}\phi(z_1,\bar z_1)\partial_{z_0}\phi(z_0,\bar z_0)\rangle}\right|_{\text{finite}} = -\frac{(z_1-z_0)^2}{(z-z_1)^2(z-z_0)^2}, \tag{15}$$

or, in Lorentzian spacetime

$$\boxed{\langle\mathcal{E}(t,x)\rangle_{\partial\phi} = \frac{4\varepsilon^2}{\left((x-t)^2+\varepsilon^2\right)^2}.} \tag{16}$$

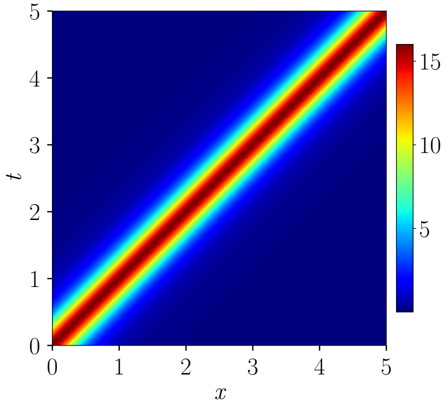
The energy perturbation has a localized soliton-like shape and propagates along the lightcone.

The total energy is given by

$$E = \frac{2\pi}{\varepsilon}. \tag{17}$$

Simplest free field theories

Two-dimensional flat space



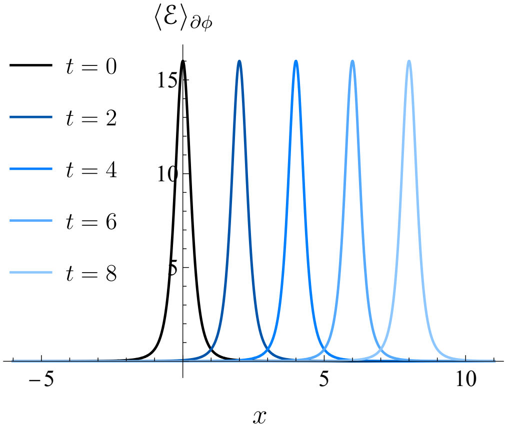
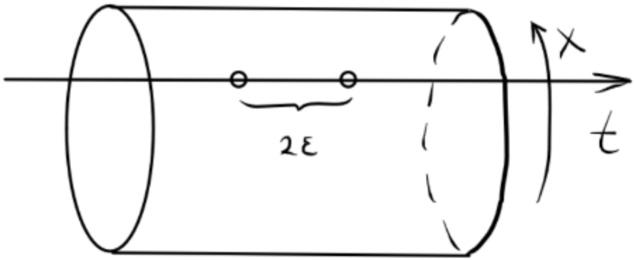


Figure. *Left:* Energy density evolution following the local $\partial\phi$-quench in free massless scalar field theory, $\varepsilon = 0.5$. The soliton-like perturbation of the characteristic width 2ε propagates along the lightcone $x = t$ and does not dilute with time. *Right:* The same evolution; each line in the figure corresponds to a configuration that the perturbation has at a particular time.

Simplest free field theories

Living on a cylinder



CFT result for the energy density evolution on a cylinder after the local quench by a primary operator $O_{(1,0)}$ is given by

$$\boxed{\langle \mathcal{E}(t,x)\rangle_{O_{(1,0)}} = \frac{\pi^2}{3L^2} - \frac{4\pi^2}{L^2}\cdot\frac{\sinh^2\left[\frac{2\pi\varepsilon}{L}\right]}{\left(\cos\left[\frac{2\pi(t+x)}{L}\right] - \cosh\left[\frac{2\pi\varepsilon}{L}\right]\right)^2}.} \tag{18}$$

Now we will derive the same result by a straightforward calculations using Wick's theorem.

SIMPLEST FREE FIELD THEORIES

LIVING ON A CYLINDER

2-point function for a massless scalar on a cylinder is the Green's function of the Klein-Gordon operator with periodic boundary conditions

$$\begin{cases} \Delta K(\vec{x}_1 - \vec{x}_2) = -4\pi\delta^{(2)}(\vec{x}_1 - \vec{x}_2), \\ K(\tau, x + L) = K(\tau, x), \end{cases} , \tag{19}$$

where $\Delta = \partial_\tau^2 + \partial_x^2$, and $\vec{x}_1 = \{\tau_1, x_1\}$, $\vec{x}_2 = \{\tau_2, x_2\}$ are Euclidean 2-vectors. Formal solution for 2-point function of the operator ϕ is

$$\langle \phi(\tau, x)\phi(0,0)\rangle = \frac{2\pi\sqrt{\tau^2}}{L} - \ln\left[\cosh\left(\frac{2\pi\sqrt{\tau^2}}{L}\right) - \cos\left(\frac{2\pi x}{L}\right)\right] + \text{IR divergence}. \tag{20}$$

