Existence of planar H-loops via Hardy's inequality

Roberta Musina
University of Udine, Italy

Given a continuous function on $\mathbb{R}^2$, we study the existence of non-constant, 2π-periodic solutions to the 2-dimensional Hamiltonian system

$$u'' = |u'|\,\mathrm{H}(u)iu'. \qquad (\mathcal{P})$$

Any non-constant solution to $(\mathcal{P})$ parametrizes a closed planar curve having prescribed curvature H at each point. Our interest in $(\mathcal{P})$ is motivated also by its relations with Arnold's problem on H-magnetic geodesics.

Problem $(\mathcal{P})$ admits a variational formulation; under reasonable assumptions on the prescribed (non-constant) curvature H, the associated energy functional has a nice Mountain-Pass geometry. However, due to the groups of dilations and translations in $\mathbb{R}^2$, the Palais-Smale condition fails to hold, and in fact there could exists unbounded Palais-Smale sequences.

We will present an existence result which is strongly based on a Hardy type inequality for functions of two variables.

This is joint work with Gabriele Cora (Università di Torino, Italy).