Exploring postselection-induced quantum phenomena with time-bidirectional state formalism

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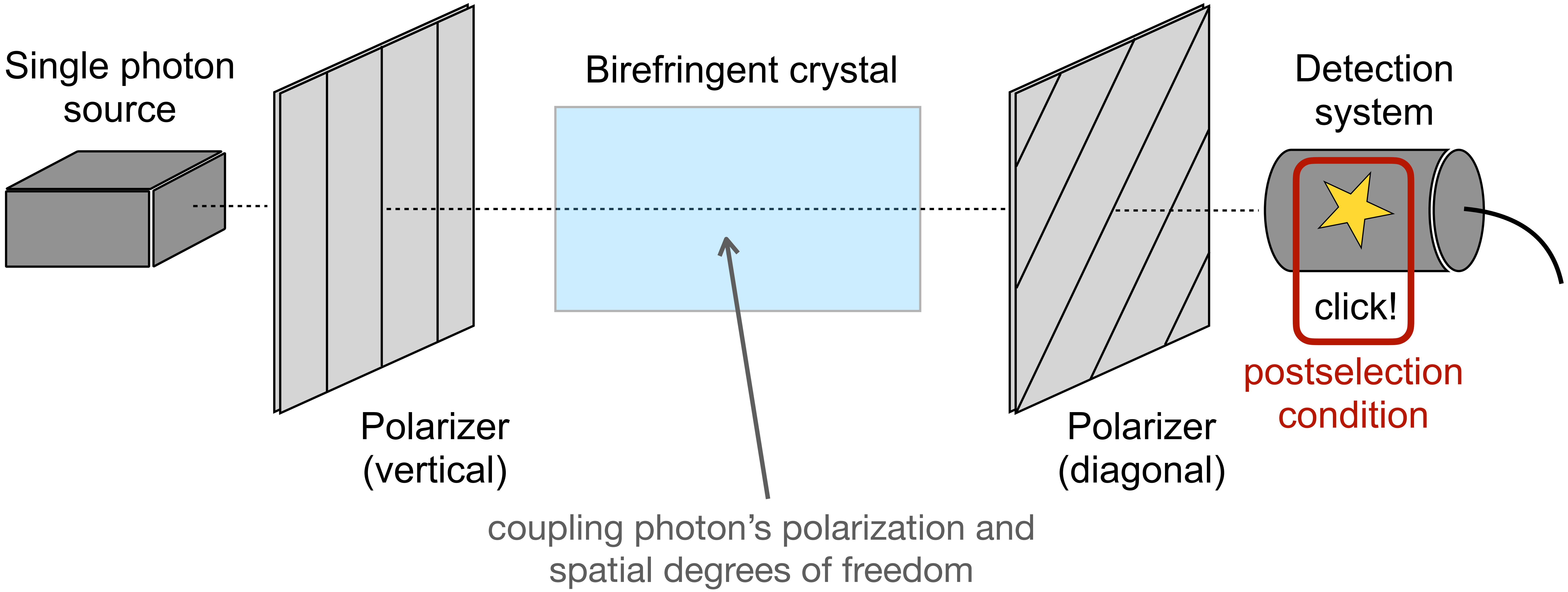
Outline

1. Introduction and motivation
2. Derivation of time-bidirectional states (TBS)
3. Tomographical reconstruction of a TBS
4. Experimental demonstration of employing TBS formalism

Introduction and motivation

Introduction

Postselection (in quantum physics) is a consideration of a realization of experiment given that a <u>particular measurement</u>, performed in the experiment, provides some <u>particular (prefixed) outcome</u>. Conceptually it means a consideration of <u>conditional probability distributions</u> of observables' outcomes or fixing a «boundary condition in time».



Applications of the postselection

Quantum metrology

P. B. Dixon et al., “Ultrasensitive beam deflection measurement via interferometric weak value amplification,” PRL **102**, 173601 (2009).
K. Lyons et al., “Power-recycled weak-value-based metrology,” PRL **114**, 170801 (2015).
D. R. M. Arvidsson-Shukur et al.“Quantum advantage in postselected metrology,” Nat. Com. **11**, 1–7 (2020).
…

Quantum complexity theory and contextuality

S. Aaronson, “Quantum computing, postselection, and probabilistic polynomial-time,” Proc. R. Soc. A: Math. **461**, 3473–3482 (2005).
M. S. Leifer and R. W. Spekkens, “Pre-and post-selection paradoxes and contextuality in quantum mechanics,” PRL **95**, 200405 (2005).
R. Kunjwal et al., “Anomalous weak values and contextuality: robustness, tightness, and imaginary parts,” Phys. Rev. A **100**, 042116 (2019).
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Quantum computation

A. W. Harrow et al. “Quantum algorithm for linear systems of equations,” PRL **103**, 150502 (2009).
E. Knill "Quantum gates using linear optics and postselection", Phys. Rev. A **66** (2022).
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Quantum communication

D. R. M. Arvidsson-Shukur and C. H. W. Barnes, “Quantum counterfactual communication without a weak trace,” Phys. Rev. A **94**, 062303 (2016).
A. Wander, E. Cohen, and L. Vaidman, “Three approaches for analyzing the counterfactuality of counterfactual protocols,” Phys. Rev. A **104**, 012610 (2021).
D.A. Kronberg "Vulnerability of quantum cryptography with phase–time coding under attenuation conditions", TMP, **214** (1), 140 (2023).
…

Two-state formalism

In order to properly describe quantum states in setups with postselection, it was proposed to use a «twofold» object, which both captures preparation (or preselection) and postselection.

History of the development

1) Two-state vector [1]: $(|\psi\rangle, \langle\phi|)$
2) Generalized two-state vector [2]: $\underline{\Psi} = \sum_i c_i(|\psi_i\rangle, \langle\phi_i|)$.
3) Two-state density vector [3]: $\eta = \sum_r p_r(\underline{\Psi}^r \otimes \underline{\Psi}^{r\dagger})$.
4) Mixed two-state vector [4]: $(\rho_{\text{post}}, \rho_{\text{pre}})$.
5) Time-bidirectional state [5]: $\eta \in \mathcal{S}(\mathcal{H} \otimes \mathcal{H}^\dagger)$

[1] Y. Aharonov, P. G. Bergmann, J. L. Lebowitz, Phys. Rev. **134**, B1410 (1964).
[2] Y. Aharonov, L. Vaidman, J. Phys. A Math. **24**, 2315 (1991).
[3] R. Silva, Y. Guryanova, N. Brunner, et al. Phys. Rev. A **89**, 012121 (2014).
[4] L. Vaidman, A. Ben-Israel, J. Dziewior, et al. Phys. Rev. A **96**, 032114 (2017).
[5] EOK Phys. Rev. A **107**, 032419 (2023).

Derivation of time-bidirectional states

Definitions and notations

$\mathscr{H}_{\mathrm{X}}$ – finite-dimensional complex Hilbert space assigned to particle X.

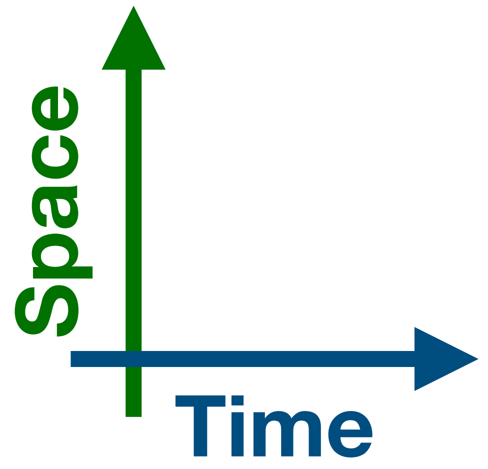
$\mathscr{L}(\mathscr{H})$ – set of linear operators acting on $\mathscr{H}$.

$\mathscr{U}(\mathscr{H}) := \{U \in \mathscr{L}(\mathscr{H}) \,|\, U^{\dagger}U = \mathbf{1}\}$ – set unitary operators acting on $\mathscr{H}$.

$\mathscr{S}(\mathscr{H}) := \{\rho \in \mathscr{L}(\mathscr{H}) \,|\, \mathrm{Tr}\rho = 1,\ \rho \geq 0\}$ – set of density operators acting on $\mathscr{H}$.

$\mathscr{E}(\mathscr{H}) := \{E \in \mathscr{L}(\mathscr{H}) \,|\, 0 \leq E \leq \mathbf{1}\}$– set of «effects» acting on $\mathscr{H}$.

Generalized postselection experiment



Alice

Bob

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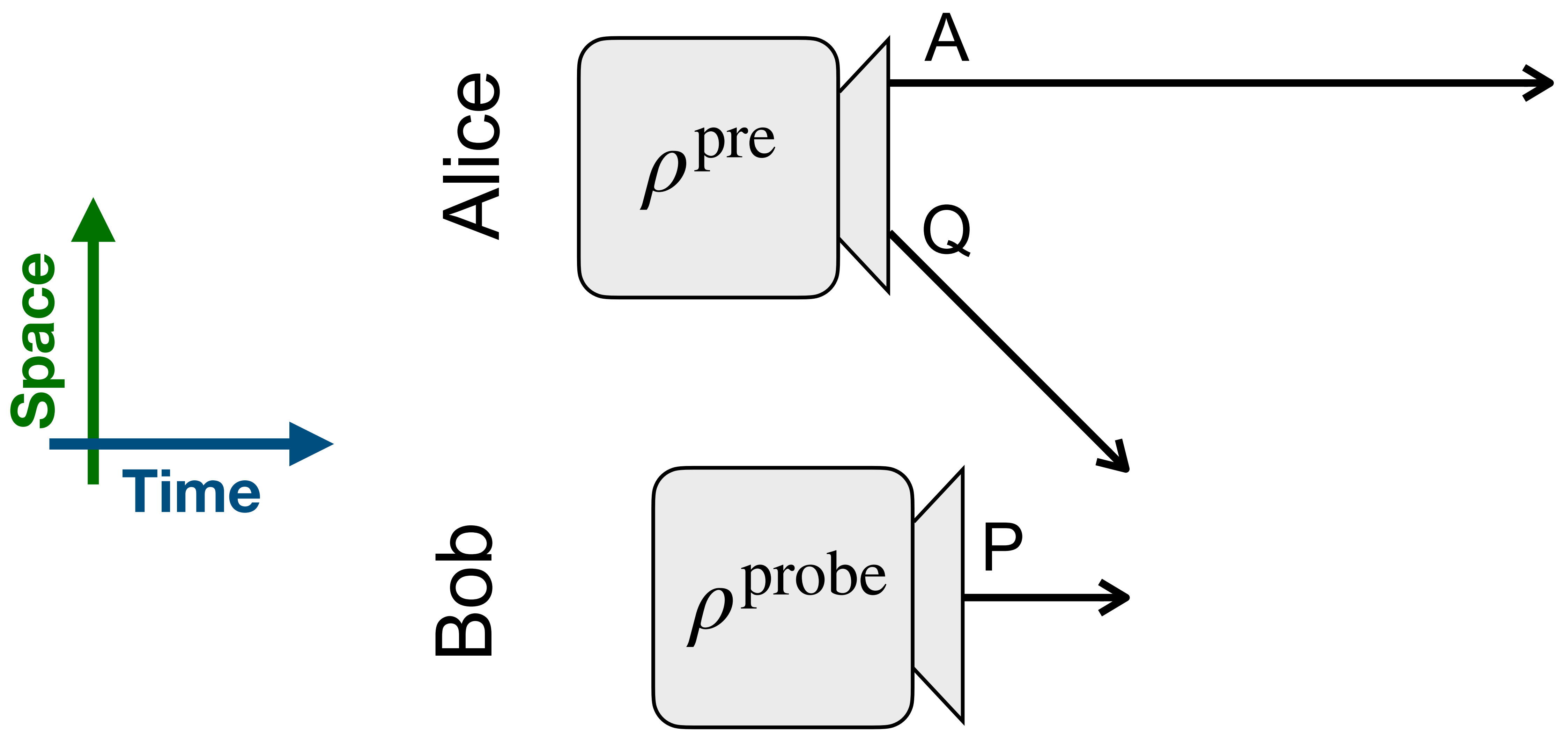

Q: the main quantum particle under consideration

A: ancillary particle that keeps possible initial entanglement with Q)