2-point function of the operator $\partial\phi$ does not possess IR divergences

$$\langle \partial\phi(t, x)\partial\phi(0,0)\rangle = \left(\frac{\pi}{L}\right)^2 \sinh^{-2}\left[\frac{\pi(t+x)}{L}\right]. \tag{21}$$

Simplest free field theories

Living on a cylinder

$$\boxed{\langle \mathcal{E}(t,x)\rangle_{\partial\phi} = \frac{\pi^2}{3L^2} - \frac{4\pi^2}{L^2}\cdot\frac{\sinh^2\left[\frac{2\pi\varepsilon}{L}\right]}{\left(\cos\left[\frac{2\pi(t+x)}{L}\right]-\cosh\left[\frac{2\pi\varepsilon}{L}\right]\right)^2}.} \tag{22}$$

The evolution of the energy density after the local quench on a cylinder has the form of a localized perturbation freely winding around the cylinder.

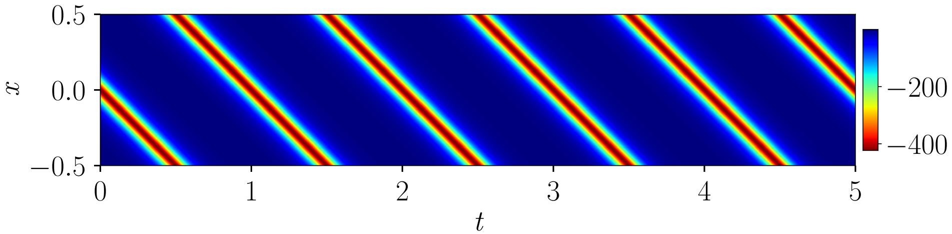


Figure. Energy density evolution after the local $\partial\phi$-quench in CFT_2 on a cylinder; $\varepsilon = 0.1$, $L = 1$.

Straightforward calculation

Basic steps

Summing up, the procedure of calculating observables on excited states generated by the operator local quenches is as follows:

- regularize composite operators by the point-splitting scheme;
- calculate the regularized 4-point correlation function;
- take the limit of merging points in the composite operator and extract the finite part. This reduces the correlator to a 3-point function after merging the points of the regularized composite operator;
- analytically continue to Lorentzian signature;
- "prepare" the excited state by taking the points of the quenching operators as $(\pm i\varepsilon, \vec{0})$.

Straightforward calculation

Advantages

We have illustrated that the straightforward calculation based on Wick's contractions gives the same results as the CFT_2 techniques.

However, only few QFTs possess conformal symmetry. The advantage of the straightforward calculation is that it allows to study the dynamics of an arbitrary QFT with any underlying symmetry group.

In the following, we obtain new results for **a higher-dimensional massless free field theory**, as well as for **a massive field theory in flat space** and on **a cylinder**.

MASSLESS THEORY IN $d > 2$ DIMENSIONS

Free massless scalar field theory is an example of a higher-dimensional CFT.

The $d = 2$ and $d > 2$ theories share a lot of similarities. However, higher-dimensional theories possess **well-defined two-point functions of the operator ϕ** in contrast to the $d = 2$ case.

The Euclidean action of a free massless scalar field in $d > 2$ dimensions reads

$$S = \frac{1}{8\pi} \int d^d x \left((\partial_\tau \phi)^2 + \partial \phi^i \partial \phi_i \right) , \tag{23}$$

where $i = 1, \ldots, d$.

The energy density is given by

$$\mathcal{E}(t, x^i) = \frac{1}{4} \left(-(\partial_\tau \phi)^2 + \partial^i \phi \partial_i \phi \right) . \tag{24}$$

MASSLESS THEORY IN $d > 2$ DIMENSIONS

The energy density dynamics after the ϕ-quench is described by

$$\boxed{\langle \mathcal{E}(t, x^i) \rangle_{\phi,d} = \frac{(d-2)\Gamma\left(\frac{d}{2}\right)}{\pi^{\frac{d}{2}-1}} \cdot \frac{(2\varepsilon)^{d-2}\left(\varepsilon^2 + t^2 + \rho^2\right)}{\left[\left(\rho^2 - t^2\right)^2 + 2\varepsilon^2\left(\rho^2 + t^2\right) + \varepsilon^4\right]^{\frac{d}{2}}},} \tag{25}$$

where $\rho = \sqrt{x^i x_i}$.

