THE STRUCTURAL DEFINITION OF LOGICAL NEGATION

THROUGH THE DOUBLY NEGATED PROPOSITIONS

Antonino Drago

University “Federico II” of Naples, Italy - drago@unina.it

SCAN, June 17th, Moscow

Outline

1. The structure of the foundations of logic:

   the decisive role played by a doubly negated proposition.

II. There exist two kinds of theoretical organization. The definition of Negation depends on them.

III. The problem of defining Negation

   in intuitionist logic.

Conclusions

Part I

The structure of the foundations of logic:

The decisive role played by a doubly negated proposition.

Doubly negated propositions

- Against idealistic notions in logic and mathematics Intuitionists and Constructivist claimed that a proposition is true only if supported by factual evidence; otherwise, it is replaced by its doubly negated proposition (DNP).
- In the following each doubly negated proposition whose corresponding affirmative one lacks of evidence or even differs in meaning from it will be considered a DNP.
- In intuitionist logic two negations do not affirm. Hence, a DNP belongs to this kind of logic.

Consequences of the failure of the double negation law

Some instances:

- "Without contradiction" ≠ "provably consistent".
- "Acquitted for insufficient evidence of guilty" ≠ "for a loyally correct behavior ". (Court's sentence):
- "All enemies of my enemy are my friends" ≠ "All friends of my friend are my friends".

Great novelty: It is easy to recognize within a text the DNPs and hence author's use of intuitionist logic.

It is enough to take into account all negative words of the text and then to verify when a couple of negation constitutes a DNP which plays an essential role in the author's aurguing.

As a fact, several scientific texts made an essential (not rethorical) use of DNPs.

The logical versions of a same proposition within the two main kind of logic

CLASSICAL LOGIC	*INTUITIONIST LOGIC*
Two versions:	***Three*** versions:
The only true one is *A*, which opposes to ¬*A* ; and vice versa.	¬¬*A* is not false; *A* and ¬*A* are not true and do not mutually oppose.
A	*A*
¬*A*	¬*A*
¬¬*A*=*A*	***¬¬A*** *(≠A)*

A law separating two kinds of logic

- In the second half of 20th Century scholars of mathematical logic privileged the double negation law with respect to the excluded middle law in the role of differentiating classical logic from most kinds of non-classical logic.
- First of all intuitionist logic, where two negations do not affirm: $\neg\neg A \neq A$. (Prawitz and Melmnaas 1968; Prawitz 1976; Dummett 1976).

This law is a part of the foundations of general logic.

No DNP in an axiomatic theory

- Along two millennia a clear presentation of a scientific theory was commonly equated to a theoretical organization of a deductive kind from few axioms, whether or not they were supported by evidence (**AO**).
- It does not include a DNP: no DNP comes from an affirmative axiom, whose doubly negated proposition is equivalent to its affirmative proposition; no DNP is an axiom, because the failure of the double negated law forbids to derive from it any affirmative proposition.

The question arises of how DNPs may compose a theory.

Authors presenting their theories in a not-axiomatic way

- Lazare Carnot's mechanics (1783),
- Sadi Carnot's thermodynamics (1824),
- Lobachevsky's theory of parallel lines (1826),
- Galois' theory of algebraic equations (1832),
- Klein's theory of geometries as groups (1872),
- Einstein's theory of special relativity (1905),
- Einstein's first theory of quanta (1905),
- Kolmogorov's theory of LEM's consistency (1924/25),
- Kolmogorov's formalization of intuitionist logic (1932),
- Markov's theory of computable numbers (1961).

The problem-based theoretical organization

A comparative analysis on their original texts shows the characteristic features of a common model of their theoretical organizations:

i) The premise is the common knowledge on the field.

ii) A general problem is declared; in Lobachevsky's: how many parallel line exist; in Kolmogorov's 1932: how to formalize intuitionist logic; etc..

I call it a problem-based organization (**PO**).

DNPs, *ad absurdum* arguments and their conclusion

iii) The theory is aimed at discovering a new scientific method capable to solve the given problem.

iv) The theory argues by composing DNPs into an *ad absurdum* argument of a weak kind, i.e. its final proposition is a DNP (AAA). This DNP may work as a premise for a next AAA in a chain of AAAs.

v) The conclusion of the final AAA is a universal predicate, $\neg\neg UP$, representing a possible solution for all cases of the stated problem.

Translation into an affirmative conclusion and its testing

vi) Afterwards, an author of such a PO theory feels to have collected enough reasoning evidence to be justified in promoting his conclusion $\neg\neg UP$ to the corresponding affirmative predicate *UP*, although this change is not allowed by his initial intuitionist logic.

This step is an appeal to the rationality of the world: the author feels that his arguing has to apply to reality..