P: probe particle used for indirect measurement of Q

→ quantum particles

⇢ classical information

$$\rho^{\mathrm{pre}} \in \mathcal{S}(\mathcal{H}_{\mathrm{A}} \otimes \mathcal{H}_{\mathrm{Q}})$$

$$\rho^{\mathrm{probe}} \in \mathcal{S}(\mathcal{H}_{\mathrm{P}})$$

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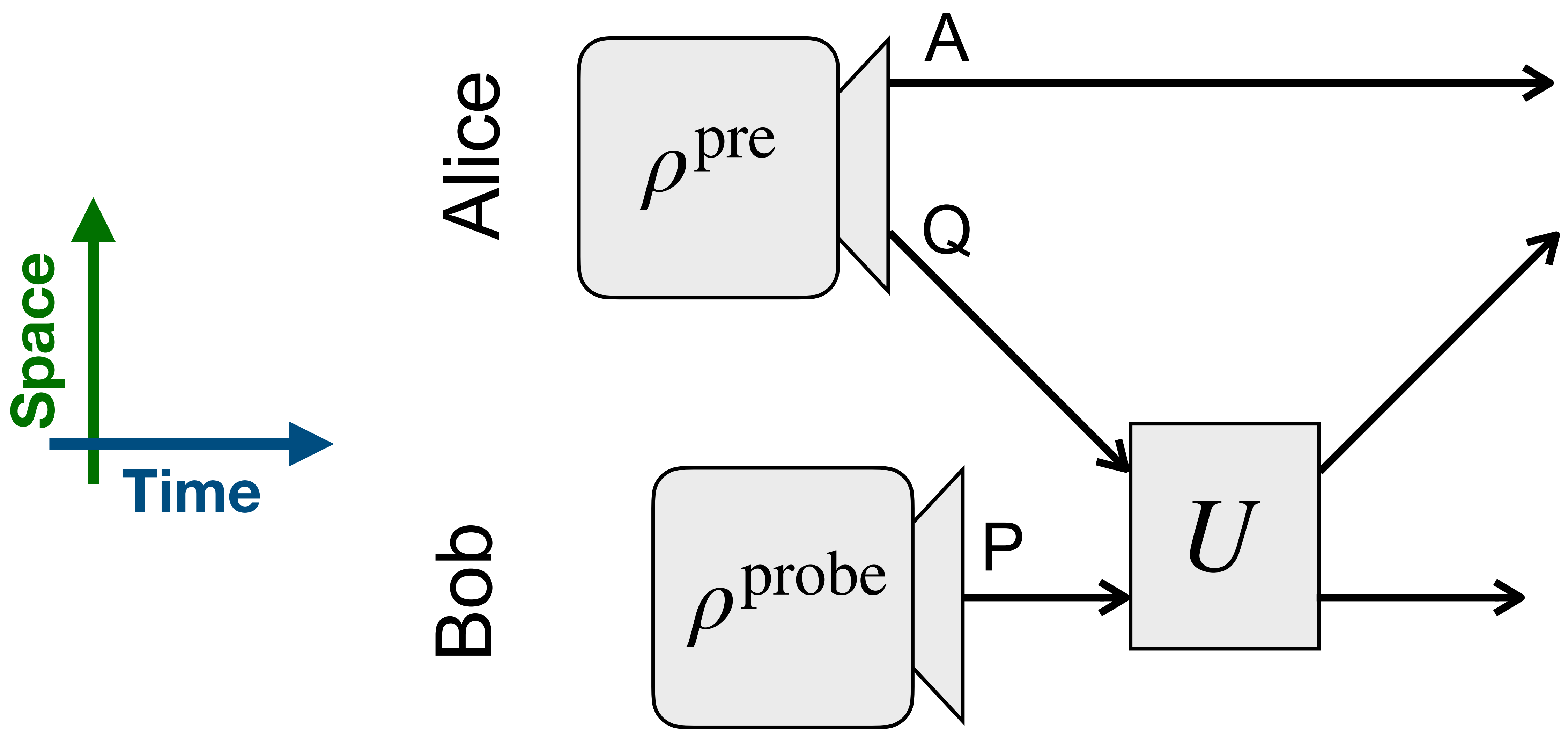

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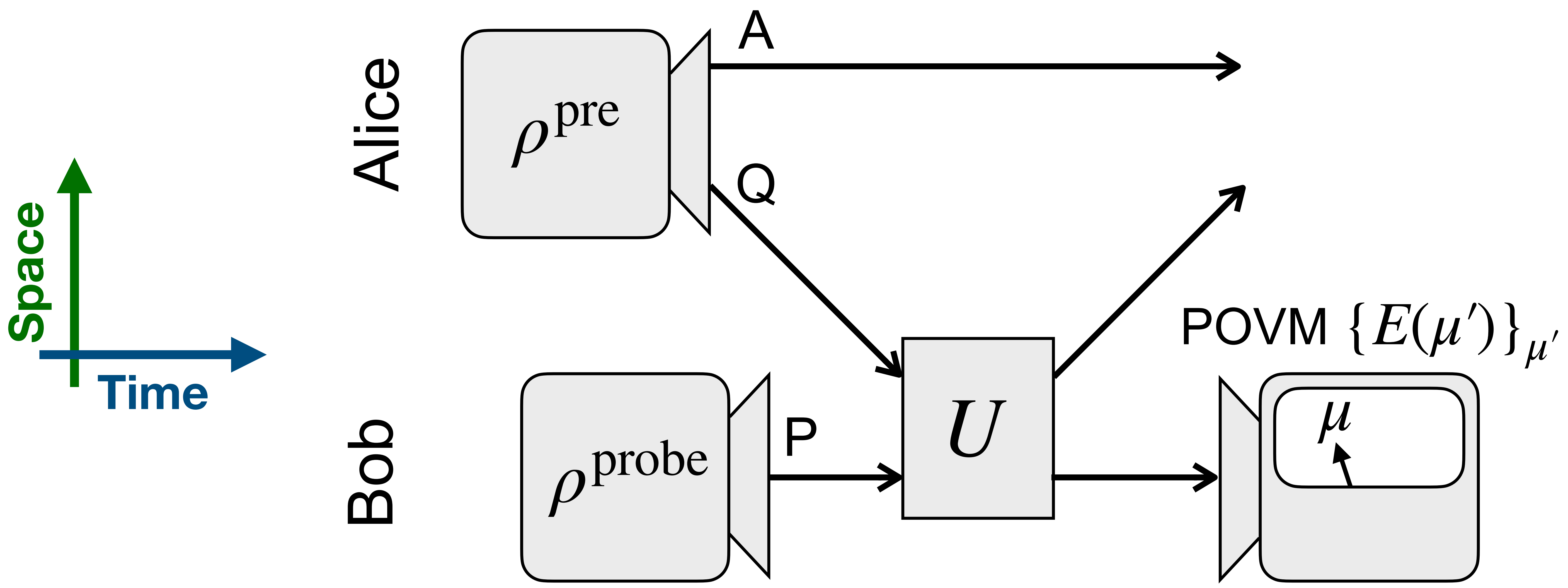

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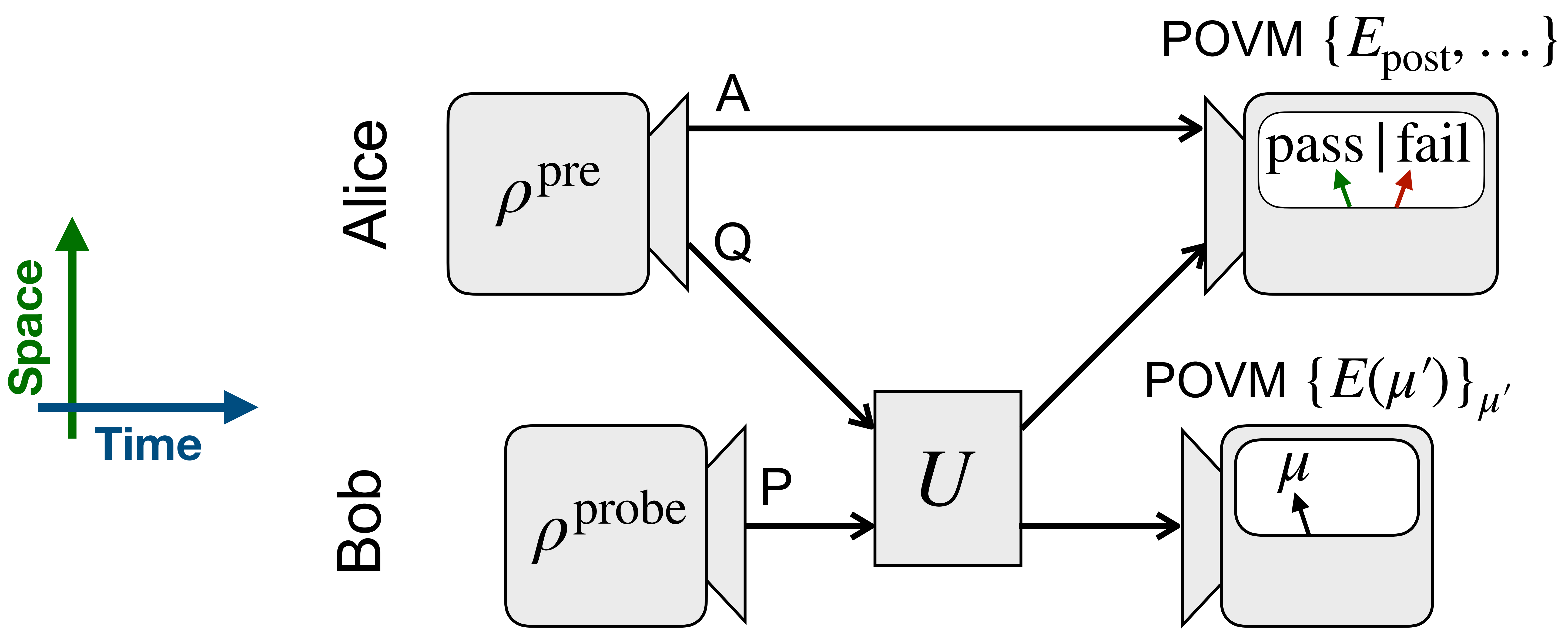

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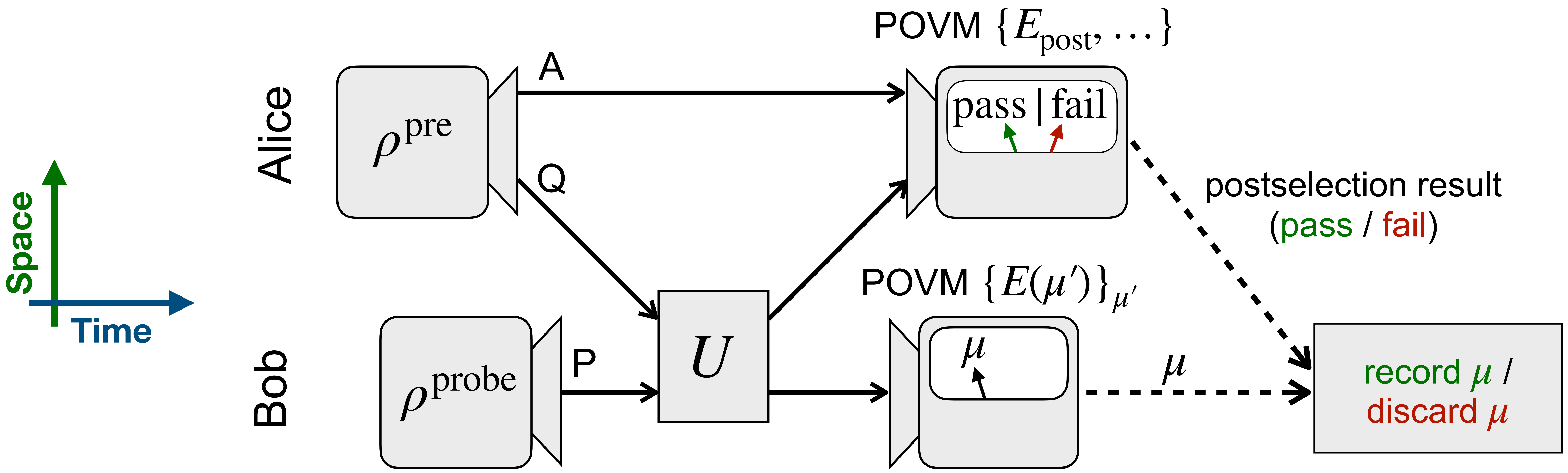

μ	E_{post}
-1	yes
~~2~~	~~no~~
~~4~~	~~no~~
5	yes
~~3~~	~~no~~

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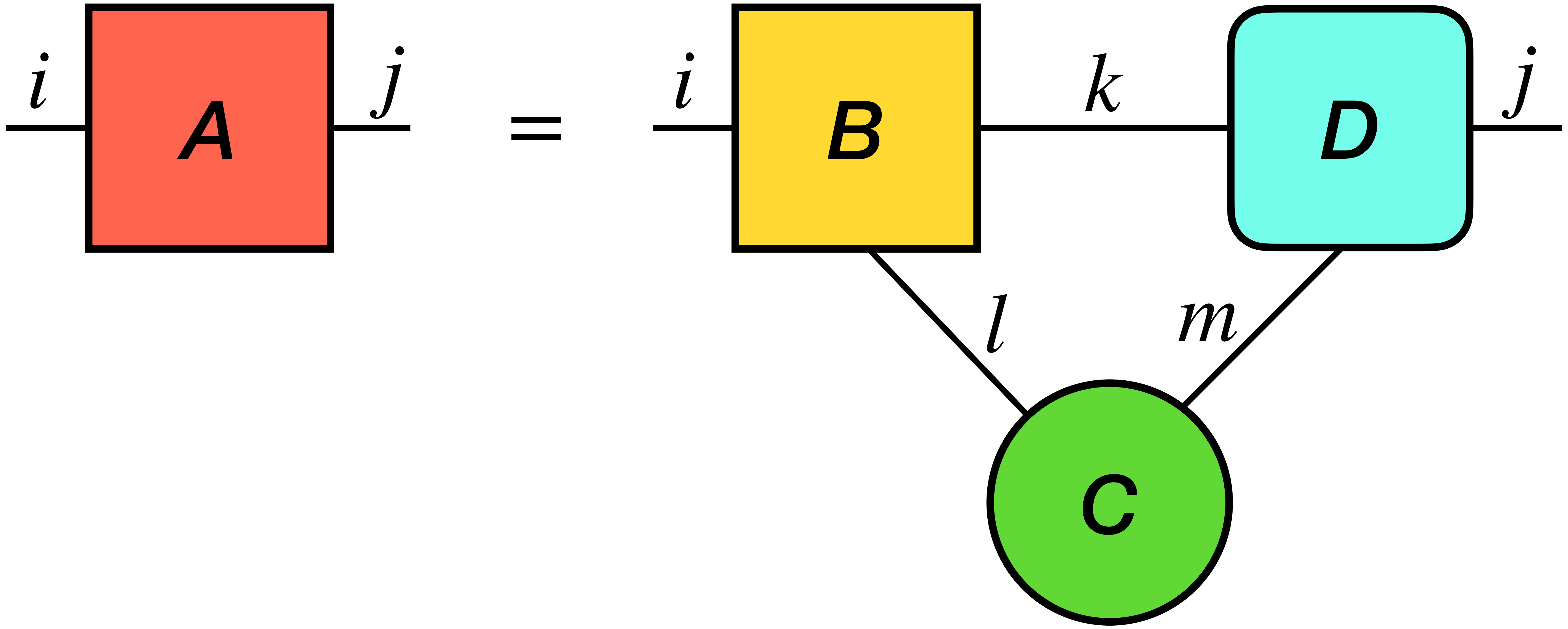
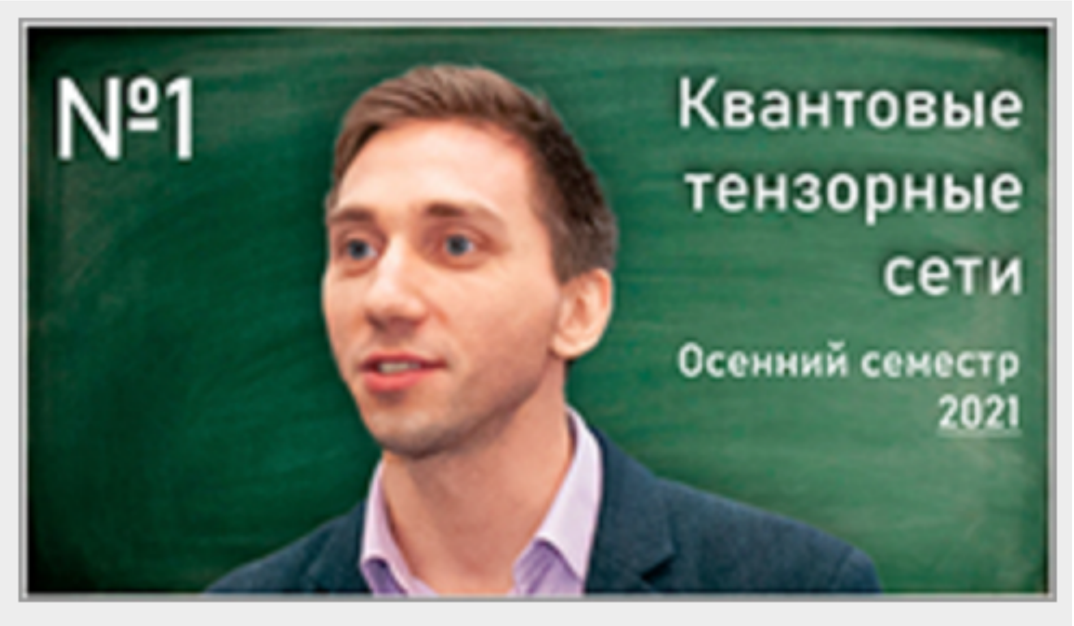
Tensor networks formalism