Note that in contrast to the massless case in $2d$, in which the perturbation does not dissipate over time, in the case of $d > 2$, the amplitude of the perturbation decays as $t^{2(1-d)}$ (or $\rho^{2(1-d)}$).

The total energy is

$$E = \int d^{d-1}x \, \langle \mathcal{E}(t, x^i) \rangle_{\phi,d} = \frac{(d-2)\pi}{\varepsilon}. \tag{26}$$

ADDING MASS

FLAT SPACE. $\partial\phi$-QUENCH IN $d = 2$

In massive scalar field theory, the ϕ-quench is well-defined in contrast to the massless case (it does not possess IR-divergences).

However, to compare with CFT_2 results, let us start with the $\partial\phi$-quench and explore how the presence of mass m affects the energy evolution.

The Euclidean action is given by

$$S = \frac{1}{8\pi} \int d\tau\, dx \left((\partial_\tau \phi)^2 + (\partial_x \phi)^2 + m^2 \phi^2\right) . \tag{27}$$

The energy in Euclidean and holomorphic coordinates reads

$$\begin{aligned} \mathcal{E}(\tau, x) &= \frac{1}{4}\left(-(\partial_\tau \phi)^2 + (\partial_x \phi)^2 + m^2 \phi^2\right) \\ &= \frac{1}{2}\left(\partial\phi(z,\bar{z})\right)^2 + \frac{1}{2}\left(\bar{\partial}\phi(z,\bar{z})\right)^2 + \frac{1}{4} m^2 \phi^2(z,\bar{z}). \end{aligned} \tag{28}$$

Adding mass

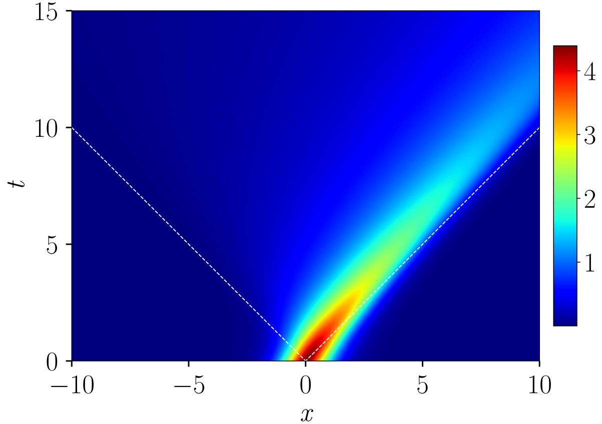
Flat space. $\partial\phi$-quench in $d = 2$

Straightforward calculation gives for the energy density evolution following the $\partial\phi$-quench

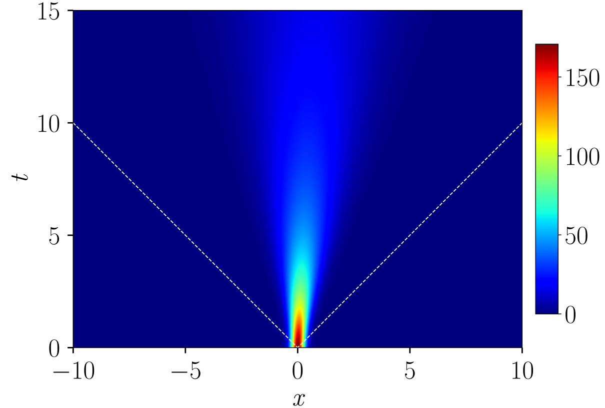
$$\langle \mathcal{E}(t,x)\rangle_{\partial\phi} = \frac{m^2}{2K_2(2\varepsilon m)}\left[\frac{2}{\varepsilon^2+(t-x)^2}\left|\sqrt{(\varepsilon-it)^2+x^2}\,K_1\left(m\sqrt{(\varepsilon-it)^2+x^2}\right)\right|^2 + \frac{\varepsilon^2+(t+x)^2}{\varepsilon^2+(t-x)^2}\left|K_2\left(m\sqrt{(\varepsilon-it)^2+x^2}\right)\right|^2+\left|K_0\left(m\sqrt{(\varepsilon-it)^2+x^2}\right)\right|^2\right]. \tag{29}$$

ADDING MASS

FLAT SPACE. $\partial\phi$-QUENCH IN $d = 2$



(a) $m = 1$



(b) $m = 15$

Figure. Energy density evolution following the local $\partial\phi$-quench in massive scalar field theory. The left figure corresponds to $m = 1$ and the right one to $m = 15$, and $\varepsilon = 1.5$ is fixed for both figures. Dotted lines mark the lightcone.