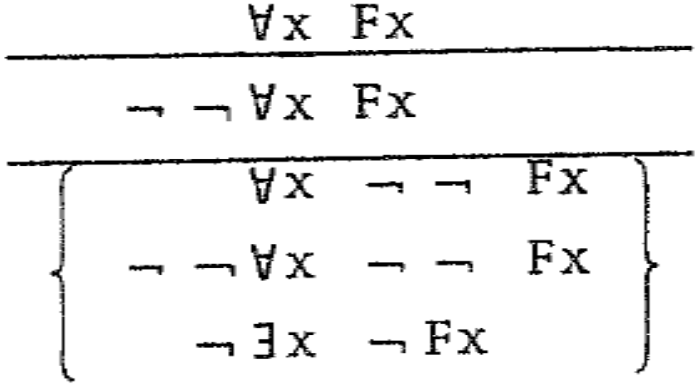
It is the application of Leibniz's principle of sufficient reason

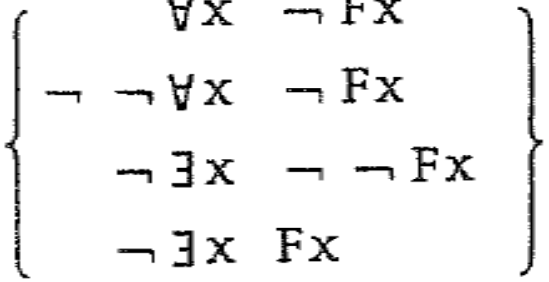
vii) Apparently, previous logical step on $\neg\neg UP$ is an application of Leibniz's principle of sufficient reason: $\neg\exists x\ \neg P(x) => \forall x\ P(x)$; whose antecedent is itself a universal DNP ("Nothing is without reason") and the consequent is the corresponding affirmative proposition ("Everything has a reason").

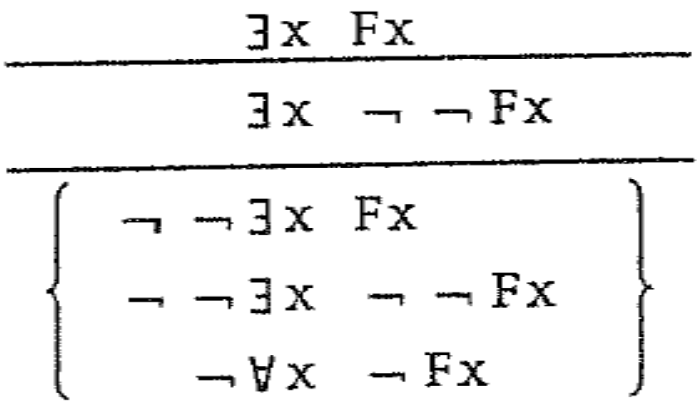
In the square of opposition this step translates into the main classical thesis A ("S is F") the intuitionist version of it, A^I ("not (S is not F)"). Dummett's table (Dummett 1977, p. 29) makes apparent that this single translation is enough for translating the entire intuitionist predicate logic into the classical one (e.g. the similar change of thesis E^I is obtained by mere negation of thesis A^I; the change of thesis I^I is obtained by doubly negating thesis A^I).

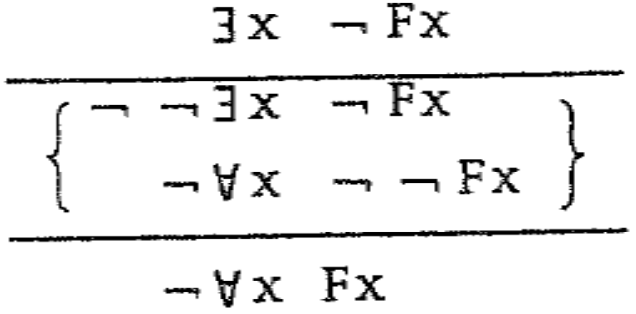
Predicate intuitionist calculus as a square of opposition

(**M. Dummett**: *Elements of intuitionism*, Claredon, 1977, p. 29)









Two reality's constraints for the inverse translation

Surprisingly, it was not noticed that the translation performed by PSR constitutes *the inverse translation* of the so-called 'negative translation' (KGK translation) of CPL into IPL. (T&vD p. 56ff)

However, whereas the latter one is always allowed (under rules about the way to add the two negations to a predicate), the application of PSR, by carrying from a hypothetical to a real world, requires, more than arguments, to be supported by reality criteria. As Markov stated, it is valid under two constraints on the predicate ¬¬*UP*: it is 1) derived from an AAA and 2) decidable (Markov 1961, p. 5).

viii) Then *UP* is an hypothesis for solving the given problem. From *UP*, precisely because it is an affirmative proposition, one can draw all possible derivations to be tested against reality (Drago 2012).

Part II

There exist two kinds of theoretical organization.

The definition of Negation depends from them.

The meaning of a PO theory

It argues through DNPs and AAAs for discovering a new method. The conclusion of the final AAA suggests a surmise $\neg\neg UP$; which, being a DNP, cannot be tested with reality; the application of PSR translates it into an affirmative proposition *UP* in order to test with reality all its possible derivations.

If the reality tests are succssful, the theory is true.

There exists an alternative model of a theoretical organization

- A PO is governed by intuitionist logic and deserves the same importance of AO.
- Hence, a single DNP severs two entirely different (logical and scientific) worlds, i.e. a world of assured truths, as those drawn from assured principles-axioms, and a world of an inductive searching for discovering a new method solving a basic problem.