$$A_{ij} = \sum_{k,l,m} B_{ikl} C_{lm} D_{kmj}$$

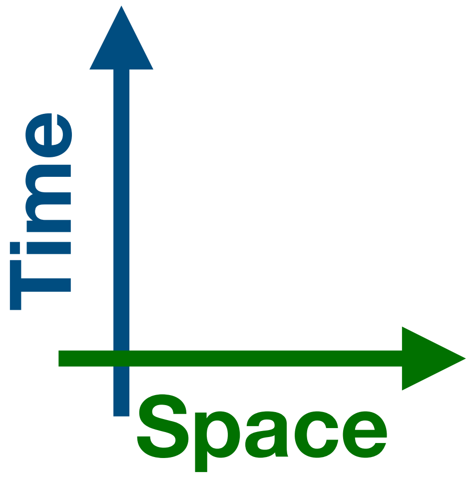


С.Ф. Филиппов «Квантовые тензорные сети»
https://www.mathnet.ru/conf1854



Calculations with tensor networks

$$\Pr(\mu, E_{\text{post}}) =$$



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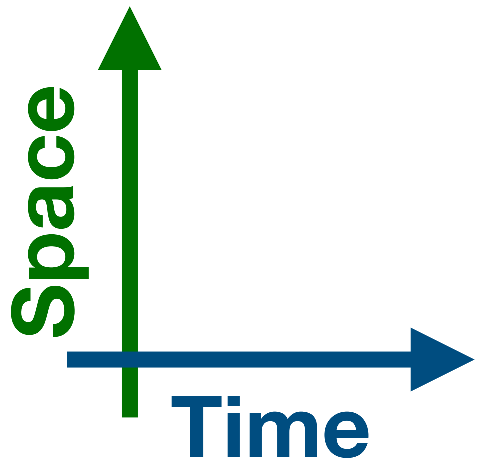

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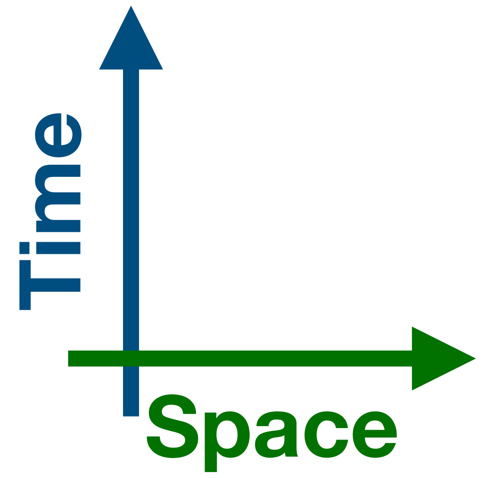

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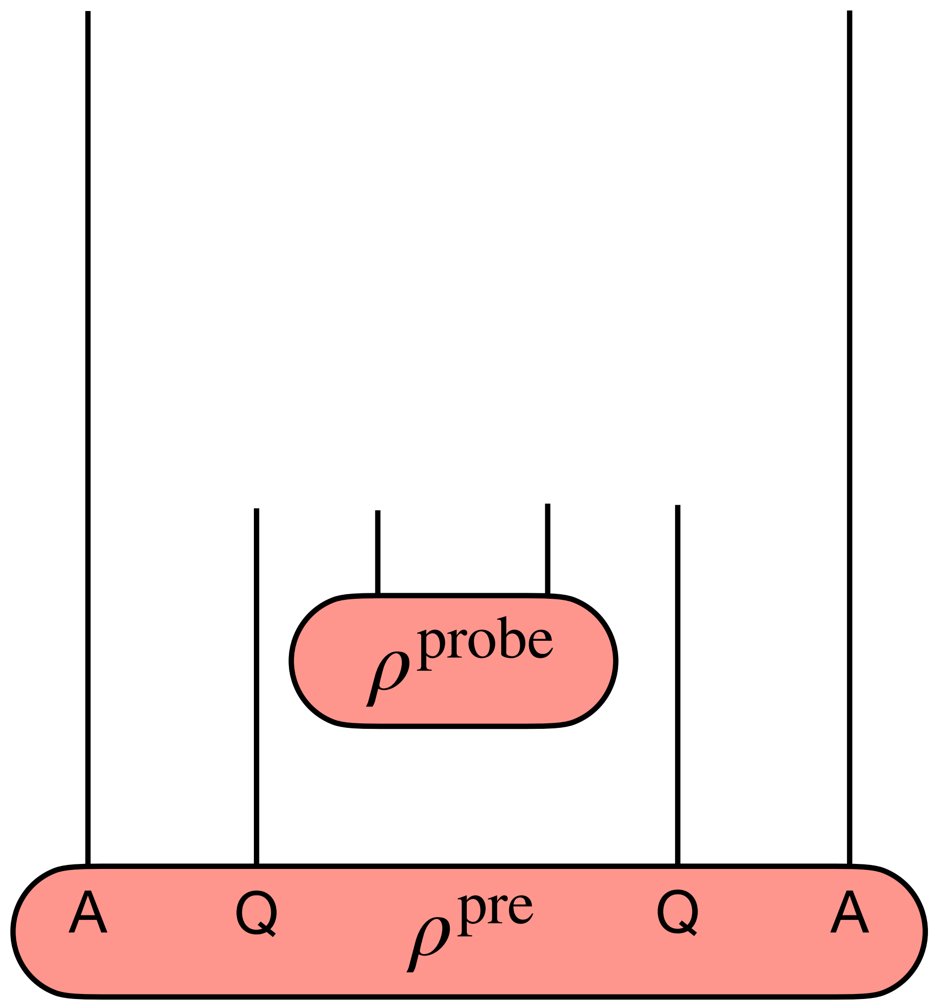

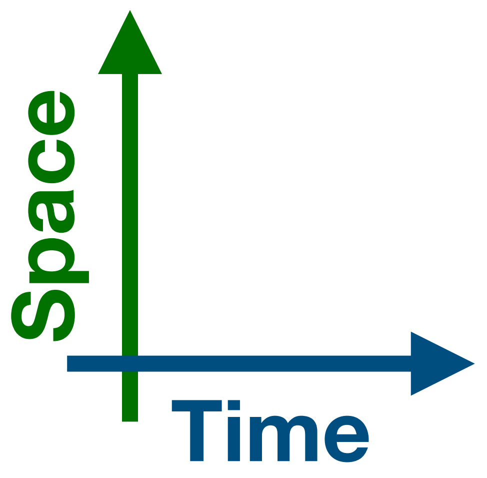

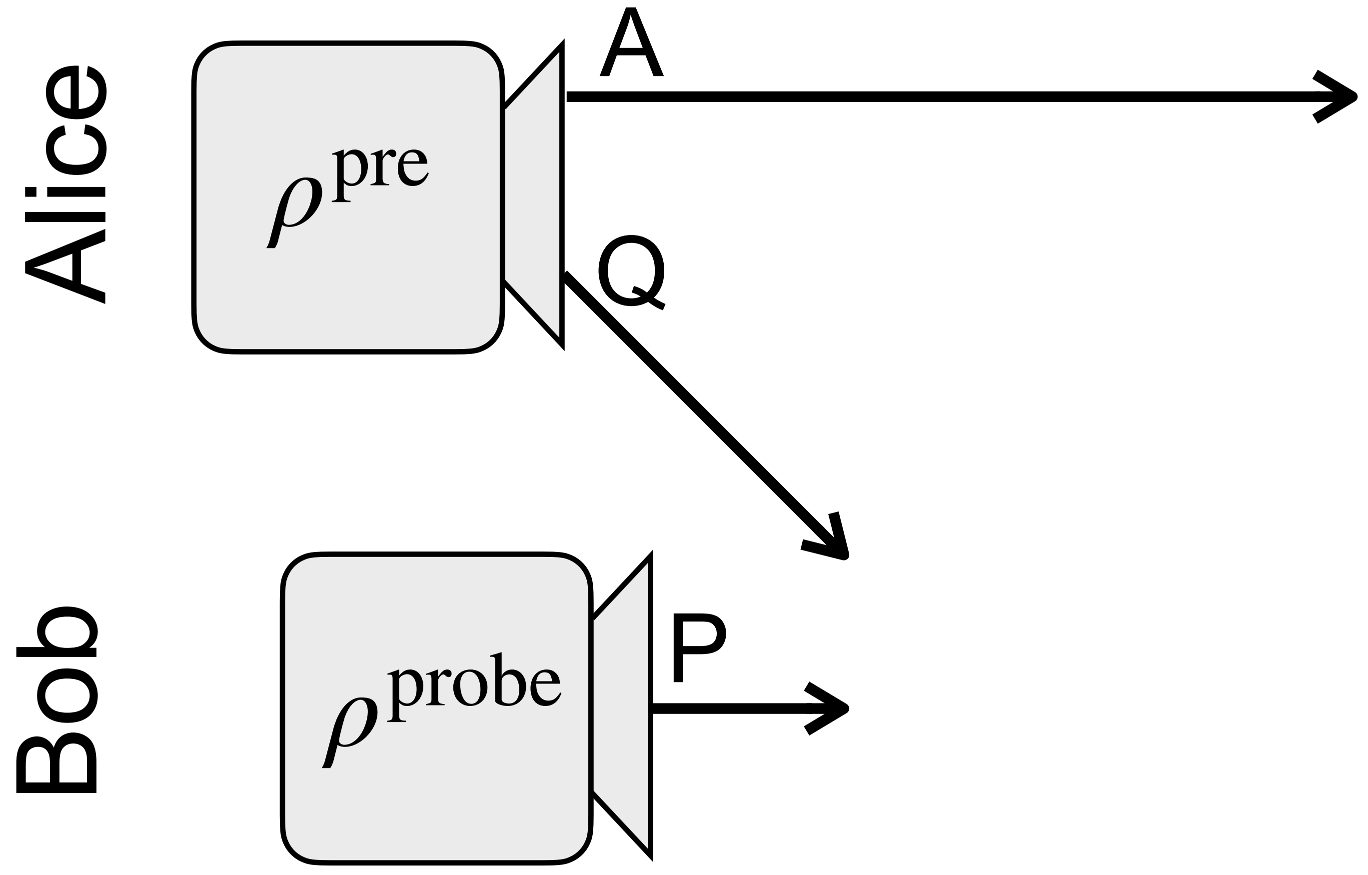

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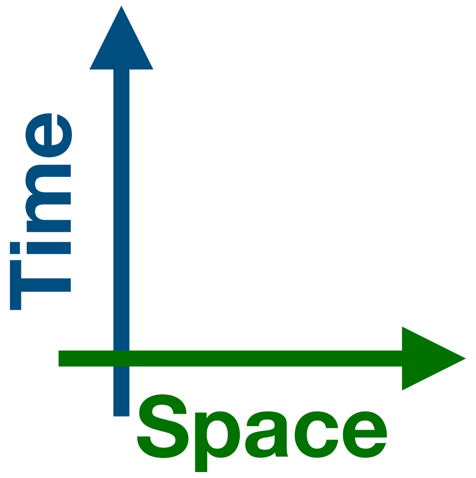