Adding mass

Flat space. $\partial\phi$-quench in $d = 2$. Fourier images

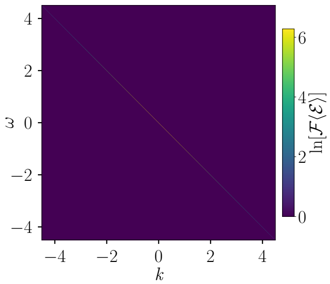
To compare the results between CFT_2 and massive theory, let us consider the energy density after $\partial\phi$-quench in massless theory in momentum space $(\omega,\, k)$

$$\begin{aligned}\langle\mathcal{E}(\omega,k)\rangle_{\partial\phi} =& -2\pi e^{\varepsilon\omega}\omega\,\theta(-\omega)\delta(\omega+k)+\frac{2\pi}{\varepsilon}e^{\varepsilon\omega}\,\theta(-\omega)\delta(\omega+k)\\ &+2\pi e^{-\varepsilon\omega}\omega\,\theta(\omega)\delta(\omega+k)+\frac{2\pi}{\varepsilon}e^{-\varepsilon\omega}\,\theta(\omega)\delta(\omega+k),\end{aligned} \tag{30}$$

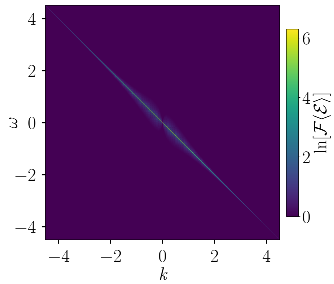
We see the localization of modes along the lightcone with the exponential suppression corresponding to larger ω.

Adding mass

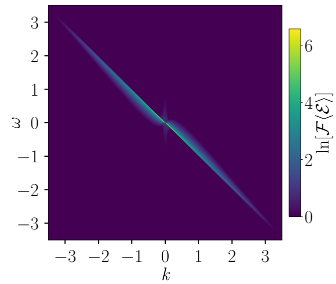
Flat space. $\partial\phi$-quench in $d = 2$. Fourier images



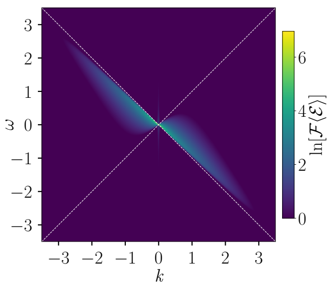
(a) massless (CFT)



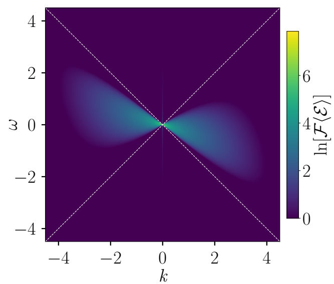
(b) $m = 0.1$



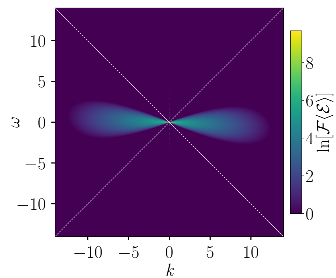
(c) $m = 0.5$



(d) $m = 1$



(e) $m = 3$



(f) $m = 20$

Adding mass

Flat space. $\partial\phi$-quench in $d = 2$. Fourier images

Comparison of the massive and massless cases:

- **In the massless case**, the Fourier image of the energy density is proportional to $\delta(\omega + k)$.
- **In the massive case**, the configuration rolls off the lightcone part $\omega = -k$ and gets more localized around $\omega = 0$ as m increases.
- With the further growth of mass, the configuration takes a dumbbell-like shape localizing along the line $\omega = 0$ and dissipating rapidly for large k.