Classical logic of an AO	**Intuitionist logic of a PO**
Ideas as fixed and concrete	Processes of investigation
Negation as the mirror image of the affirmation	Negation as not opposite to the affirmation
Bivalence law: either true (excl.) or false	Double negation different from affirmation (and negation)
Deduction through implication	Induction of a new method
Direct proofs	(Weak) Indirect, *ad absurdum* proofs
Theory deduced from few axioms	Theory aimed at solving a general problem
Principle of non-contradiction	Principle of sufficient reason

Classical logic of an AO	**Intuitionist logic of a PO**
Logic as calculus	Logic as Language
Truth as fixed, absolute idea (Plato's tale of prisoner's view in a cave)	Truth as *Alétheia* (privative alfa and léthos = hiding)) = *disvealing*: a process
Thus, even the definition of negation must be referred to the kind of organization of its theory.	

Definition of Negation within AO and PO

- Within an AO, ***governed by classical logic***, a doubly negated proposition is exactly equal to the corresponding affirmative proposition. Hence, a negative proposition is the mirror opposite to the corresponding affirmative one.
- Within a PO, ***governed by intuitionist logic***, both negation and affirmation are at most partially true; only the DNP is not false. This is a new kind of negation.
- No surprise if scholars, only considering classical logic, met insurmountable difficulties in defining negation in general terms (Horn and Wansing 2015, Gabbay 2020).

Part III

The problem of defining Negation in intuitionist logic

Brouwer's definition of Negation

- Also the definition of intuitionist negation was difficult.
- "Brouwer [papers 1923a-c]… expressed negation as reasoning that leads to an absurdity…" (Franchella 1994, p. 258).
- But, is there in all cases of a negation a proof of its leading to a contradiction (absurdity)?

The debate on the subject

- After a century the debate on this question is still inconclusive (Sundholm 1994; Raatikainen 2004; 2013).
- The latter author distinguished three attitudes; *i*) *strict actualism*, i.e., to require the actual existence of the contradiction proof in all cases; *ii*) *possibilism*, i.e. the proof is no more than possible; *iii*) an intermediated case of *liberalized actualism*.
- His conclusion: “We have examined the three basic choices there are for the intuitionist theory of truth,… and found all them wanting.” (Raatikainen 2004, p. 143)

Two organizations, two definitions of Negation

My suggestion: the question is still unresolved because referred to Heyting's AO formluation of an axiomatic intuitionist logic; i.e. an AO whose are mere variations of the classical ones.

In such a case the negation is equipped by the existence of a direct proof of its absurdity no matter of the idealistic nature of its existence.

Instead, Kolmogorov (1932) offered an almost exactly PO formulation of intuitionist logic (Drago 2021)

Kolmogorov's PO formulation of intuitionist logic (1932)

- Problem: to state a "new *calculus of problems*" (p. 238)
- The methodological principle. (p. 323)
- Numerous DNPs (21).
- 4 AAAs. E.g.: "[¬Ts:] LEM cannot be found on the list of problems solved by the reader", otherwise [⊥ :] he would "be omniscient".(p. 332),
- The (implicit) application of PSR obtains:

"*The calculus of problems is formally identical with the Brouwerian intuitionist logic*".(p. 331)

Representative examples of $A, \neg A, \neg\neg A$

The corresponding negative to a DNP of a PO does not oppose the positive. That is easily verified in these examples:

- equality, non-equality, equivalent (non-difference)
- consistency, non-consistency, non contradiction (for ex., the principle of)
- truth, non truth, alétheia (dis-vealing)
- goodness, violence, non-violence.

Hence, negative may lead to absurdity for only its part concerning its partial opposition to positive.

Hence, an intuitionist negation suggests a mere hypothesis of the existence of a proof of its absurdity (Possibilism).

Kolmogorov's criticism and conclusion

- Kolmogorov's objection (1924/25, 420-422): Browuer presupposes to have defined [besides the implication,] negation in either the law of contradiction or an AAA.
- His definition of negation is different from Brouwer's: "to obtain a contradiction provided a solution of *a* is given". It is a verification avoiding the problem existence of the absurdity proof (Like L. Carnot's version of inertia principle: "Once...")
- His conclusion: "*For any universal proposition it is in general meaningless to consider its negation as a determinate proposition.*" (1932, p. 333)

Conclusions

The nature of the definition of negation is not of an objective kind, or worst, as in Brouwer's sense, of subjective kind, rather it is of a structural kind. It depends from the kind of theoretical organization which it belongs to.

Rightly Carnap (1943) distrusted the "logical inferentialism", i.e. the commonly shared opinion that all logical constants can be uniquely defined through a local analysis of inference relations.

Moreover, Dummett's interpretation of intuitionism, called "meaning-as-use", has to be extended beyond the use of single inferences, to the use of the two theoretical organizations.

Kolmogorov rejected Brouwer's definition of negative proposition. Thus, the BHL interpretation of intuitionism is an a posteriori artifact.

THANKS !

