Dislocations of soliton lattices: experiment and theory

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Abstract

The main motivation of this talk has became the recent paper [1] published in PRL. This is experimental paper performed in Department of Applied Mathematics, University of Colorado, Boulder.

What was written in the abstract to this paper:

The observation of traveling breathers (TBs) with large-amplitude oscillatory tails realizes an almost 50-year-old theoretical prediction [E.A. Kuznetsov and A.V. Mikhailov, Stability of stationary waves in nonlinear weakly dispersive media, Zh. Eksp. Teor. Fiz. **67***, 1717 (1974) [E.A. Kuznetsov and A.V. Mikhailov, Sov. Phys. JETP* **40***, 855 (1975)] and generalizes the notion of a breather. Two strongly nonlinear TB families are created in a core-annular flow by interacting a soliton and a nonlinear periodic (cnoidal) carrier. Bright and dark TBs are observed to move faster or slower, respectively, than the carrier while imparting a phase shift. Agreement with model equations is achieved. Scattering of the TBs is observed to be physically elastic. The observed TBs generalize to many continuum and discrete systems.*

In our paper [2], in the framework of the KDV equation, such traveling breathers were called as dislocations of soliton lattice. This solution was constructed by means of the dressing method suggested by A.B. Shabat (1972) to the KDV case on the base of the Marchenko equation and later got development in a series of papers by V.E. Zakharov and A.B. Shabat. First time we together with Sasha Mikhailov [2] applied the dressing procedure to stability analysis of cnoidal wave for the KDV equation. Remarkably that this wave can be represented as soliton lattice that one allows to understand the nonlinear behavior of a the KDV soliton propagating along the cnoidal wave. In fact, propagation of solitons on the cnoidal wave background was a prototype of breather solitons in the NLS. It was the first example of application of the IST to the non-vanishing potentials. Later this idea was exploited in studies of the nonlinear stage of modulation instability in 1D NLS [4] where first time the breather-type soliton solution oscillating on the condensate was constructed.

In the experiment [1], the two-fluid system can be described by the so-called conduit equation. Our analysis shows that this equation in this experiment transforms with a good accuracy into the KDV equation. In fact, this experiment confirms the theoretical prediction of our paper.

At the end of this talk I'll consider my results devoted to stability of nonlinear waves in the integrable systems including the KP [3] and NLS [4–6] equations where the idea of [2] got its development.

[1] Yifeng Mao, Sathyanarayanan Chandramouli, Wenqian Xu, and Mark A Hoefer, *Observation of Traveling Breathers and Their Scattering in a Two-Fluid System*, Physical Review Letters, **131**, 147201 (2023).

[2] E.A. Kuznetsov, A.V. Mikhailov, *Stability of stationary waves in nonlinear weakly dispersive media.* ZhETF, **40**, 855, (1974) [Sov. Phys. JETP **40**, 855 (1975)].

[3] E.A. Kuznetsov, M.D. Spector, G.E. Falkovich, *On the Stability of Nonlinear Waves in Integrable Models.* Physica **10D**, 379 (1984).

[4] E.A. Kuznetsov, *Solitons in parametrically unstable plasma*, DAN SSSR **236**, 575 (1977) [Sov. Phys. Dokl., **22**, 507-508 (1977)].

[5] E.A. Kuznetsov, M.D. Spector, *Modulation instability of soliton train in the fiber communication systems*, Teor. Mat. Fiz. (Theor. Math. Phys.), **120**, 222-236 (1999).

[6] E.A. Kuznetsov, *Fermi-Pasta-Ulam recurrence and modulation instability* Pis'ma ZhETF, **105**, no.2 pp. 108-109 (2017) [JETP Letters, **105**, 125–129 (2017)].