NONLOCAL de SITTER GRAVITY AND ITS EXACT COSMOLOGICAL SOLUTIONS

Branko Dragovich
Institute of Physics, University of Belgrade, and
Mathematical Institute, Serbian Academy of Sciences
and Arts, Belgrade, Serbia

III International Conference
Mathematical Physics, Dynamical Systems and Infinite Dimensional Analysis (V.S. Vladimirov - 100, L.D. Kudryavtsev - 100, O.G. Smolyanov - 85)

5.07–13.07.2023, Dolgoprudny,
Moscow Region, Russia, online

Contents

Based on joint work with I. Dimitrijevic, Z. Rakic and J. Stankovic:
JHEP 12 (2022) o54; arXiv:2206.13515 [gr-qc].

1. Introduction

Standard Model of Cosmology

- General Relativity (GR) is classical theory of gravitation at all scales from the Solar system to the universe as a whole: $R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = 8\pi G T_{\mu\nu}$.
- At the current cosmic time the universe consists of 68 % of dark energy (DE), 27 % of dark matter (DM) and only 5 % of ordinary matter.
- DE = Λ, DM = CDM, ordinary matter of known elem. particles.
- DE causes accelerated expansion of the universe, DM is responsible for galaxy dynamics.

——————————- However ——————————

- DE and DM are not yet discovered in any experiment.
- GR is not confirmed on galaxy and larger cosmic scales without DE and DM. GR – singularities,problems with quantization.
- There is a sense to look for a modified gravity.
- There are many directions modify GR.
- Here we consider nonlocal approach to modification of GR.

2. Nonlocal de Sitter gravity

- Einstein equation and EH action

$$R_{\mu\nu} - \frac{1}{2} R\, g_{\mu\nu} = 8\pi G\, T_{\mu\nu}$$

$$S = \int d^4x \frac{\sqrt{-g}}{16\pi G} R + \int d^4x \sqrt{-g}\, \mathcal{L}(matter)$$

- What does mean modification of GR?

$$R \to f(R, \Lambda, R_{\mu\nu}, R^{\alpha}_{\mu\beta\nu}, \Box, \ldots), \quad \Box = \nabla^{\mu}\nabla_{\mu} = \frac{1}{\sqrt{-g}} \partial_{\mu} \sqrt{-g} g^{\mu\nu} \partial_{\nu}$$

2. Nonlocal de Sitter gravity

- There is no theoretical principle that could tell us in what direction to make modification of GR. Hence, many attempts!
- $f(R)$ modified gravity

$$S = \int d^4x \frac{\sqrt{-g}}{16\pi G} f(R) + \int d^4x \sqrt{-g}\, \mathcal{L}(matter)$$

- nonlocal modified gravity

$$S = \int d^4x \frac{\sqrt{-g}}{16\pi G} f(R, \Box, \Box^{-1}, ...) + \int d^4x \sqrt{-g}\, \mathcal{L}(matter)$$

2. Nonlocal de Sitter gravity

- Our nonlocal de Sitter gravity model

$$S = \frac{1}{16\pi G} \int \Big(R - 2\Lambda + \sqrt{R - 2\Lambda}\, \mathcal{F}(\Box)\, \sqrt{R - 2\Lambda} \Big) \sqrt{-g}\, d^4x$$

where $\mathcal{F}(\Box) = \sum_{n=1}^{+\infty} \left(f_n \Box^n + f_{-n} \Box^{-n} \right)$ and Λ is cosmological constant. Motivation: string theory, UV and IR.

- Simple and natural construction of nonlocal term

$$R - 2\Lambda = \sqrt{R - 2\Lambda}\, \sqrt{R - 2\Lambda} \to \sqrt{R - 2\Lambda}\, \mathcal{F}(\Box)\, \sqrt{R - 2\Lambda}$$

We consider nonlocal modification without matter sector, but we obtain effect of dark matter and dark energy at the cosmological scale.

2. Nonlocal de Sitter gravity

Action for a class of models:

$$S = \frac{1}{16\pi G} \int_M \Big(R - 2\Lambda + P(R)\mathcal{F}(\Box)Q(R) \Big) \sqrt{-g}\, d^4x$$

where $P(R)$ and $Q(R)$ are some differentiable functions of scalar curvature R.