Foundation of (Intuitionist) Logic

- Since the problem of what is a negation plays a crucial role inside the theory of intuitionist logic, one may re-formulate this logic according to a PO whose basic problem is this one.
- Intuitive, primitive notion of negation.
- "What means a negation?". For a chain of *n*-times negations of a same propoition is possible: this infinite chain is surely meaningless. It is necessary stop this chain. The simplest way is to stop at the second negation and to ask: is it equivalent ot the affirmation, or not? Here a fork appears. 1) Either the double negation is equivalent to the affirmation anad hence an implication appears for the first time. 2) or it is not equivalent and three possibilities hold.
- Two main kinds of logic; CL and ML. Il occurs not before a theory is concluded.

- **NO???**However, the proof has to be an AAA, because at the moment we have no other certainty in derivation.
- Moreover, the *absurdum* cannot be different from the negation of the autonomy of DNP. Hence there is a proof only if not denying $\neg\neg A$, hence = A. Thus, we come to classical logic. The absurd is collapsed into CL Ma Brouwer absurdity of absurdity: theor.s!!! E poi CL e Il coesistono!
- . ---+ unity, division, *advaita* ---+ friend, enemy, non-enemy (enemy of my enemy) --- loving me, hating me, not-hating me. self, other, not-other.

- In sum, Negation as definied after double negation is in a some kind of opposition to the positive.
- Brouwer’s definition of negation is an imitation of classical definition, but it is inadequate to intuitionist logic.

Change of definition within a PO

- However, the application of the PSR achieves an affirmative conclusion and changes the kind of theoretical organization.
- If testing the affirmative proposition is successful, then also the proofs of contradiction of negations exist; being now the theory in classical logic, these proofs are always assumed as actual, albeit also in an idealistic sense.

So, also in a PO we have two methods obtaining a same notion, intuitionist negation, but attributing to it two different meanings.

The logical versions of a same proposition within the two main logics

CLASSICAL LOGIC	*INTUITIONIST LOGIC*
Two versions:	***Tree*** versions:
The only true is *A*, which opposes to ¬*A* ; and vice versa	The only true is ¬¬*A*, which does not oppose to either *A or* ¬*A*; neither the last ones mutually oppose
A	*A*
¬*A*	¬*A*
¬¬*A*=*A*	***¬¬A*** *(≠A)*

We have to overcome ***five*** prejudices

1. A double negation is a characteristic feature of **primitive** languages **!!!** (Horn 2001; Horn 2012)
2. *Ad absurdum* proofs can be always **inverted into direct** proofs (Gardiès 1991).
3. There exists **only** the Axiomatic model of Organisation of a scientific theory (AO) (Hilbert 1899).
4. The principle of sufficient reason is a **useless** metaphysical claim (Kant).
5. Carnap: Religious thinking is "**entirely out of line** with the systematic way of thinking in the present century".

Which **method** for studying logic in general?

- Instead of the bivalence law as first, to assume **the divide** between classical logic and almost all non-classical logics: $A \neq \neg\neg A$
- To consider each kind of logic as performing reasonings by means of not only direct proofs but also **indirect** (*reductio ad absurdum*) **proofs**.
- To consider each kind of logic as **organizing a theory** not only in an axiomatic way but also **for solving a general problem**.(e.g. Kolmogorov's 1924/25: why the use of LEM does not lead to contradiction?)

Foundation of Intuitionist Logic

- A next problem is how to argue according to each alternatives of the branch? Being in the foundation of logic the indirect proofs only are possible. Hence one has to make clear the conclusion of this proof. Again two alternatives are possible, according to the previous choices. Either an affirmative, through the DNL, and hence the inversion of the indirect proof into the direct one, and hence the development of classical logic (provided that the or and the and are added or the sheffer strokes) o the DN is not equal to te affirmative one and hience this conclusion obliges to argue through a chain of indirect proofs, etc. etc. .

$A \neq \neg\neg A$ as the borderline between the kinds of logics

Prawitz' and Dummett's studies recognised as the most accurate distinction of intuitionistic logic (and weaker logics) from classical logic the failure of double negated law (not the failure of the equivalent law of excluded middle, LEM). (D. Prawitz and P.-E. Melmnaas: “A survey of some connections between calssical intuitionistic and minimal logic”, in A. Schmidt and H. Schuette (eds.): *Contributions to Mathematical Logic*, North-Holland, Amsterdam, 1968, pp. 215-229. M. Dummett: *Principles of Intuitionism*, Clarendon Press, Oxford, 1975).

Consequences of the failure of the double negation law

The meaning of each of the following **doubly negated propositions** is not preserved by dropping out the two negations, owing to the lack of evidence supporting the resulting statement. i.e they are not semantically equivalent (**DNP**s):

"All enemies of my enemy are my friends" ≠ "All friends of my friend are my friends". Court's sentence: "Acquitted for insufficient evidence of guilty" ≠ "for a loyally correct behavior ".

"Without contradiction" ≠ "provably consistent".

Leibniz enounced his celebrated principle in a logically exact way: "Nothing is without reason, [≠] or everything has its reason, although we are not always capable of discovering this reason…"

Great novelty: It is easy to recognize within a text the DNPs and hence **author's use of intuitionist logic**.

There is an essential (not rethorical) use of DNPs.

Investigation of two classical oriental texts through the DNPs and intuitionist logic

Laurence Horn has remarked that Eastern word "non-violence" is a double negated word which is not equivalent to an affirmative word; since in this case the double negation law fails, this word pertains to intuitionist logic. (Horn L.R. (1989), *The Natural History of Negation*, Chicago U.P, p. 84.)