Adding mass

Flat space. ϕ-quench in $d = 2$

The energy density evolution following the ϕ-quench is given by

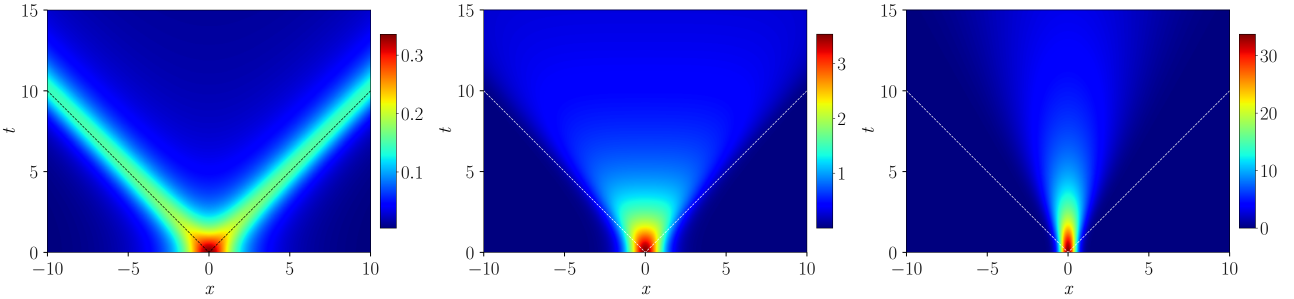
$$\boxed{\begin{aligned}\langle \mathcal{E}(t,x)\rangle_\phi = \frac{m^2}{K_0(2\varepsilon m)}\Bigg[&(\varepsilon^2+t^2+x^2)\left|\frac{K_1\left(m\sqrt{(\varepsilon-it)^2+x^2}\right)}{\sqrt{(\varepsilon-it)^2+x^2}}\right|^2\\ &+\left|K_0\left(m\sqrt{(\varepsilon-it)^2+x^2}\right)\right|^2\Bigg].\end{aligned}} \tag{31}$$

In contrast to the $\partial\phi$-quench, the ϕ-quench generates a perturbation that propagates along both sides of the lightcone.

ADDING MASS

FLAT SPACE. ϕ-QUENCH IN $d = 2$

Critical regime (double-hill $\rightarrow$ plateau) at $m_{\text{crit}} = \dfrac{3}{2\varepsilon}$.

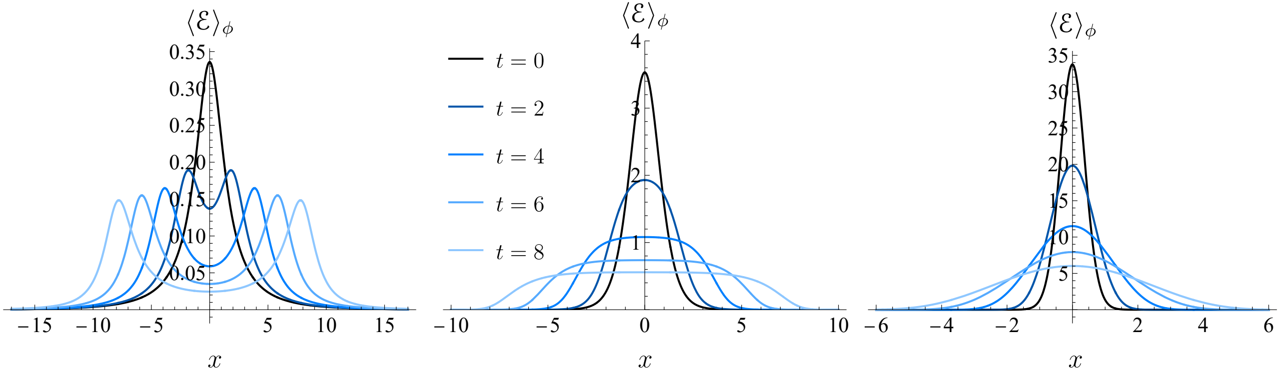


(a) $m = 0.1$ **(b)** $m = m_{\text{crit}} = 1$ **(c)** $m = 5$

Figure. *Top:* Energy density evolution after the local ϕ-quench in massive scalar field theory. The left figure corresponds to $m = 0.1$, the middle — to $m = m_{\text{crit}} = 1$, and the right — to $m = 5$; $\varepsilon = 1.5$ is fixed for each figure. Dotted lines mark the lightcone.