Equations of motion (EoM):

$$G_{\mu\nu} + \Lambda g_{\mu\nu} - \frac{1}{2} g_{\mu\nu} P(R)\mathcal{F}(\Box)Q(R) + (R_{\mu\nu} - K_{\mu\nu})\,\Phi$$
$$+ \frac{1}{2}\sum_{n=1}^{\infty} f_n \sum_{\ell=0}^{n-1} \big(g_{\mu\nu} g^{\alpha\beta} \partial_\alpha \Box^\ell P(R) \partial_\beta \Box^{n-1-\ell} Q(R)$$
$$- 2\partial_\mu \Box^\ell P(R) \partial_\nu \Box^{n-1-\ell} Q(R) + g_{\mu\nu} \Box^\ell P(R) \Box^{n-\ell} Q(R) \big) = 0,$$

where $K_{\mu\nu} = \nabla_\mu \nabla_\nu - g_{\mu\nu}\Box$, $\Phi = P'(R)\mathcal{F}(\Box)Q(R) + Q'(R)\mathcal{F}(\Box)P(R)$, and $'$ denotes derivative on R.

2. Nonlocal de Sitter gravity

A way to solve EoM

- $P(R) = Q(R) = \sqrt{R - 2\Lambda}$
- $\Box\sqrt{R-2\Lambda} = q\sqrt{R-2\Lambda}, \quad \Box^{-1}\sqrt{R-2\Lambda} = q^{-1}\sqrt{R-2\Lambda}, \quad q \neq 0$
  $\mathcal{F}(\Box)\,\sqrt{R-2\Lambda} = \mathcal{F}(q)\,\sqrt{R-2\Lambda}$
- Very simple form of EoM

$$(G_{\mu\nu} + \Lambda g_{\mu\nu})\,(1 + \mathcal{F}(q)) + \frac{1}{2}\mathcal{F}'(q) S_{\mu\nu}(\sqrt{R-2\Lambda}, \sqrt{R-2\Lambda}) = 0$$

where

$$S_{\mu\nu}(P,P) = g_{\mu\nu}\big(\nabla^\alpha P\,\nabla_\alpha P + P\Box P\big) - 2\nabla_\mu P\,\nabla_\nu P, \quad P = \sqrt{R-2\Lambda}$$

- Equations of motion satisfied with conditions:
  $\mathcal{F}(q) = -1$ and $\mathcal{F}'(q) = 0.$

3. Exact cosmological solutions

- The universe is homogeneous and isotropic space at cosmic scale with FLRW metric

$$ds^2 = -dt^2 + a^2(t)\Big(\frac{dr^2}{1-kr^2} + r^2 d\theta^2 + r^2 \sin^2\theta\, d\phi^2\Big)$$

- We have to solve equation: $\Box\sqrt{R-2\Lambda} = q\sqrt{R-2\Lambda}$

$$\Box = -\frac{\partial^2}{\partial t^2} - 3H(t)\frac{\partial}{\partial t}, \quad H(t) = \frac{\dot a}{a},$$
$$R(t) = 6\left(\frac{\ddot a}{a} + \Big(\frac{\dot a}{a}\Big)^2 + \frac{k}{a^2}\right), \quad k \in \{0, +1, -1\}.$$

- Then $\mathcal{F}(\Box)\sqrt{R-2\Lambda} = \mathcal{F}(q)\sqrt{R-2\Lambda}$.

3. Exact cosmological solutions

- Equations of motion

$$(G_{\mu\nu} + \Lambda g_{\mu\nu})\,(1 + \mathcal{F}(q)) + \frac{1}{2}\mathcal{F}'(q) S_{\mu\nu}(\sqrt{R - 2\Lambda}, \sqrt{R - 2\Lambda}) = 0$$

have solutions if $\mathcal{F}(q) = -1, \quad \mathcal{F}'(q) = 0.$

- An example of nonlocal operator

$$\mathcal{F}(\Box) = -\frac{1}{2e}\Big(\frac{\Box}{q}\, e^{\frac{\Box}{q}} + \frac{q}{\Box}\, e^{\frac{q}{\Box}}\Big), \quad q = \zeta\Lambda \neq 0,$$

where ζ is dimensionless parameter depending of a concrete cosmological solution.