I recognized intuitionist logic within two oriental classical texts.

- Sun Tzu: *Art of War*, ~500 B.C. (Covone G. and Drago A. (2000), "L'*Arte della guerra* in Sun Tzu", *Quaderni Asiatici*, n. 52, pp. 47-62).
- M.K. Gandhi: *Hind Swaraj*, Amhedabad, 1909
- (Drago A. (2011), "*Hind Swaraj*: "A birth of a new model of development", in S.K. Joseph & B. Mohandaya (eds.): *Reflections on Hind Swaraj*, Inst. Gandhian Studies, Wardha, pp. 73-143).

The DNPs in *The Art of War*

One finds out several DNPs. E.g.

III 1. In the practical art of war… to shatter and destroy it is not so good.

III 2. supreme excellence consists in breaking the enemy's resistance without fighting

IV 2. To secure ourselves against defeat lies in our own hands, but the opportunity of defeating the enemy is provided by the enemy himself.

IV 3. Thus the good fighter is able to secure himself against defeat….

IV 13. He wins his battles by making no mistakes.

IV 14. Hence the skillful fighter puts himself into a position which makes defeat impossible, and does not miss the moment for defeating the enemy.

Their sequence gives back the entire content of the book.

Although: "Technically, classical China had semantic theory but no logic." (H. Chad: "Logic in China", in *Routledge Enc. Phil*., 2021, Article Summary) **!!!**

DNPs in Gandhi's *Hind Swaraj* (1909)

Also this text includes many doubly negated propositions whose sequence gives back the entire book's content.

- "[My method] is the reverse of resistance by arms. When I refuse to do a thing which is repugnant to my conscience, I use soul-force.
- It is a mark of wisdom not to kick away the very step from which we have risen higher.(p. 19)
- I can never subscribe to the statement that all Englishmen are bad." (p. 20)

Gandhi's book includes also (weak) *ad absurdum* arguments (AAA), the only ones allowed by intuitionist reasoning:

- "If [¬Ts] the story of the universe had commenced with wars, [⊥] not a man would have been found alive today." (p. 67)
- "[¬Ts] An eye for an eye only [⊥] ends up making the whole world blind", => [¬ ¬Ts] "No eye for an eye", that is "Non-violence".
- By applying PSR: [Ts] "Benevolence makes world brotherly".

This sequence constitutes an entire theory.

Misleading western appraisal on eastern reasoning

- Eastern reasoning includes classical reasoning **and also** intuitionist reasoning, which yet West considered extraneous from reality.
- By scrutinizing Eastern reasoning according to only classical logic incorrectly Western scholars concluded that it is not at the level of Western logic.
- The difference was more quantitative in nature than qualitative.

Ancient Greek word for truth: un-veiling

Past Western appraisal was incorrect also because in ancient times also Western logical reasoning was of the intuitionist kind.

The ancient Greek word for thuth was “ἀλήθεια (alétheia)” which is composed by a privative ἀ and λήθω (to be hidden); hence, **un-veiling**, i.e. a double negation without an equivalent affirmative word, since it means a process, not a fixed idea.

‘Alétheia’ vs. ‘Veritas’

This original meaning was then changed (maybe by Plato) into an abstract, eternal entity which to believe in (in Italian language wedding ring is called “vera”, the feminine of “vero”, true).

Afterwards, Greeks improved classical logic to perform **mechanical implications**, as it occurs in Aristotle’s syllogistic and in Euclid’s geometry. Therefore they assumed classical logic as the most productive one.

In next centuries Western culture chose classical logic in an exclusive way (see e.g. Kant’s prejudice).

The exception of Christian dogmas

Christianity is a Western religion born in the middle-East.

Along ten centuries it elaborated two basic dogmas. **They are DNPs**:

1) Christ had divine and human natures "without mutation and without confusion", "without separation and without division" (Calcedonian Council 451).

2) God is Unity and at the same time is Trinity. "Athanasius' creed" (8th Century) is full of double negations. "… we worship one God in Trinity, and Trinity in Unity; neither confounding the Persons; nor dividing the Essence….; Father un-created… un-limited…, non-mortal…. neither created, nor begotten…; in this Trinity none is before, or after another; none is greater, or less than another…

The logical divergence of these dogmas was solved by considering them as oxymora of an irrational and a-logical religious thinking.

Re-emergence of intuitionist logic within basic scientific principles

Yet, since the 18th Century intuitionist logic re-emerged within

- Stevin's statement "… the motion will not have an end;… that is absurd“ (quoted in R. Dugas: *Histoire de la Mécanique*, Griffon, Neuchatel, 1950, p. 121) ≠ "All motions have an end“ (we cannot determine at which exact point a moving body will stop, because before the motion the exact friction law on the path is unknown). Great part of mechanics and thermodynamics originated fom this principle.
- Although calculus suggested to conceive matter as infinitesimals elements Chemists stated: "Matter is not divisible at infinity" (H. Guerlac: "Quantisation in Chemistry", *ISIS*, (1961) p. 206) ≠ "Matter is divisible in a finite way" because the atoms were experimentally proved not before 20th Century.