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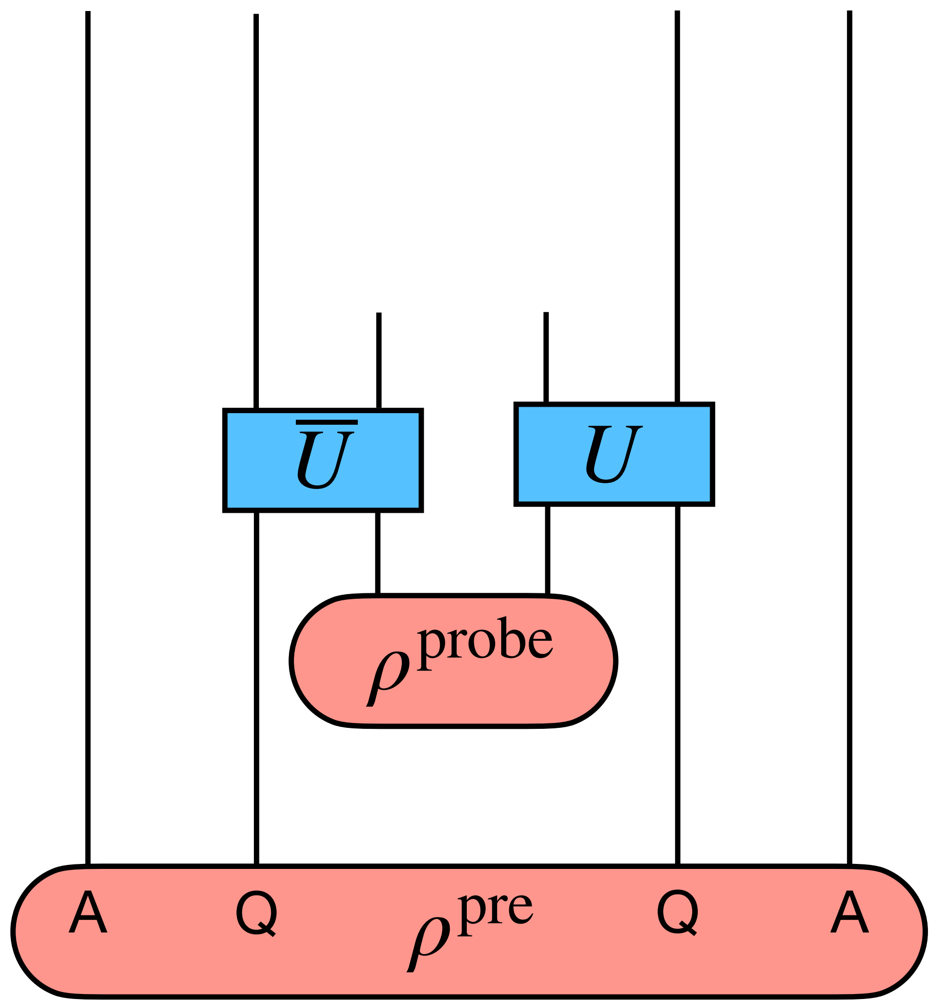

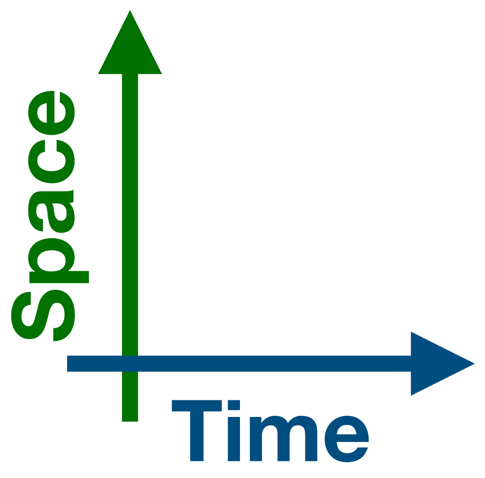

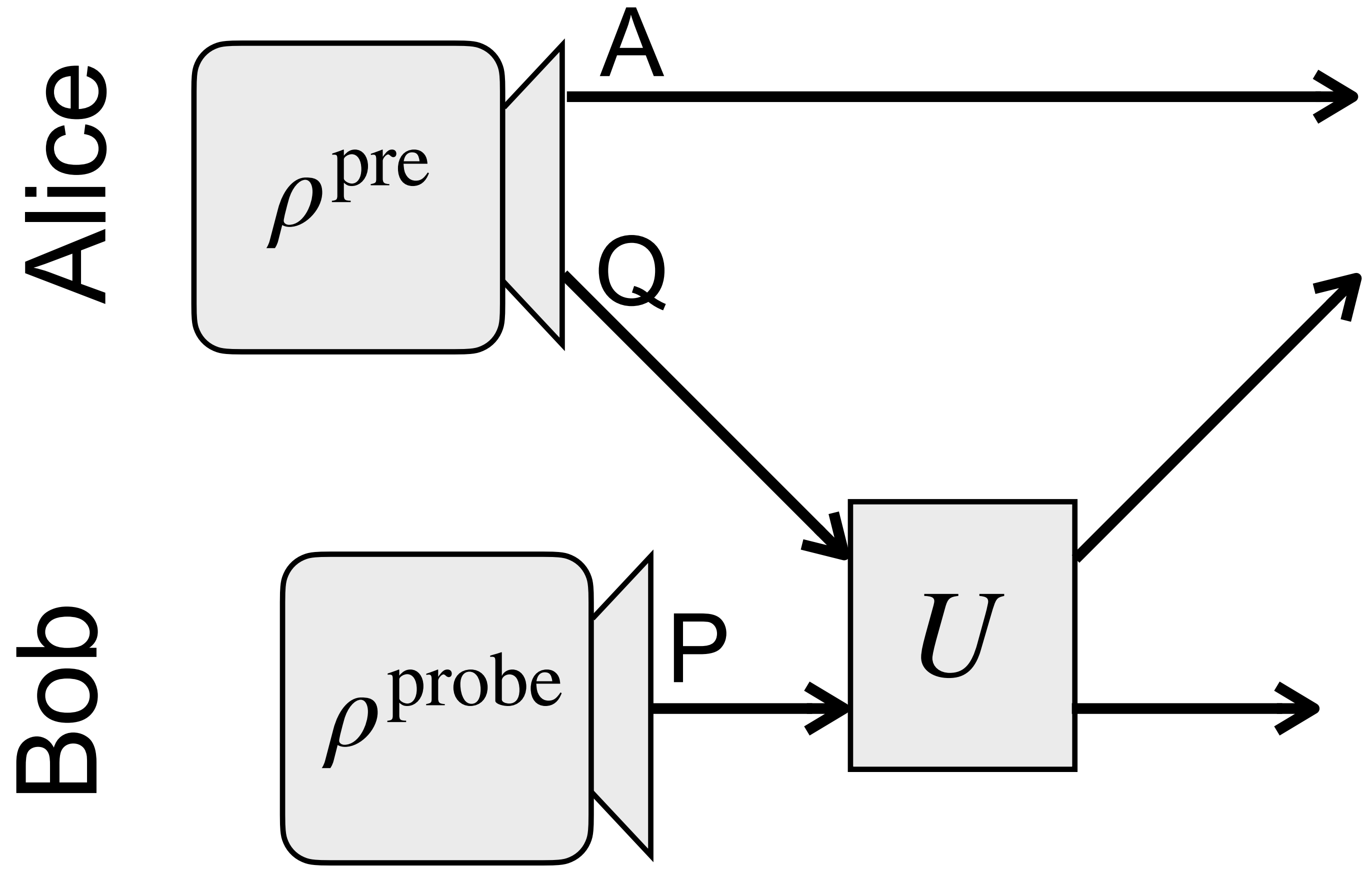

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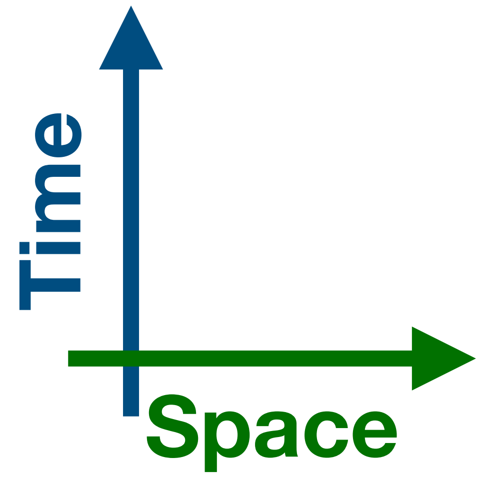

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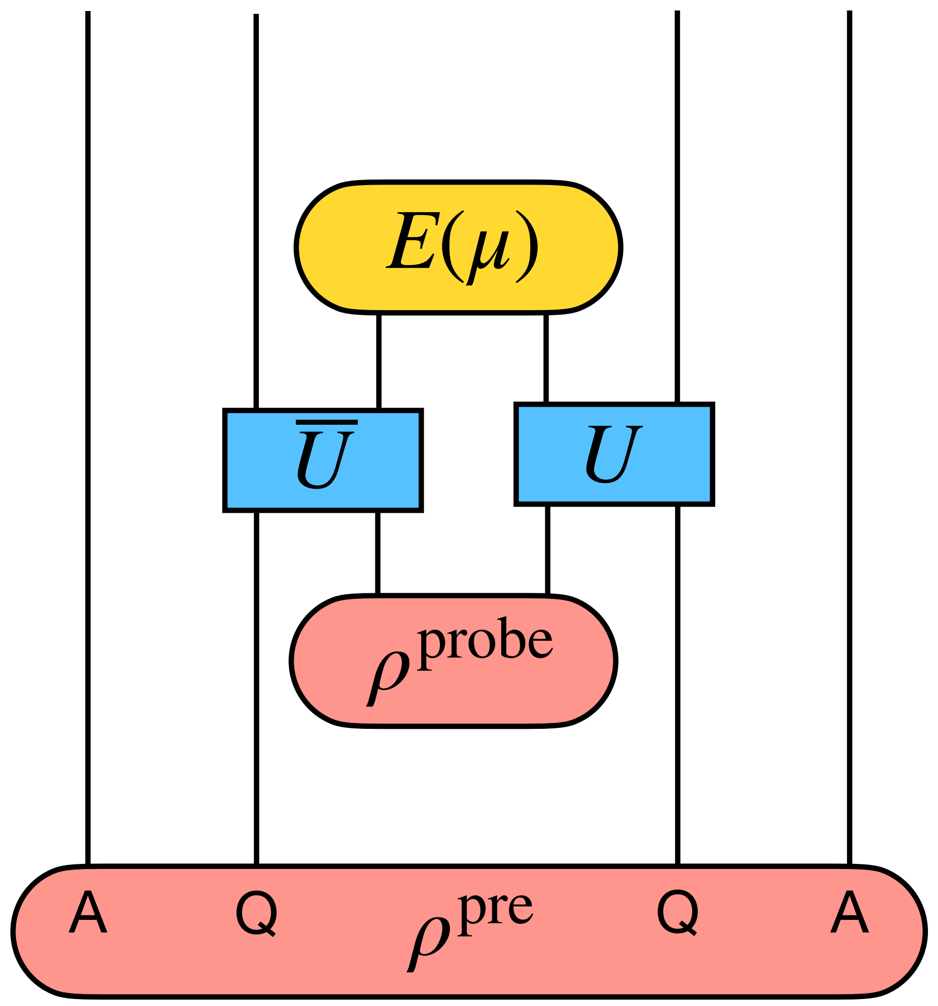