3. Exact cosmological solutions

- $a_1(t) = A\, t^{\frac{2}{3}}\, e^{\frac{\Lambda}{14} t^2}, \quad (k = 0,\ \Lambda \neq 0)$
- $a_2(t) = A\, e^{\frac{\Lambda}{6} t^2}, \quad (k = 0,\ \Lambda \neq 0)$
- $a_3(t) = A\, \cosh^{\frac{2}{3}} \left(\sqrt{\frac{3\Lambda}{8}}\, t\right), \quad (k = 0,\ \Lambda > 0)$
- $a_4(t) = A\, \sinh^{\frac{2}{3}} \left(\sqrt{\frac{3\Lambda}{8}}\, t\right), \quad (k = 0,\ \Lambda > 0)$
- $a_5(t) = A \left(1 + \sin \left(\sqrt{-\frac{3\Lambda}{2}}\, t\right)\right)^{\frac{1}{3}}, \quad (k = 0,\ \Lambda < 0)$
- $a_6(t) = A \left(1 - \sin \left(\sqrt{-\frac{3\Lambda}{2}}\, t\right)\right)^{\frac{1}{3}}, \quad (k = 0,\ \Lambda < 0)$
- $a_7(t) = A\, \sin^{\frac{2}{3}} \left(\sqrt{-\frac{3\Lambda}{8}}\, t\right), \quad (k = 0,\ \Lambda < 0)$
- $a_8(t) = A\, \cos^{\frac{2}{3}} \left(\sqrt{-\frac{3\Lambda}{8}}\, t\right), \quad (k = 0,\ \Lambda < 0)$
- $a_9(t) = A\, e^{\pm\sqrt{\frac{\Lambda}{6}}\, t}, \quad (k = \pm 1,\ \Lambda > 0)$
- $a_{10}(t) = A\, \cosh^{\frac{1}{2}} \left(\sqrt{\frac{3\Lambda}{2}}\, t\right), \quad (k = \pm 1,\ \Lambda > 0)$
- $a_{11}(t) = A\, \sinh^{\frac{1}{2}} \left(\sqrt{\frac{3\Lambda}{2}}\, t\right), \quad (k = \pm 1,\ \Lambda > 0)$
- + many anisotropic cosmological solutions.

4. Discussion: Case $a_1(t) = A\, t^{\frac{2}{3}}\, e^{\frac{\Lambda}{14}t^2}$

The Planck 2018 data for the ΛCDM universe are:

- $H_0 = (67.40 \pm 0.50)$ km/s/Mpc – Hubble parameter;
- $\Omega_m = 0.315 \pm 0.007$ – matter density parameter;
- $\Omega_\Lambda = 0.685$ – Λ density parameter;
- $t_0 = (13.801 \pm 0.024) \cdot 10^9$ yr – age of the universe;
- $w_0 = -1.03 \pm 0.03$ – ratio of pressure to energy density.
- $\Lambda = 3H_0^2\Omega_\Lambda = 0.98 \cdot 10^{-35} s^{-2}$.

Solution $a_1(t) = A\, t^{\frac{2}{3}}\, e^{\frac{\Lambda}{14}t^2}, \quad (k = 0,\ \Lambda \neq 0)$

- mimics dark matter $t^{\frac{2}{3}}$ and dark energy $e^{\frac{\Lambda}{14}t^2}$
- $\Lambda_1 = 1.05 \cdot 10^{-35}\, s^{-2}$ from $H_0 = \frac{2}{3}t_0^{-1} + \frac{1}{7}\Lambda t_0$.
- $\bar{\rho}_1(t_0) = \frac{3}{8\pi G}\left(H_0^2 - \frac{\Lambda_1}{3}\right) = 2.26 \times 10^{-30}\ \frac{g}{cm^3}$.
- $\rho(t_0) = \frac{3}{8\pi G}\left(H_0^2 - \frac{\Lambda}{3}\right) = 2.68 \times 10^{-30}\ \frac{g}{cm^3}$.
- $\rho_c = \frac{3}{8\pi G}H^2(t_0) = 8.51 \times 10^{-30}\ \frac{g}{cm^3}$.
-