ADDING MASS

FLAT SPACE. ϕ-QUENCH IN $d = 2$

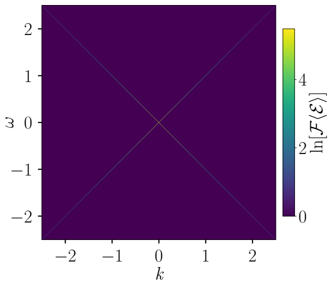


(a) $m = 0.1$ **(b)** $m = m_{\text{crit}} = 1$ **(c)** $m = 5$

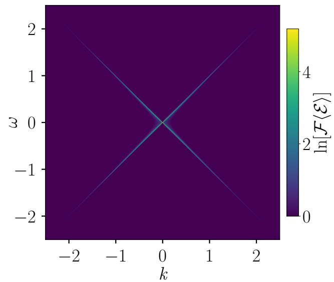
Figure. Spatial energy density distribution after the local ϕ-quench for fixed time moments. The left figure corresponds to $m = 0.1$, the middle — to $m = m_{\text{crit}} = 1$, and the right — to $m = 5$; $\varepsilon = 1.5$ is fixed for each figure.

ADDING MASS

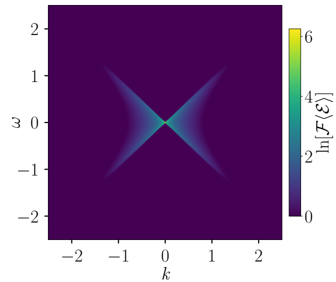
FLAT SPACE. ϕ-QUENCH IN $d = 2$. FOURIER IMAGE



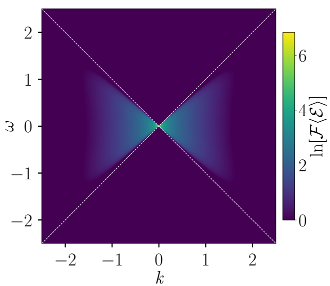
(a) massless



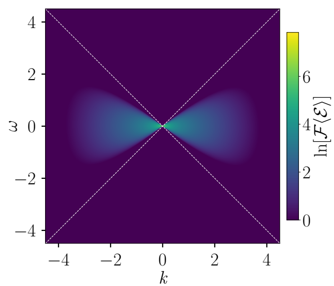
(b) $m = 0.1$



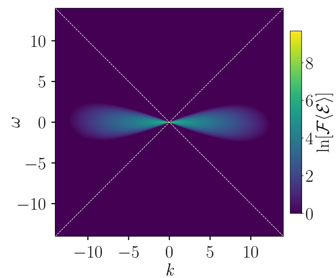
(c) $m = 0.5$



(d) $m = 1$



(e) $m = 3$



(f) $m = 20$

ADDING MASS

FLAT SPACE. ϕ-QUENCH IN $d > 2$

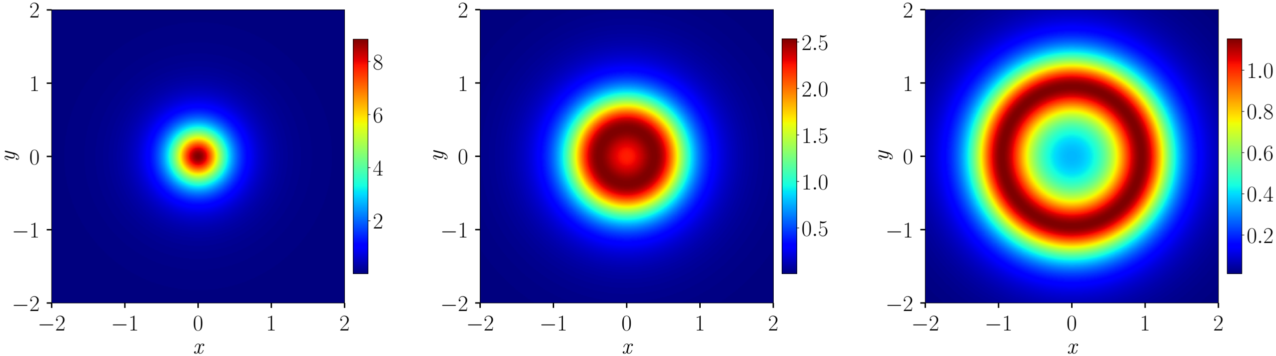
The energy density evolution following the ϕ-quench in $d > 2$ dimensions is given by

$$\boxed{\begin{aligned} \langle \mathcal{E}(t, x^i) \rangle_{\phi, d} = & \frac{m^{\frac{d}{2}+1} \varepsilon^{\frac{d}{2}-1}}{\pi^{\frac{d}{2}-1} K_{\frac{d}{2}-1}(2\varepsilon m)} \left| \left((\varepsilon - it)^2 + \rho^2 \right) \right|^{\frac{d}{2}} \\ & \times \left(\left(\varepsilon^2 + t^2 + \rho^2 \right) \left| K_{\frac{d}{2}} \left(m \sqrt{(\varepsilon - it)^2 + \rho^2} \right) \right|^2 \right. \\ & \left. + \left| \sqrt{(\varepsilon - it)^2 + \rho^2}\, K_{\frac{d}{2}-1} \left(m \sqrt{(\varepsilon - it)^2 + \rho^2} \right) \right|^2 \right) . \end{aligned}} \tag{32}$$

Qualitatively, the perturbation propagates as radially symmetric waves.