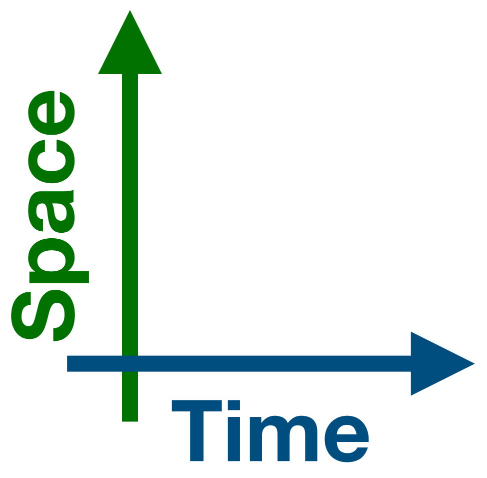

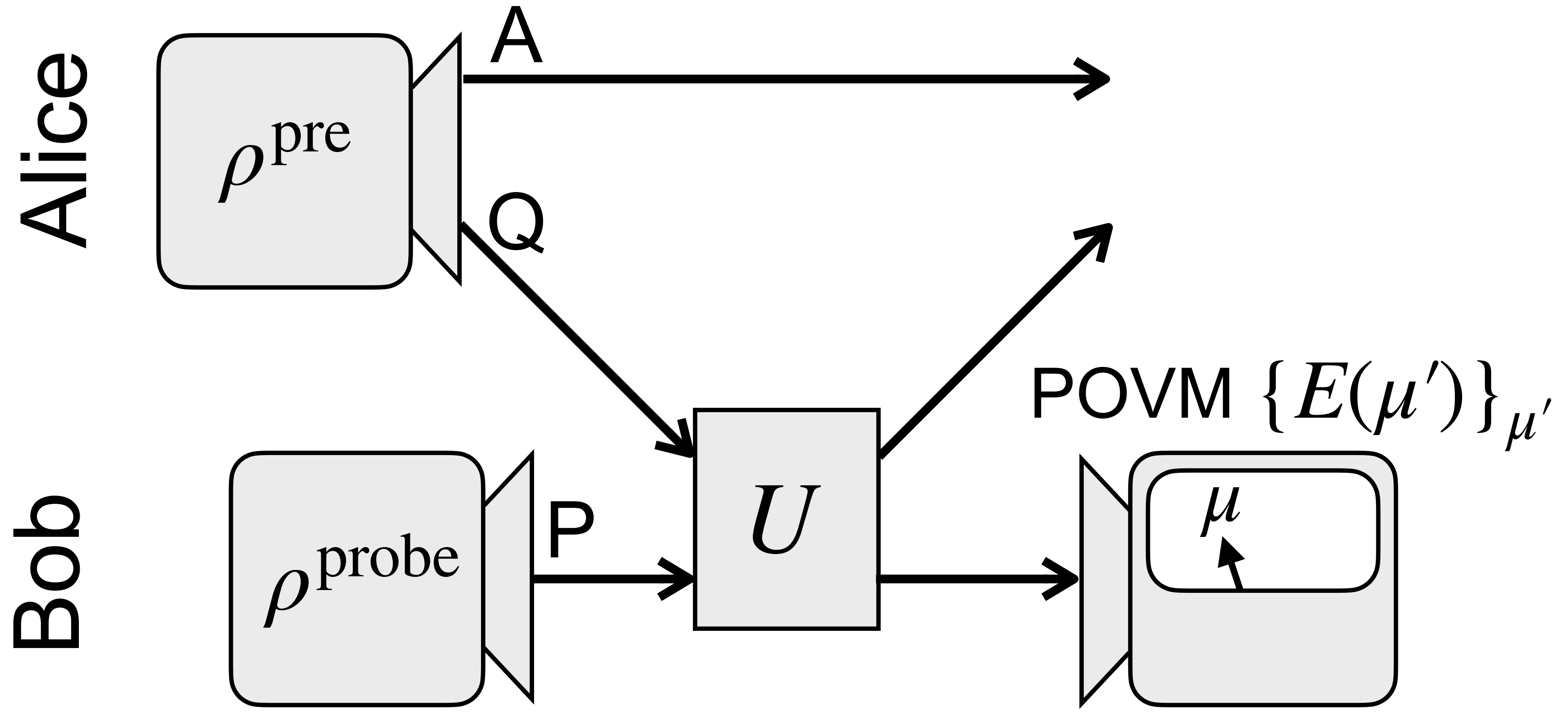

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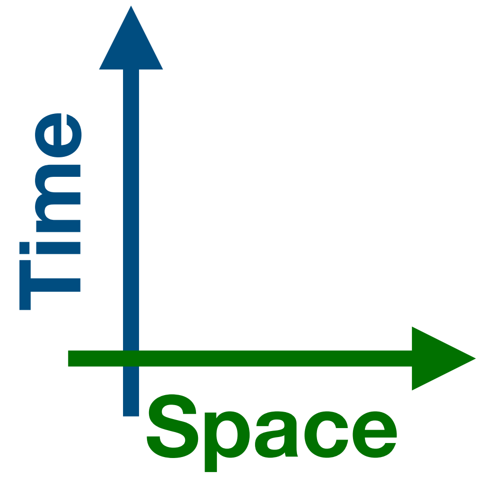

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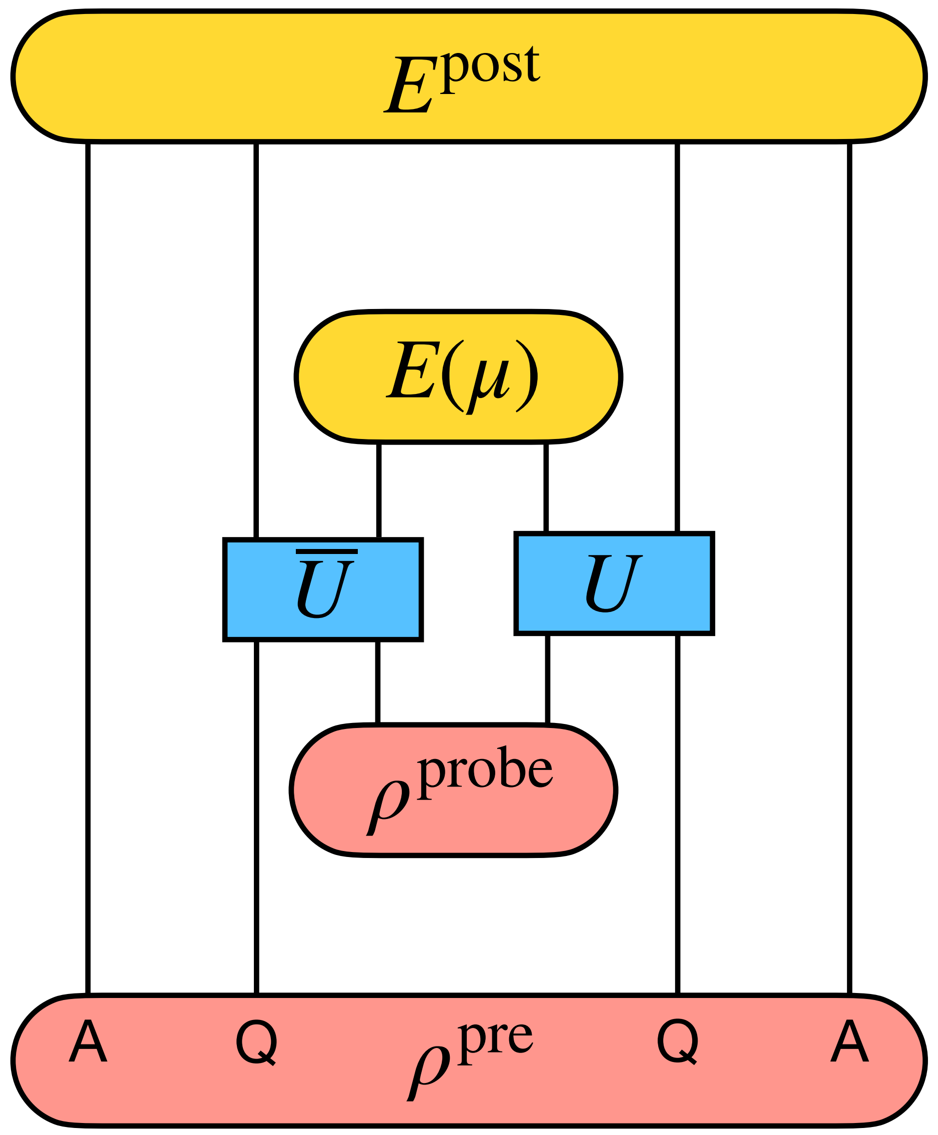
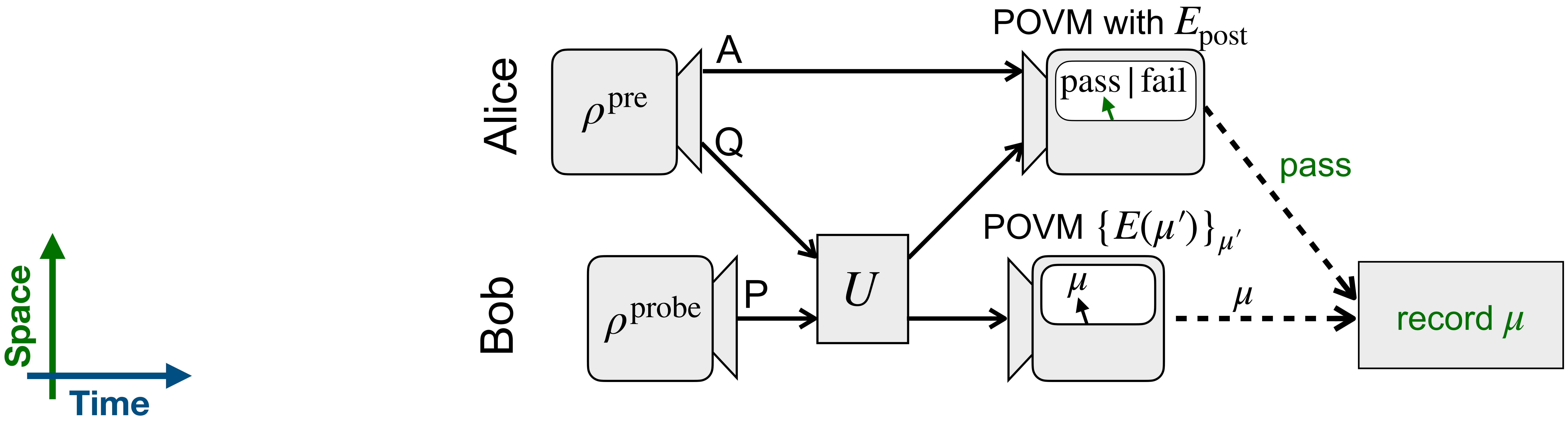


$$\sum\nolimits_{\dots} \rho^{\text{pre}}_{ii';kk'} \rho^{\text{probe}}_{mm'} U^{i;m}_{l;j} \overline{U}^{i';m'}_{l';j'} E(\mu)^{ll'} E^{jj';kk'}_{\text{post}}$$



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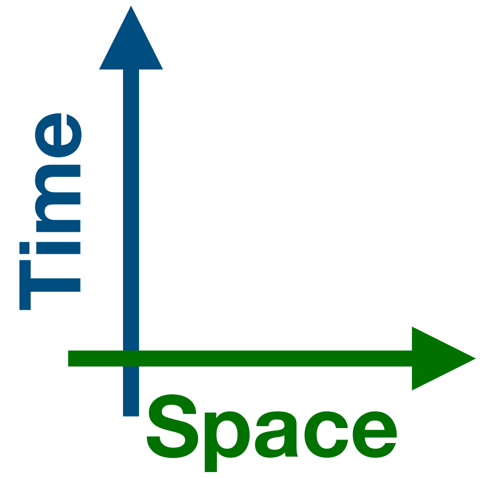

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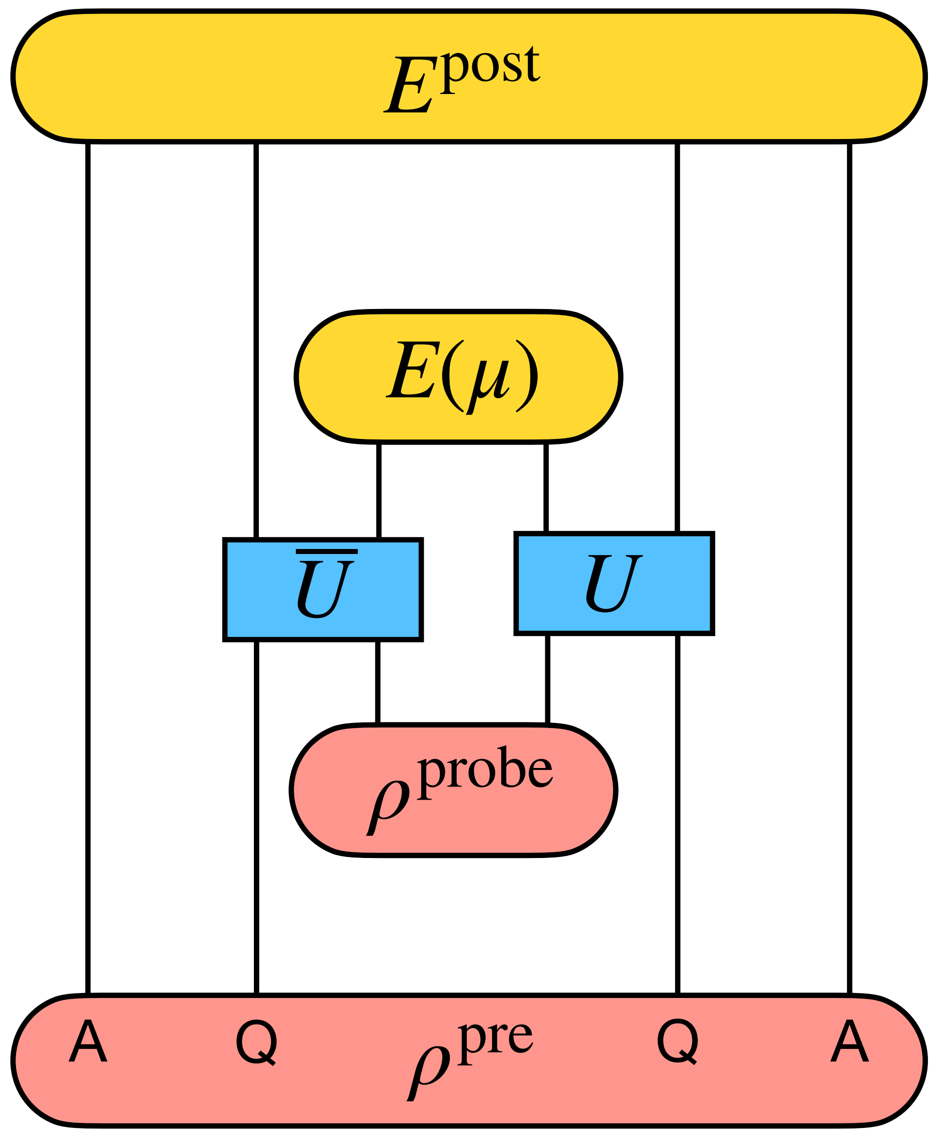

