Review of the behaviour of a model of type many predators - one prey

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This is a review of most recent results on bifurcation and chaos in a known predator-prey system. The system of n competing predators feeding on the same prey is of the type

$$X_i' = p_i\,\varphi_i(S)\,X_i - d_i\,X_i, \quad i = 1, ..., n, \tag{1a}$$

$$S' = H(S) - \sum_{i=1}^{n} q_i\,\varphi_i(S)\,X_i, \tag{1b}$$

where the variable S represents the prey and the variables X_i represent the predators. They are, of course, non-negative. The function φ_i is assumed non-decreasing.

We consider the case where

$$H(S) = r\,S\,\left(1 - \frac{S}{K}\right),\ \varphi_i(S) = \frac{S}{S + A_i}, \tag{2}$$

and where the parameters r, K and A_i are positive.

The dynamics in the coordinate planes representing one of the predators and the prey is well known and there is no more than one cycle. The system has no equilibrium, where predators coexist (in non-degenerate cases). But the predators can coexist in a cyclic and complicated way.

We here give conditions on parameters for extinction of one predator and for the possibility of coexistence.

Below we consider more detailed the case with only two predators. Very little is known about the behaviour when more than two predators coexist. We show numerical results for different kinds of coexistence, periodic or more complicated and discuss possible bifurcations.

In some cases it is possible to use a one dimensional model map for the Poincaré map on a subset of $s = const$ as a good approximation. It has the form

$$f(v) = b + v - \frac{k}{1 + e^v},$$

where $v = \ln(x_2/x_1)$ is a quantity for the ratio of the predators in transformed coordinates x_1 and x_2. This map can have two attractors for small parameter regions, but no more. We formulate interesting open questions for the behaviour of this map. Two attractors in the case the model is working can also be found for small parameters region in the original system.

If this model map is not working there might be more attractors. We give an example of four coexisting attractors and discuss their bifurcations. In these cases there is often observed spiral-like chaos, not well understood yet.

Literature

- Söderbacka G. J.Model map and multistability for a two predator–one prey system. Differential Equations and Control Processes. 2023. N 1. pp 12-23.