More scientific principles as DNPs

- " Infinitesimal quantities are therefore not chimerical (= not real) beings" (L. Carnot ." Dissertation sur l'infini mathématique" (1781), in C.C. Gillispie: *Lazare Carnot Savant*, Vrin., Paris, 1979, 251-298, § 13, p. 256).
- "…there is thus a quantity which is not altered (≠ constant) by an impact ... "(L. Carnot: *Éssai sur les machines en général*, Dijon, 1783, 44).
- *Inertia principle*: "A body, once put to rest, by itself (= if not by others) could not come out." (L. Carnot: *Principes fondamentaux de l'équilibre et du mouvement*, Deterville, Paris, 1803, p. 49).
- "All forms of energy are equivalent [= not different]".
- "The hypothesis [of two parallels] does not lead to contradiction" (N.I. Lobachevskii: *Geometrische Untersuchungen der Theorie der Parallellineen*, (as an Appendix to R. Bonola Non-Euclidean Geometry, Dover, 1955, prop. 22)

An exceptional use of DNPs in Kolmogorov's 1925 paper

The theory is organized for solving a basic problem (**PO**): "Why the [formalist[use of LEM does not lead to contradiction"?

The paper makes use of two separated yet parallel logical paths, one based on classical propositions, ("truths") the other on and those obtained by doubly negating them, called "pseudotruths".

He reasoned through ad absurdum AAAs: "[Ts] The use of the principle of excluded middle never leads to a contradiction. In fact, if [¬Ts] a false formula [¬*A*] were obtained with its help, then [⊥] the corresponding formula of pseudomathematics [¬ ¬*A*] (would be proved without its help and) would also lead to a contradiction."

This is the first proof of relative consistency

A theoretical use of intuitionist logic

DNPs are recognized within each original text founding **a theory aimed at solving a problem (PO)**:

Lavoisier's and Mendeleev's classical chemistry, Thermodynamics, Lobachevsky's non-Euclidean geometry, Galois theory, Klein program on geometries, Einstein's 1905 paper on special relativity, Einstein's 1905 paper on quanta, Quantum theory (the analogies and the wave-particle complementarity are all equivalent to DNPs), Kolmogorov's foundation of intuitionist logic, Markov's theory of constructive numbers. (Drago A. (2012), "Pluralism in Logic. The Square of opposition, Leibniz's principle and Markov's principle", in J.-Y. Béziau and D. Jacquette (eds.), *Around and Beyond the Square of Opposition*,, Basel: Birckhaueser, pp. 175-189)

The existential predicate concluding a PO theory - I

- *S. Carnot*: "no change of temperature inside the bodies employed for obtaining the motrice power of heat occurs without a change in the volume". (*Réflexions sur la puissance motrice du feu*. Blanchard, 1824, p. 23).
- *Lobachevsky*: "… without leading to any contradiction in the results" (Appendix to R. Bonola, *Non-Euclidean Geometry*. Dover, 1955, p. 19).
- *Poincaré*: "If the absence of contradiction of a syllogism whose number is entire implies the absence of contradiction of the following one, one has to not fear any [= no] contradiction for each syllogism whose number is entire" (*Science et Méthode*, Flammarion, 1912, p. 187).

The existential predicate concluding a PO theory- II

- *Planck* ("second theory"): "There is, however, no method of testing its admissibility except by the investigation of its consequences, and as long as no contradiction in itself or with experiment is discovered in it, and as long no more adequate hypothesis can be advanced to replace it, it may justly claim a certain importance.." (*The Theory of Heat Radiation*, Dover, New York, 1959, p. 154).
- *Kolmogorov*: "None of the conclusions of ordinary mathematics that are based on the use outside the domain of the finitary…. can be regarded as firmly established." (*On the principle 'tertium non datur'*, J. van Heijenoort (ed.), *From Frege to Goedel*. Harvard U.P., Cambridge, 1967, p. 416).
- All such conclusions are formalised by a same formula:

$$\neg\exists x\ \neg A(x).$$

The total predicate concluding a PO theory

- *Avogadro*: "[All] The proportions among the quantities in the combinations of the substances do not seem depend other than both the relative number of molecules which combine themselves and the number of the composed molecules which result from them"], ("Essay d'une manière de determiner les masses…", *J. de Phys,, de Chimie et d'Hist. Nat.*, **73** (1811) 58-76, p. 58).
- *Einstein*: "[Every] Monochromatic radiation of low density… behaves thermodynamically as in-dependent energy quanta of magnitude $R\,\beta\nu/N$".("Ueber einen die Erzeugung….", *Am. J. Phys.*, 33 (1965) p. 372).
- Kleene's statement on *Church's thesis*: "Every general recursive function cannot conflict with the intuitive notion which is supposed to complete…"(*Introduction to Metamathematics.* Van Nostrand, Princeton, 1952, pp. 318-319)

All such conclusions are formalised by a same formula:

$$\forall x\ \neg\neg A(x).$$

The two kinds of concluding predicates of a PO theory are equivalent

- In some theories the logical formula of the conclusive predicate $\neg\neg T$ is: $\neg\exists x\ \neg A(x)$
- In some other theories its formula is $\forall x\ \neg\neg A(x)$
- Remarkably, in intuitionist predicate logic they are equivalent (last and two last predicates in top left).
- Also the logical formula of Leibniz' principle of sufficient reason (PSR) is the same