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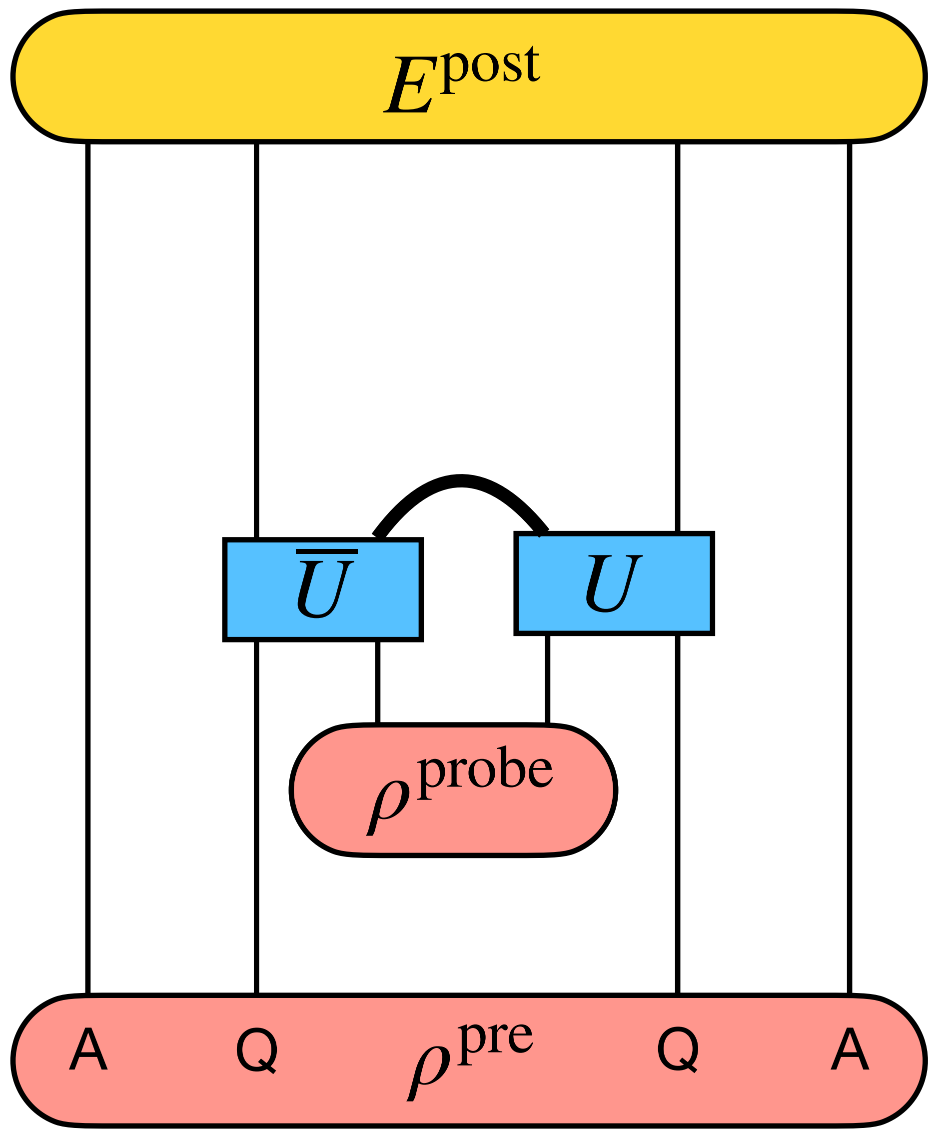
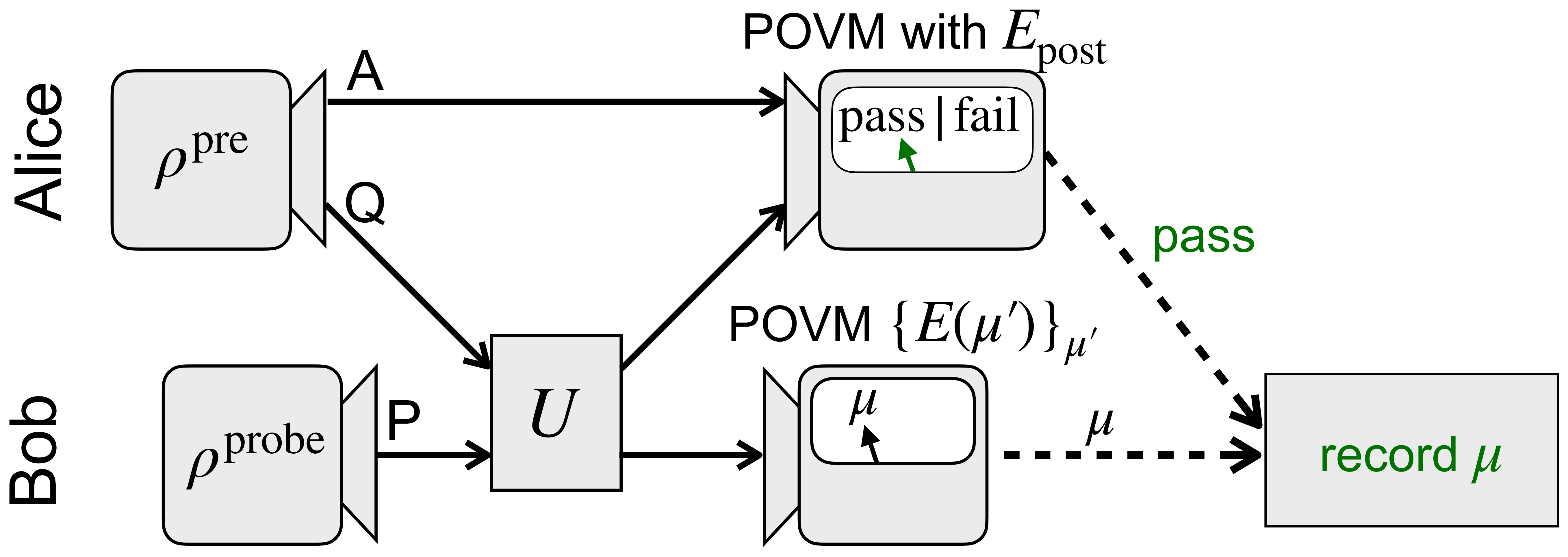


$$\sum_{\dots} \rho^{\text{pre}}_{\tilde{i}\tilde{i}';\tilde{k}\tilde{k}'} \rho^{\text{probe}}_{\tilde{m}\tilde{m}'} U^{\tilde{i};\tilde{m}}_{\tilde{l};\tilde{j}} \overline{U}^{\tilde{i}';\tilde{m}'}_{\tilde{l};\tilde{j}'} E^{\tilde{j}\tilde{j}';\tilde{k}\tilde{k}'}_{\text{post}}$$



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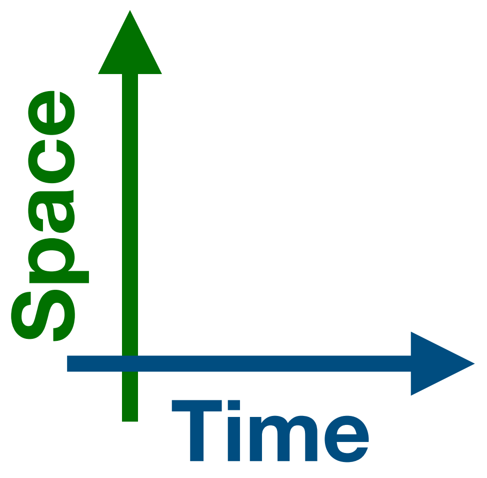

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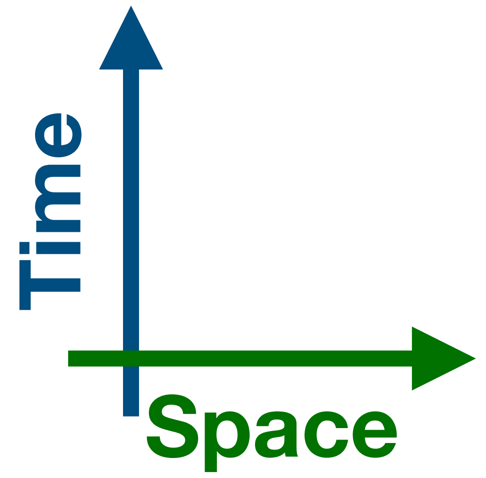

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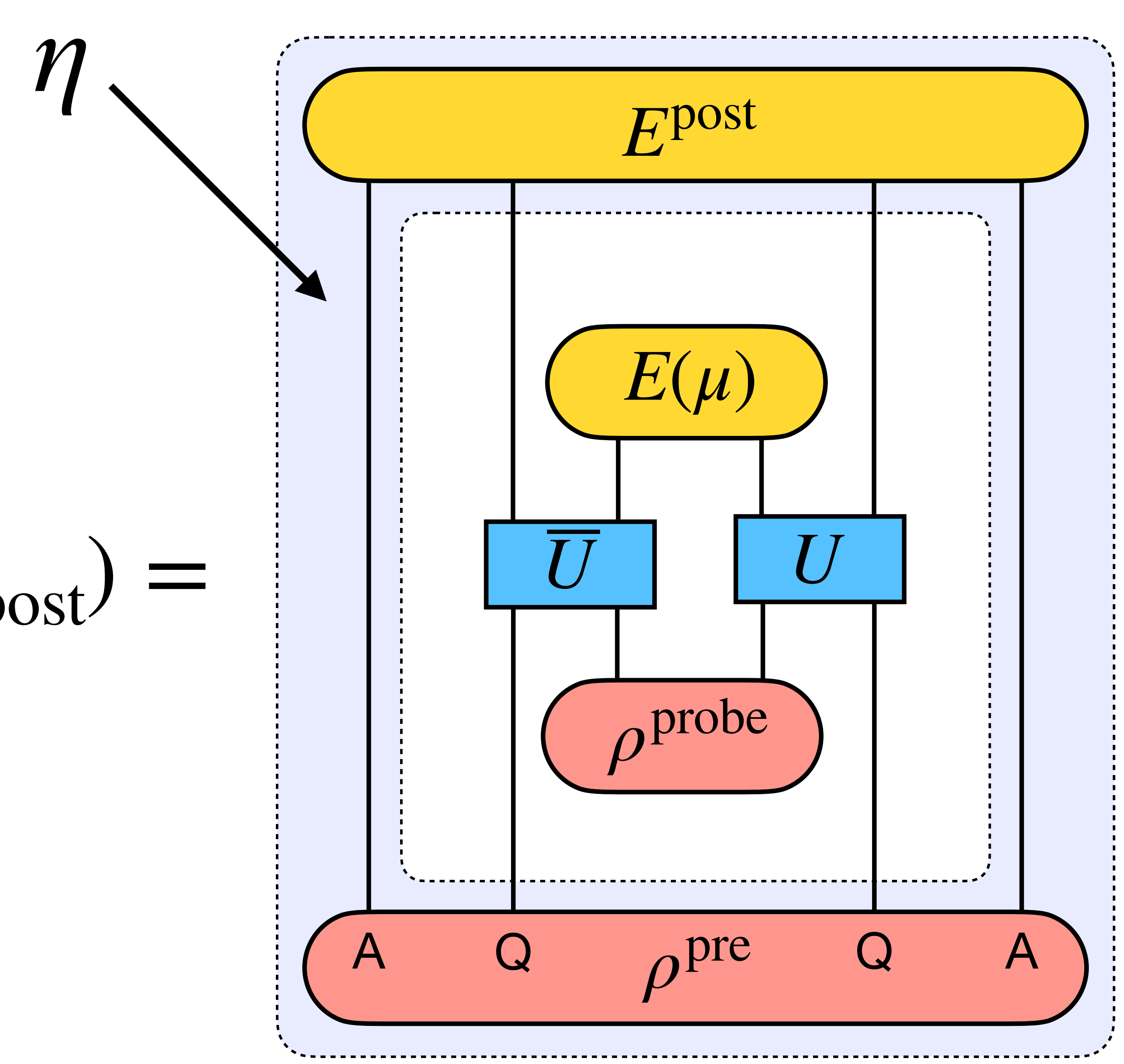

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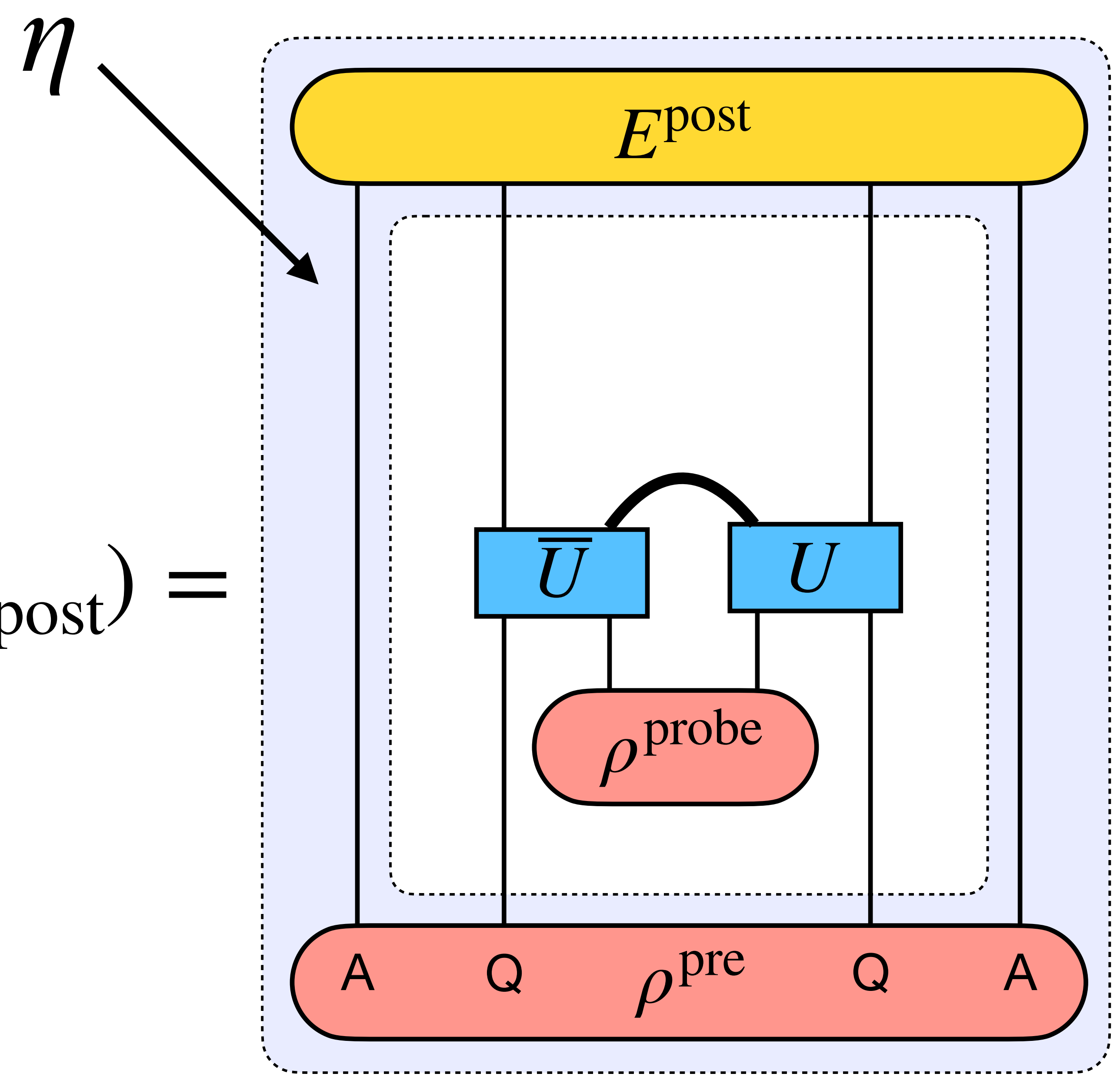