Adding mass

Flat space. ϕ-quench in $d > 2$

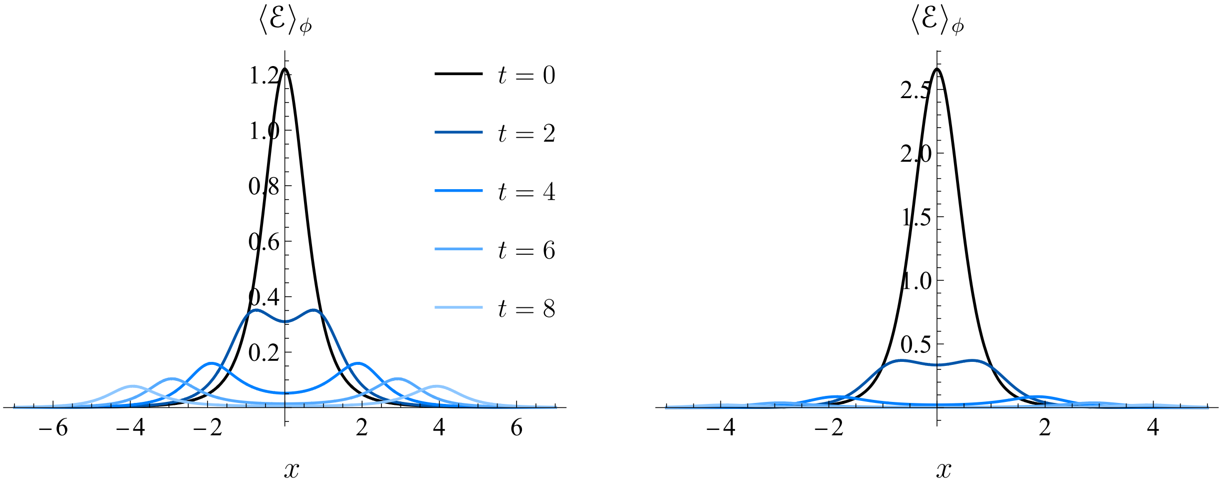


(a) $t = 0$ **(b)** $t = 0.5$ **(c)** $t = 1$

Figure. Spatial dependence of the energy density for $t = 0$ (left), $t = 0.5$ (middle) and $t = 1$ (right), following the local ϕ-quench in $d = 3$. The parameters are $\varepsilon = 1$ and $m = 0.1$.

ADDING MASS

FLAT SPACE. ϕ-QUENCH IN $d > 2$



(a) $d = 3$ **(b)** $d = 4$

Figure. Spatial energy density distribution along x-axis after the local ϕ-quench in $d = 3$ (left) and $d = 4$ (right), respectively, for fixed time moments; $\varepsilon = 1$, $m = 0.1$.

ADDING MASS

CYLINDER. ϕ-QUENCH

We are not able to obtain an analytical expression for 2-point function of a massive scalar on a cylinder.

Euclidean 2-point function on a cylinder periodic in x is the solution to the following system

$$\begin{cases} \left(-\Delta + m^2\right) K(\vec{x}_1 - \vec{x}_2) = 4\pi\delta^{(2)}(\vec{x}_1 - \vec{x}_2), \\ K(\tau, x + L) = K(\tau, x), \end{cases} \tag{33}$$

The solution is given by the following infinite sum

$$\langle\phi(\tau, x)\phi(0, 0)\rangle = \frac{2\pi}{mL} \cdot e^{-m\sqrt{\tau^2}} + \frac{4\pi}{L} \sum_{n \neq 0} \frac{e^{i\omega_n x - |\widetilde{\omega}_n|\sqrt{\tau^2}}}{2|\widetilde{\omega}_n|}\Bigg|_{\substack{\omega_n = \frac{2\pi n}{L} \\ \widetilde{\omega}_n = \sqrt{\omega_n^2 + m^2}}} . \tag{34}$$