Nothing is without reason : $\neg\exists x\ \neg A(x)$

Predicate intuitionist calculus

(M. Dummett: *Elements of intuitionism*, Claredon, 1977, p.29)

$$\frac{\forall x\ Fx}{\neg\neg\forall x\ Fx}$$

$$\left\{\begin{array}{l} \forall x\ \neg\neg\ Fx \\ \neg\neg\forall x\ \neg\neg\ Fx \\ \neg\exists x\ \neg Fx \end{array}\right\}$$

$$\left\{\begin{array}{l} \forall x\ \neg Fx \\ \neg\neg\forall x\ \neg Fx \\ \neg\exists x\ \neg\neg Fx \\ \neg\exists x\ Fx \end{array}\right\}$$

$$\frac{\exists x\ Fx}{\exists x\ \neg\neg Fx}$$

$$\left\{\begin{array}{l} \neg\neg\exists x\ Fx \\ \neg\neg\exists x\ \neg\neg Fx \\ \neg\forall x\ \neg Fx \end{array}\right\}$$

$$\frac{\exists x\ \neg Fx}{\left\{\begin{array}{l} \neg\neg\exists x\ \neg Fx \\ \neg\forall x\ \neg\neg Fx \end{array}\right\}}$$

$$\neg\forall x\ Fx$$

The crucial passage to a deductive development. Galilei

- This progressive logical dynamics corresponds to what a physicist usually does. From his questioning the experience he hints a conjecture hypothesis ¬¬T about it he reasons through AAAs (on the field of phenomena at issue) confirming the conjecture ¬¬T; eventually he translates ¬¬T into a certain, affirmative principle T, from which then he deductively develops all consequences to be tested with reality.
- *Galilei:* "Let us take it as a postulate, whose… truth will be stated [as a methodological principle] by seeing that further conclusions, built on this hypothesis [through AAAs], perfectly fit the [entire field of] experience. Having admitted [through PSR] this… [axiom-]principle, the Author moves to consider the propositions, which are deductively concluded [from it]…" (Galilei 1958, p. 191; I cancelled an improper word: "absolute").

Lobachevsky and Einstein declare the same translation

Lobachevsky verbally manifested this same translation concerning a mathematical subject: “[My supposition of two parallel lines] can likewise be admitted [as a methodological principle] without leading to any contradiction in the [previous] results and [PSR, as an axiom] founds a new geometry….” (Lobachevsky 1840, prop. 22)

Einstein : "We will raise the conjecture to the state of a [methodological] postulate“; its substance will be hereafter [through PSR] called the "[axiom-] principle of relativity“. [Einstein: “Zur Elektrodynamiker…”, 1905, p. 891, minor changes]

The change of the universal predicate cocluding through the PSR a PO theory

- Author's conversion of the predicate $\neg\neg Ts$ is of a *subjective and intuitive nature*.
- This conversion is an application of the *Principle of sufficient reason* as it is exemplified by its DNP ("Nothing is without reason") leading to an affirmative predicate statement ("There exists the wanted reason"):

$$\neg\exists x \neg A(x) => \exists x A(x)$$

- A. Markov (1961, p. 5) suggested two constraints on the conclusive predicate: 1) it is decidable;
  2) it is derived from a AAA.

PSR: A global change

- That amounts to change both logic (from IL into CL) and theory organisation (from PO into AO)
- Through previous Dummett's Table one verifies that this change through the PSR is enough to translate the entire intuitionist logic into the classical logic. That shows that **the application of PSR** constitutes **the inverse translation** of the well-known GKK double negation translation.
- **Leibniz**: "Two principles of human reason: non-contradiction and sufficient reason." (*Monadology*, 1714, §§31–32)

Resuming the new organization of a scientific theory based on a problem

- Each theory starts by putting a *universal problem* whose resolution requires a new scientific method.
- Then *a methodological principle* leads this research
- which proceeds through *doubly negated propositions* not equivalent to the corresponding affirmative ones (= DNPs); hence they belong to intuitionist logic (Dummett 1977, 24)
- The final argument is a weak *ad absurdum argument*
- whose conclusion suggests a possible resolution of the starting problem for all similar problems, i.e. *a universal predicate* $\neg\neg Ts$ (Completeness indirect proof)
- Being assured to have collected all possible evidence, the author of such a theory *converts the above predicate into Ts*
- which is then assumed as *a new hypothesis* in order to draw inside classical logic all possible consequences.

Kolmogorov's intuitionist logic (1932)

The problem: a "new *calculus of problems*". (p. 238)

- The methodological principle. (p. 323)
- 21 DNPs.
- AAAs. E.g.: "LEM [$\neg$Ts] cannot be found on the list of problems solved by the reader", otherwise [$\bot$] he would "be omniscient". (p. 332),
- By [implicitly] applying PSR he obtains
- "*The calculus of problems is formally identical with the Brouwerian intuitionist logic*".(p. 331)

Conclusion

I conclude that Western thinking bounded itself to a narrow use of logic according to the most assured and productive way of reasoning, in order to obtain an infinite number of logical results at the costs of both disqualifying as irrational the religious thinking and ignoring the birth of an alternative organization of a theory.

