

Attractors with non-invariant interior

Minkov S. S.^{1*}, Okunev A. V.¹, Shilin I. S.¹

¹*Brook Institute of Electronic Control Machines, Moscow, Russia*

^{*} *stanislav.minkov@yandex.ru*

The properties of generic endomorphisms are somewhat different from the properties of generic diffeomorphisms. It is conjectured that C^1 -generic diffeomorphisms (of a connected manifold) whose non-wandering set has a non-empty interior are transitive. In contrast, for endomorphisms there are known open examples of attractors with non-empty interior for non-transitive maps.

We build on the ideas of these examples to show that the interior of the nonwandering set or attractor can be not only non-empty, but also non-invariant, and in a persistent way. That is, we construct an open set of maps that take a point in the interior of the attractor to the boundary of it. This is another contrast with diffeomorphisms, as for a diffeomorphism the interior of an invariant compact set is always invariant. In the known examples of attractors with non-empty interior the interior also is invariant.

We focus on the smallest dimension where a robust example of this type is possible. However, the main result is also valid for any manifold of dimension higher than 2.

Our initial approach was to first construct a skew product over a circle extension with the required properties, and then perturb it in the class of endomorphisms and use the technique of Ilyashenko-Negut to regain the structure of a skew product and prove that the non-invariance properties are persistent. Later we came up with a simpler geometric argument. Nevertheless, the “main” map in the proof is a skew product.