ADDING MASS

CYLINDER. ϕ-QUENCH

$m = 0.01$:

$m = 1$:

$m = 5$:

$m = 10$:

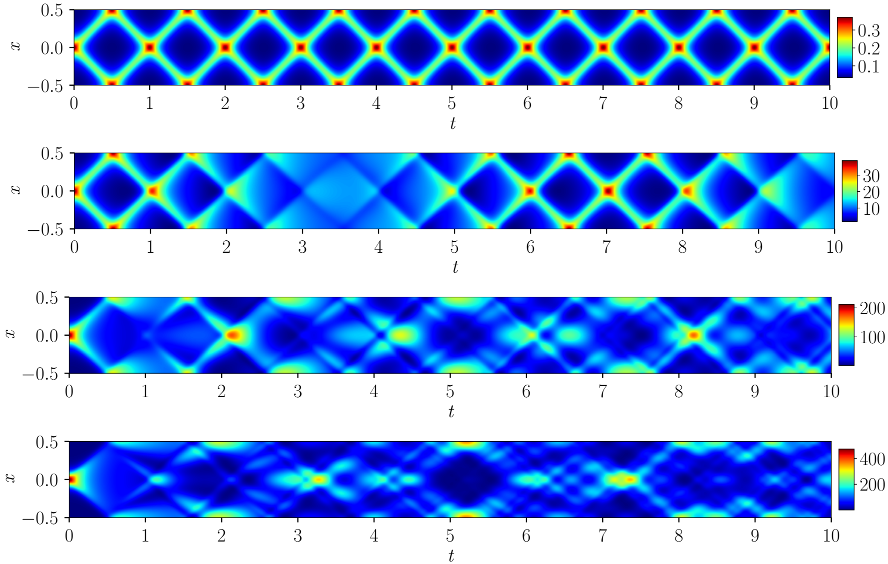


Figure. Energy density evolution after the local ϕ-quench on a cylinder. We take $\varepsilon = 0.1$, $L = 1$.

Adding mass

Cylinder. $\partial\phi$-quench

$m = 1$:

$m = 2$:

$m = 5$:

$m = 10$:

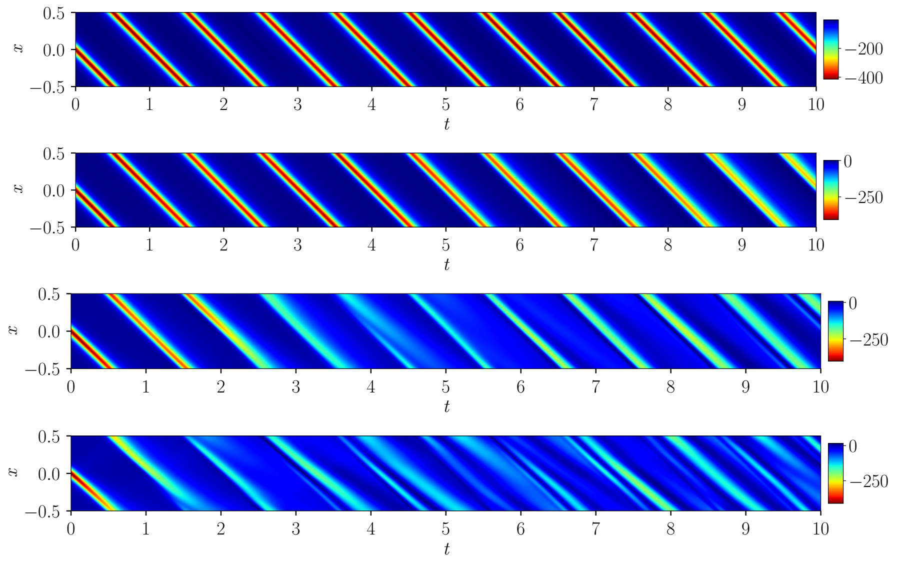


Figure. Energy density evolution after the local $\partial\phi$-quench on a cylinder. We take $\varepsilon = 0.1$, $L = 1$.

Applications and future prospects

- Local quenches in curved spacetimes (AdS, dS, etc.) — dynamics of excited states in the the Early Universe and black holes.
- Fields with interactions.
- Gauge theories with applications in quark-gluon plasma and laser physics.
- Condensed matter: graphene, Hubbard model, exotic QFTs (fractons, Lifshitz theory, ultrameric theories, unparticles), quantum Hall effect.
- Relation to quantum gravity via holography?

Thank you for your attention!