$$\Omega_{\Lambda_1} = \frac{\rho_{\Lambda_1}}{\rho_c} = 0.734, \quad \Omega_\Lambda = \frac{\rho_\Lambda}{\rho_c} = 0.685, \quad \Delta\Omega_\Lambda = \Omega_{\Lambda_1} - \Omega_\Lambda = 0.049$$

$$\Omega_m = \frac{\rho(t_0)}{\rho_c} = 0.315, \quad \Omega_{m_1} = \frac{\bar{\rho}_1(t_0)}{\rho_c} = 0.266, \quad \Delta\Omega_m = \Omega_m - \Omega_{m_1} = 0.049.$$

4. Discussion: Case $a_1(t) = A\, t^{\frac{2}{3}}\, e^{\frac{\Lambda}{14} t^2}$

Effective energy density and pressure:

- $\bar\rho = \frac{2t^{-2} + \frac{9}{98}\Lambda^2 t^2 - \frac{9}{14}\Lambda}{12\pi G}\,, \qquad \bar p = -\frac{\Lambda}{56\pi G}\big(\frac{3}{7}\Lambda t^2 - 1\big).$
- $\bar w = \frac{\bar p}{\bar\rho} \to -1 \quad \text{when} \quad t \to \infty$
- $t \to 0 : \ \bar\rho \to \infty, \quad \bar p \to \frac{\Lambda}{56\pi G}.$
- One can also compute time (t_m) when the Hubble parameter has minimum value H_m, i.e. $t_m = 21.1 \cdot 10^9$ yr and $H_m = 61.72$ km/s/Mpc.
- Beginning of the universe expansion acceleration was at $t_a = 7.84 \cdot 10^9$ yr, or in other words at 5.96 billion years ago.

4. Discussion: Case $a(t) = A\, e^{\frac{\Lambda}{6} t^2}$

$$S = \frac{1}{16\pi G} \int_M \left(R - 2\Lambda + \sqrt{R - 2\Lambda}\, \mathcal{F}(\Box)\, \sqrt{R - 2\Lambda} \right) \sqrt{-g}\, d^4 x$$

Cosmological bounce solution: $a(t) = A\, e^{\frac{\Lambda}{6} t^2}, \quad k = 0, \ \Lambda \neq 0$

- $R(t) = 2\Lambda(1 + \frac{2}{3}\Lambda t^2), \qquad H(t) = \frac{1}{3}\Lambda t$
- $\Box\sqrt{R - 2\Lambda} = -\Lambda\sqrt{R - 2\Lambda}$
- $\mathcal{F}(-\Lambda) = -1, \quad \mathcal{F}'(-\Lambda) = 0$
- $\bar{\rho} = \frac{\Lambda}{8\pi G}\left(\frac{\Lambda}{3} t^2 - 1\right), \qquad \bar{p} = -\frac{\Lambda}{24\pi G}\left(\Lambda t^2 - 1\right)$
- $\bar{w} = \frac{\bar{p}}{\bar{\rho}} \to -1$ when $t \to \infty$

4. Discussion: Case $a(t) = A\, e^{\pm\sqrt{\frac{\Lambda}{6}}t}, \quad k = \pm 1,\ \Lambda > 0$

$$S = \frac{1}{16\pi G}\int_M \Big(R - 2\Lambda + \sqrt{R-2\Lambda}\,\mathcal{F}(\Box)\,\sqrt{R-2\Lambda}\Big)\sqrt{-g}\,d^4x$$

Cosmological solution: $a(t) = A\, e^{\pm\sqrt{\frac{\Lambda}{6}}t}, \quad k = \pm 1,\ \Lambda > 0$