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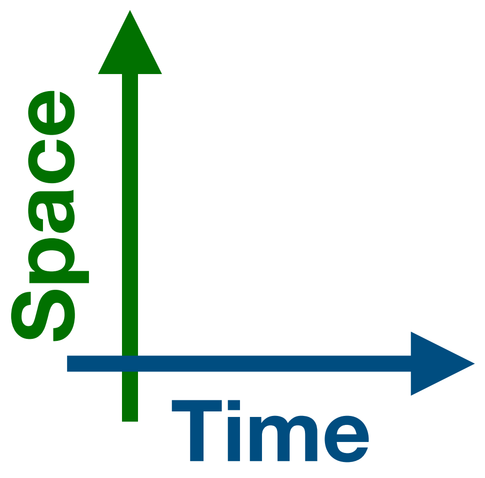

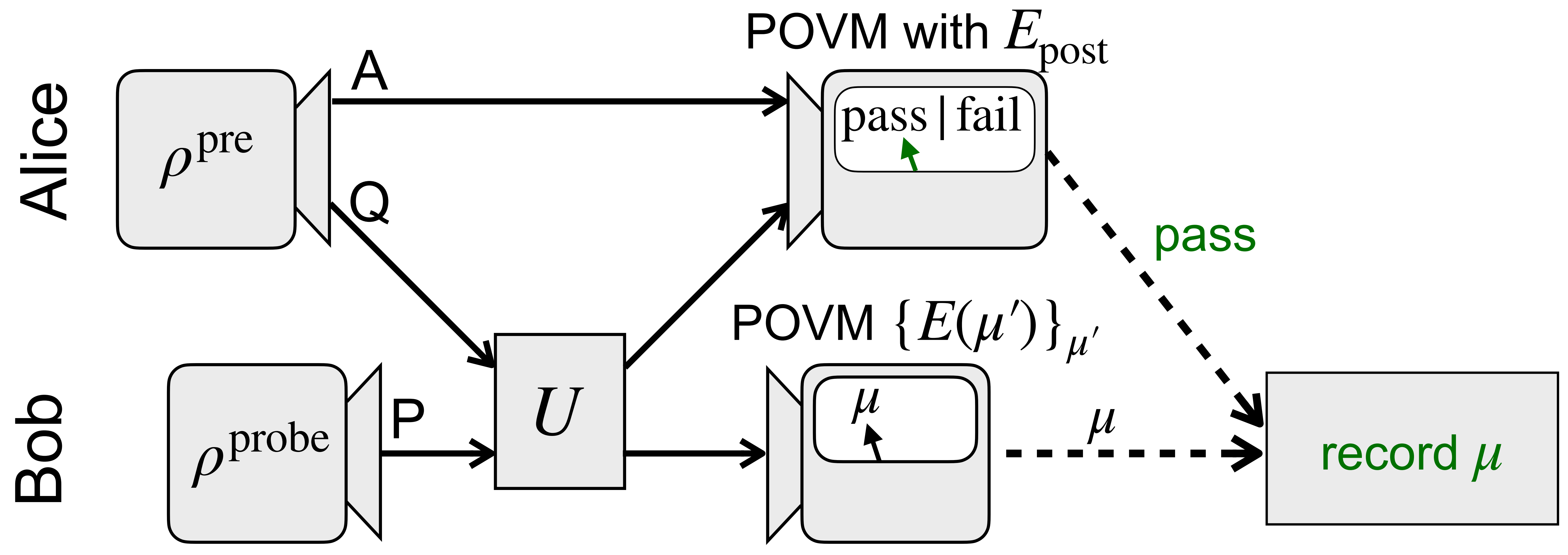

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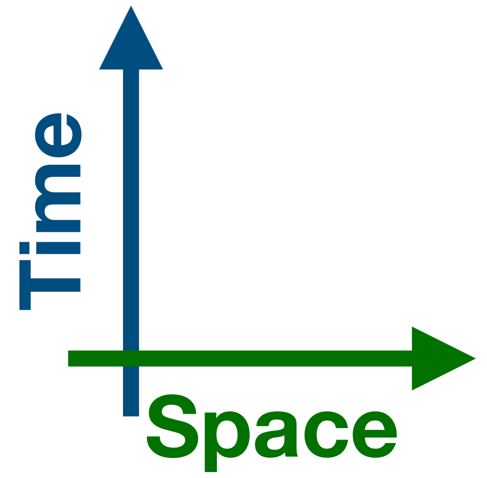

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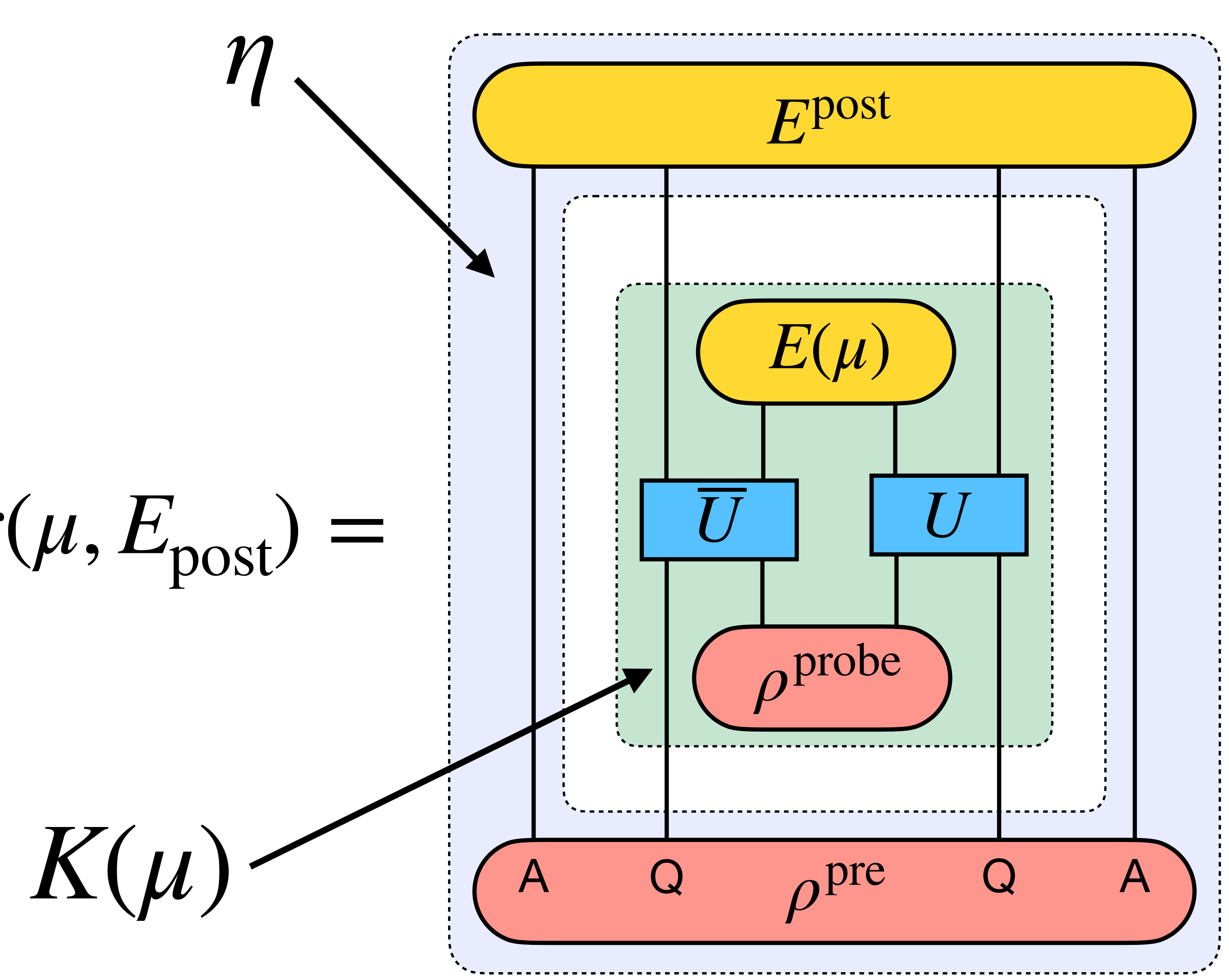
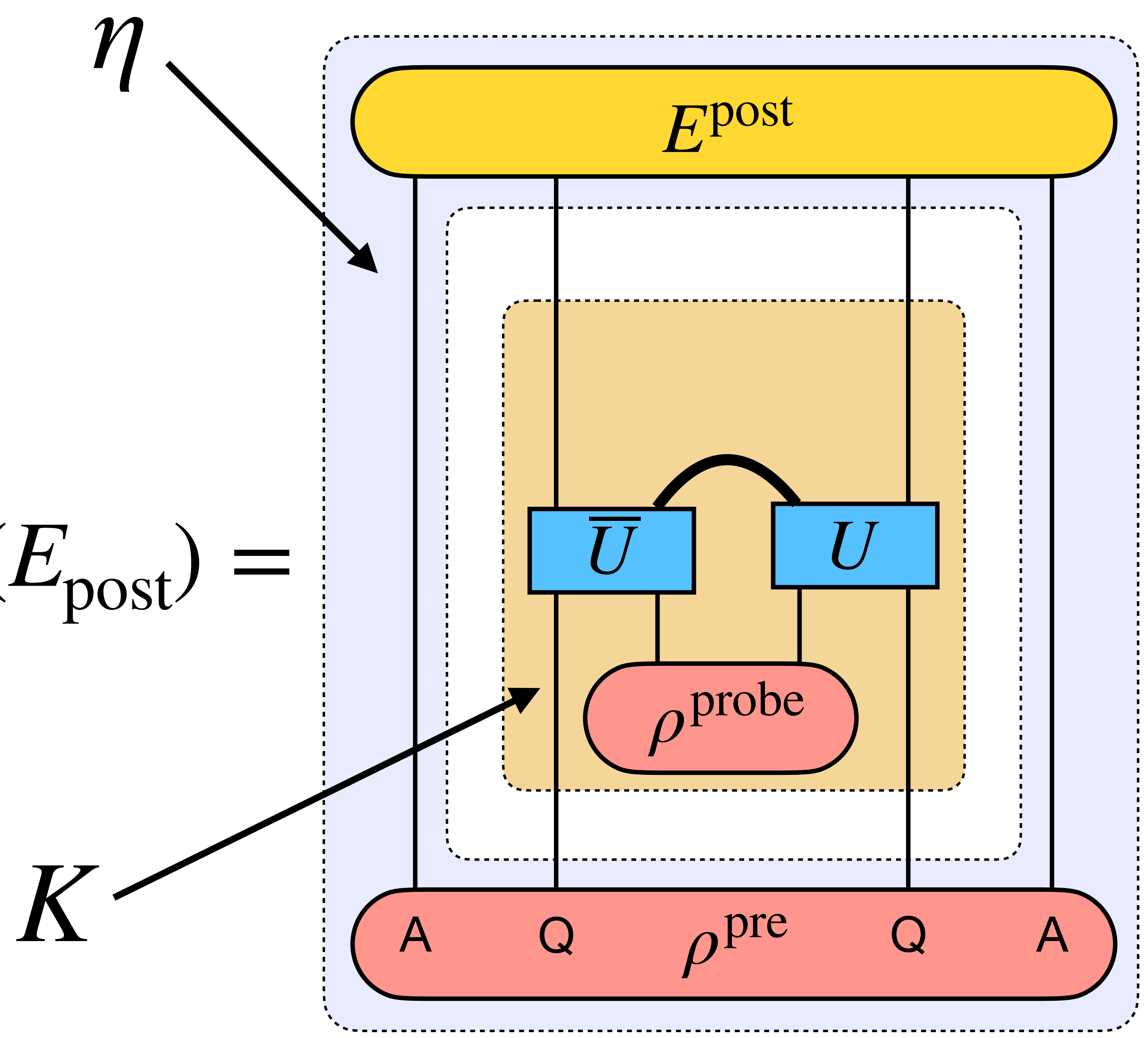
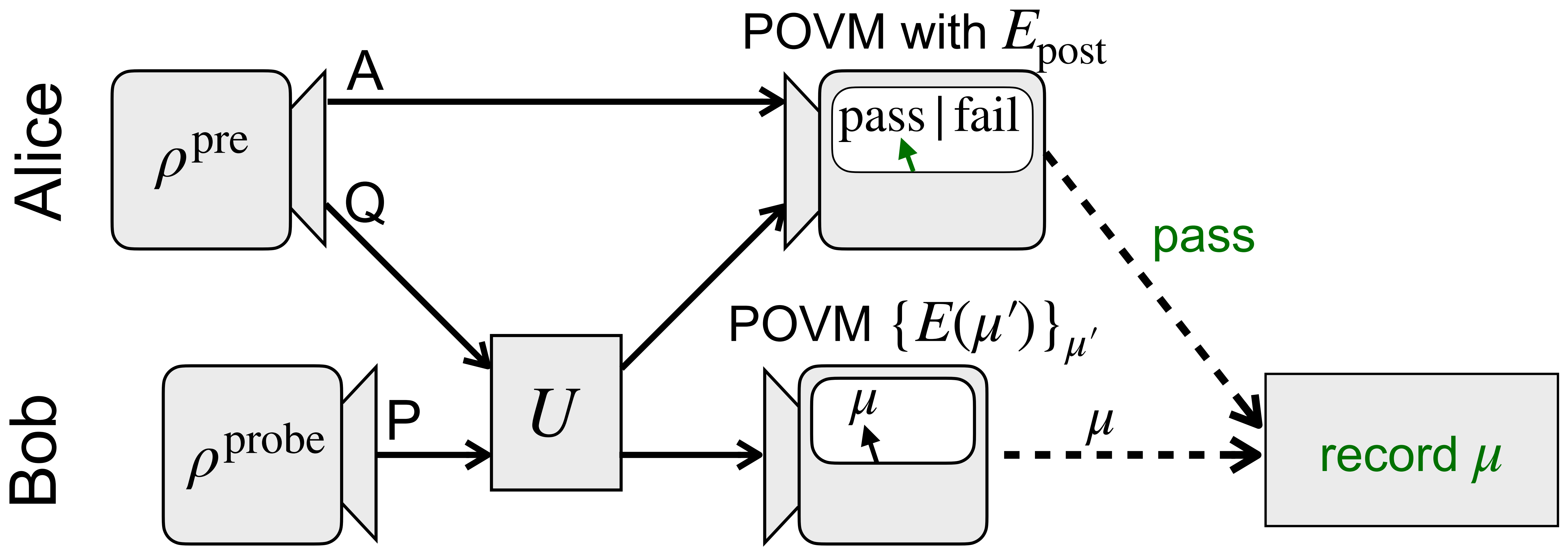