Instead, the recognition of intuitionist logic as a commonly used logic in both everyday language and scientific language introduces us into a pluralist view of logic, such as in the East always was present together with classical logic.

Eastern and Western logic reconciliated by the pluralism of logic

The time is come to re-open Western thinking to allow intuitionist logic on a par with classical logic and hence to allow an effective pluralism of the kinds of logic.

This move would reconciliate Eastern and Western ways of reasoning as sharing the same possibilities – as in the past two Western thinkers, the theologian Nicholas Cusanus and the military strategist, Karl von Clausewitz, did.

(Drago A. and Pezzullo F. (2000), "Logica e strategia. Analisi della teoria di K. von Clausewitz", *Teoria Politica*, 16, pp. 164-174).

Cusanus anticipated pluralism in logic

The cardinal Nicholas Cusanus (1401-1464) proved that one can accurately argue outside of the "Aristotelian logical sect".

His name of God was Not-Other, "which is different from the Same". In his logic "the negation does not oppose to the affirmation". (Cusanus N. (1462), *De Non-Aliud*, Engl. Transl. in https://jasper-hopkins.info/).

These two propositions represent intuitionist logical laws. He defined by doubly negating ("The sky is not-other the sky"). That anticipated Kolmogorov's translation from classical logic to intuitionist logic.

Cusanus' reasoning through AAAs

He also argued through (weak) AAAs:

> "[Ts] God defines other. For if [¬Ts] God is removed, [⊥] other does not remain" (p. 1304, n. 40).

By applying the PSR to the universal predicate ¬¬Ts concluding the last AAA, he obtained the corresponding affirmative T, from which all consequences are drawn.

In philosophy, he suggested that two are mind's faculties: *ratio*, undergoing non-contradiction principle, and *intellectus*, proceeding through *coniecturae* (DNPs of intuitionist logic).

Karl von Clausewitz's intuitionist logic in the book on military strategy

Two kinds of war: the ideal, „absolute war", aimed at „abating the adversary" through all means and ruled by an „abstract" [=classical] logic; and the „real war" which is „a bastard product" of also unforeseable factors. The latter one is „an astonishing (*Wunderbar*) Trinity" (I, 28) whose differet logic is described through several DNPs.

> ***Never*** *he wrote the current quotation: „War* ***IS*** *politics.…", but in all*
>
> *cases: „War is nothing else politics…" (p. 990) i.e. all depends from a suitable relationship between military power and political power.*

DNPs in Clausewitz *On War*

- Definition... War is nothing but a duel on a larger scale. (l, 2).
- The art of war deals with living and with moral forces, Consequently, it cannot attain the absolute [= not real], or certainty it must always leave a margin for uncertainty, in the greatest things as mucli as in the smallest.(I, 22)
- The thesis, then, must be repeated: War is an act of force, and there is no logical limit to the application of that force.(I, 3)
- neither opponent is an abstract [= not real] person to the other; not even to the extent of tliat factor in the power of resistance, nameliy the wil, which is dependent on externals. The will is not a wholly unknown factor;we can base a forecast of its state tomorrow on what it is today. (I, 7)
- In short, the absolute [= not real], so-called mathematical, factors, never find a firm basis in military calculations.
- to introduce [$\neg$Ts] the principle of moderation into the theory of war itself would always lead to [$\perp$] logical absurduty.

Ad absurdum proofs in Clausewitz' book

- - [¬Ts] Were it a complete, initrammeled, absolute [= not real] manifestation of violence (as the pure concept would require), war would of its own in-dependent [⊥] will usurp the place of policy the moment policy had brought it into being; it would then drive policy out of office and rule by the laws of its own nature.(I, 23}
- "…but if it is true that the first conception [absolute war] is never fully realized it is also true that there is no war in which the second would be exclusively applicable [i.e.] by abstracting from [= without links with] the first. (VIII, III a)
- ***The two logics at the same time**: "but if it is true that the first conception [= absolute war] is never fully [= not in part] realized, it is also true that there is no war in which the second [the real war] would be exclusively applicable [i.e.] by abstracting from [=without links with] the first. (VIII, III a)"*
- Making use of both classical and the non-classical logic Clausewitz' thinking was impenetrable to Western scholars referring to classical logic.

A contradiction-free Oneness-Trinity

- I completed an intuitionist interpretation of Christian Trinity through intuitonist logic. No contradictions.

  (Drago A. (2019), “Intuitionist reasoning in the tri-unitarian theology of Nicholas of Cues (1401-1464)”, *Journal of Logics and their Applications*, 6, n. 6, pp. 1143-1186.

À la Cusanus' intuitionist Tri-unity

(God = Oneness; Spiritus = Im-material;
¬≠ = not-other than; <==> = GKK translation and viceversa PSR)

(Drago A. (2019), "Intuitionist reasoning in the tri-unitarian theology of Nicholas of Cues (1401-1464)", *J. Logics & Appl.*, 6, n. 6, pp. 1143-1186).