- $R(t) = \frac{6k}{A^2} e^{\mp\sqrt{\frac{2}{3}\Lambda}\,t} + 2\Lambda, \qquad H = \pm\sqrt{\frac{\Lambda}{6}}$
- $\Box\sqrt{R-2\Lambda} = \frac{\Lambda}{3}\sqrt{R-2\Lambda},$
- $\mathcal{F}\left(\frac{\Lambda}{3}\right) = -1, \quad \mathcal{F}'\left(\frac{\Lambda}{3}\right) = 0$
- $\bar\rho = \frac{-\frac{\Lambda}{2} + \frac{3k}{A^2}\, e^{\mp\sqrt{\frac{2}{3}\Lambda}\,t}}{8\pi G}, \quad \bar p = \frac{\frac{\Lambda}{2} - \frac{k}{A^2}\, e^{\mp\sqrt{\frac{2}{3}\Lambda}\,t}}{8\pi G}$
- $\bar w = \frac{\bar p}{\bar\rho} \to -1, \quad -\frac{1}{3}, \quad$ when $\quad t \to \infty$

5. Conclusion

- We analyzed nonlocal de Sitter gravity model

$$S = \frac{1}{16\pi G} \int_M \Big(R - 2\Lambda + \sqrt{R - 2\Lambda}\, \mathcal{F}(\Box)\, \sqrt{R - 2\Lambda} \Big) \sqrt{-g}\; d^4 x$$

  as very simple and interesting model in several aspects.
- Model set up and EoM are relatively very simple.
- We found 11 exact cosmological (flat, closed and open) solutions. Some of them are nonsingular bounce, and also cyclic. There are also many anisotropic solutions.
- All solutions are new and do not exist in the local de Sitter case.
- The most interesting is exact vacuum cosmological solution

$$a(t) = A\, t^{\frac{2}{3}} e^{\frac{\Lambda}{14} t^2}, \quad \Lambda \neq 0, \;\; k = 0$$

  which mimics dark matter and dark energy. Computed cosmological parameters are in good agreement with observations.
- The next step is testing this model at other space-time scales.

Some relevant references

- I. Dimitrijevic, B. Dragovich, Z. Rakic, J. Stankovic, *Nonlocal de Sitter gravity and its exact cosmological solutions,* JHEP 12 (2022) 054; arXiv:2206.13515 [gr-qc].
- I. Dimitrijevic, B. Dragovich, A. S. Koshelev, Z. Rakic, J. Stankovic, *Cosmological solutions of a nonlocal square root gravity*, Phys. Lett. B 797 (2019) 134848; arXiv:1906.07560 [gr-qc].
- I. Dimitrijevic, B. Dragovich, A. S. Koshelev, Z. Rakic, J. Stankovic, *Some cosmological solutions of a new nonlocal gravity model*, Symmetry 12, 917 (2020), arXiv:2006.16041 [gr-qc].
- I. Dimitrijevic, B. Dragovich, Z. Rakic and J. Stankovic, *New cosmological solutions of a nonlocal gravity model*, Symmetry **2022** 14 (2022) 3, arXiv:2112.06312 [gr-qc].
- I. Dimitrijevic, B. Dragovich, Z. Rakic, J. Stankovic, *Variations of infinite derivative modified gravity,* Springer Proc. in Mathematics & Statistics **263** (2018) 91–111.
- I. Dimitrijevic, B. Dragovich, J. Grujic, A.S. Koshelev, Z. Rakic, *Cosmology of modified gravity with a nonlocal* $f(R)$, Filomat 33 (2019) 1163–1178, arXiv:1509.04254[hep=th].
- I. Dimitrijevic, B. Dragovich, A. S. Koshelev, J. Stankovic, Z. Rakic, *On nonlocal modified gravity and its cosmological solutions,* Springer Proc. in Mathematics & Statistics **191** (2016) 35–51.
- T. Biswas, T. Koivisto, A. Mazumdar, *Towards a resolution of the cosmological singularity in non-local higher derivative theories of gravity*, JCAP **1011** (2010) 008, arXiv:1005.0590v2 [hep-th].
- I Ya. Aref'eva, L. V. Joukovskaya and S. Y. Vernov, *Bouncing and accelerating solutions in nonlocal stringy models,* JHEP 07 (2007) 087; hep-th/0701184 .
- S. Capozziello and F. Bajardi, *Nonlocal gravity cosmology: An overview*, Int. J. Mod. Phys. D **31** (2022) 2230009, arXiv:2201.04512 [gr-qc].

THANK YOU FOR YOUR ATTENTION!