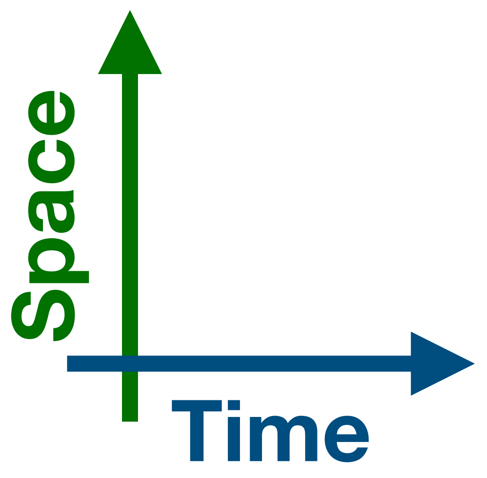
$$\sum_{\dots} \rho^{\mathrm{pre}}_{ii';kk'} \rho^{\mathrm{probe}}_{mm'} U^{i;m}_{l;j} \overline{U}^{i';m'}_{l';j'} E(\mu)^{ll'} E^{jj';kk'}_{\mathrm{post}}$$

$$\Pr(E_{\mathrm{post}}) =$$



$$\sum_{\dots} \rho^{\mathrm{pre}}_{\tilde{i}\tilde{i}';\tilde{k}\tilde{k}'} \rho^{\mathrm{probe}}_{\tilde{m}m'} U^{\tilde{i};\tilde{m}}_{\tilde{l};\tilde{j}} \overline{U}^{\tilde{i}';\tilde{m}'}_{\tilde{l};\tilde{j}'} E^{\tilde{j}\tilde{j}';\tilde{k}\tilde{k}'}_{\mathrm{post}}$$





$$\rho^{\mathrm{pre}} \in \mathcal{S}(\mathcal{H}_{\mathrm{A}} \otimes \mathcal{H}_{\mathrm{Q}})$$

$$\rho^{\mathrm{probe}} \in \mathcal{S}(\mathcal{H}_{\mathrm{P}})$$

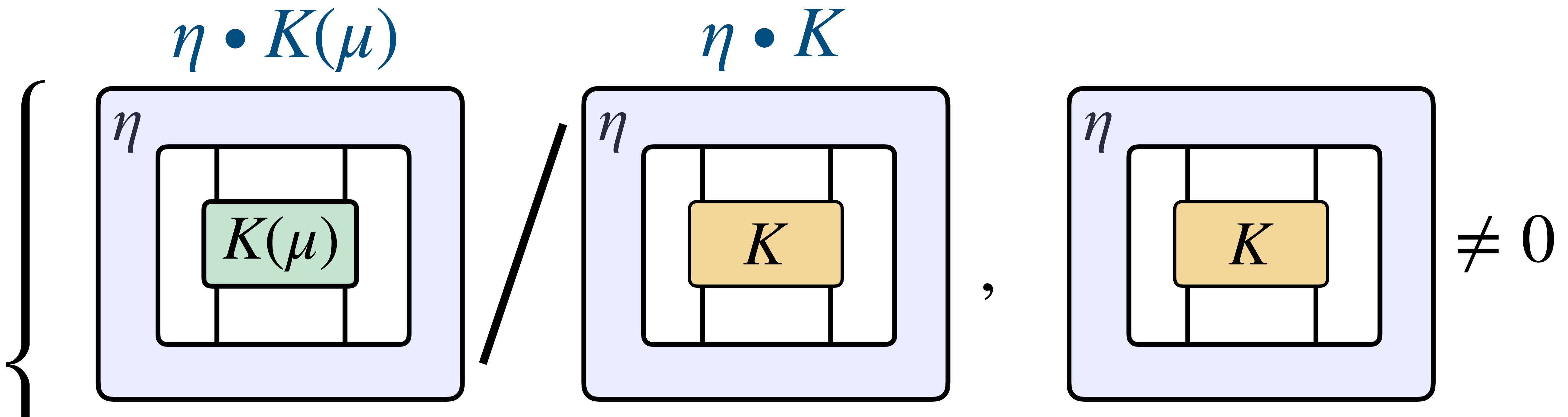
$$U \in \mathcal{U}(\mathcal{H}_{\mathrm{Q}} \otimes \mathcal{H}_{\mathrm{P}})$$

$$E(\mu) \in \mathcal{E}(\mathcal{H}_{\mathrm{P}})$$

$$E_{\mathrm{post}} \in \mathcal{E}(\mathcal{H}_{\mathrm{A}} \otimes \mathcal{H}_{\mathrm{P}})$$

Probability to obtain μ given postselection condition

$$\Pr(\mu\,|\,E_{\text{post}}) = \frac{\Pr(\mu, E_{\text{post}})}{\Pr(E_{\text{post}})} =$$



$0,\quad$ otherwise

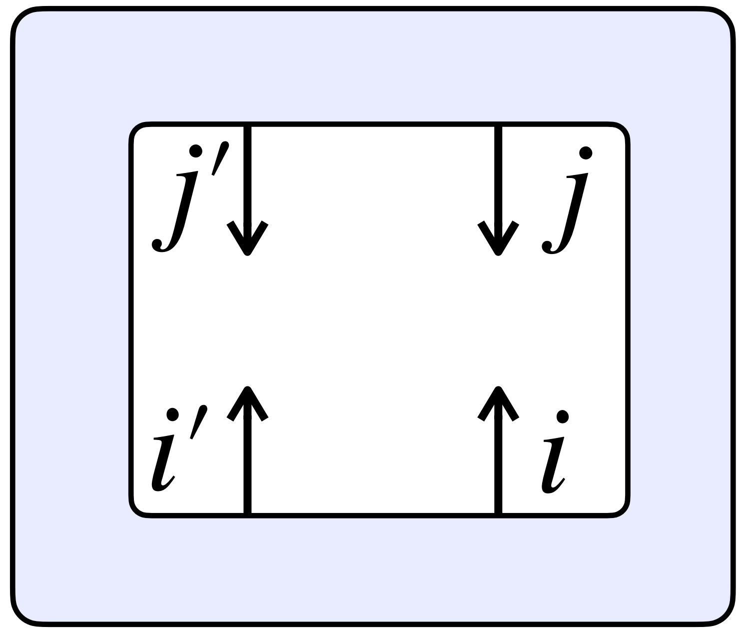
$\eta \in \mathscr{H}_{\mathrm{Q}} \otimes \mathscr{H}^{\dagger}_{\mathrm{Q}} \otimes \mathscr{H}^{\dagger}_{\mathrm{Q}} \otimes \mathscr{H}_{\mathrm{Q}}$ — time-bidirectional state captures Alice's pre- and postselection procedures.

$K(\mu) \in \mathscr{H}^{\dagger}_{\mathrm{Q}} \otimes \mathscr{H}_{\mathrm{Q}} \otimes \mathscr{H}_{\mathrm{Q}} \otimes \mathscr{H}^{\dagger}_{\mathrm{Q}}$ — operation μ outcome tensor — captures condition for obtaining given outcome μ by Bob.

$K \in \mathscr{H}^{\dagger}_{\mathrm{Q}} \otimes \mathscr{H}_{\mathrm{Q}} \otimes \mathscr{H}_{\mathrm{Q}} \otimes \mathscr{H}^{\dagger}_{\mathrm{Q}}$— operation tensor — captures an effect of Bob's operations on postelection made by Alice (CPTP map).

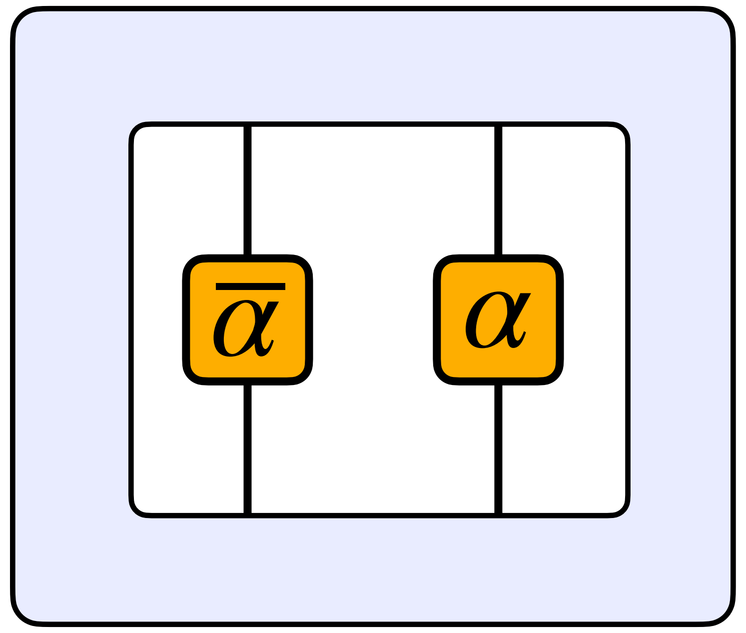
Some facts about time-bidirectional states (TBS)

$\eta =$



$\in \mathscr{H}_{\mathrm{Q}} \otimes \mathscr{H}_{\mathrm{Q}}^{\dagger} \otimes \mathscr{H}_{\mathrm{Q}}^{\dagger} \otimes \mathscr{H}_{\mathrm{Q}}$

$\forall \alpha \in \mathscr{L}(\mathscr{H}_{\mathrm{Q}}) : \langle\langle \alpha \,|\, \eta \,|\, \alpha \rangle\rangle :=$



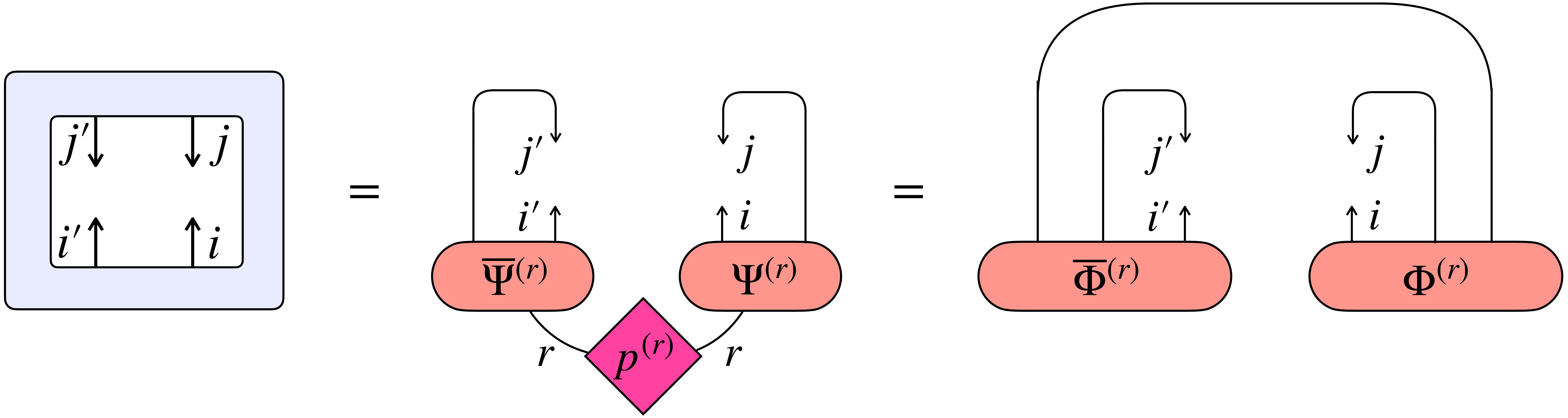
≥ 0 «semipositivity condition»

Preparation and purification of time-bidirectional states

Claim 1. Within the considered generalized postselection experiment, for any $\eta \in \mathscr{H}_{\mathrm{Q}} \otimes \mathscr{H}^{\dagger}_{\mathrm{Q}} \otimes \mathscr{H}^{\dagger}_{\mathrm{Q}} \otimes \mathscr{H}_{\mathrm{Q}}$ satisfying the condition $\forall \alpha \in \mathscr{L}(\mathscr{H}_{\mathrm{Q}}) : \langle\langle \alpha | \eta | \alpha \rangle\rangle \geq 0$, and for any indirect measurement made by Bob, given by $\{K(\mu)\}_{\mu}$, with $K(\mu) \in \mathscr{H}^{\dagger}_{\mathrm{Q}} \otimes \mathscr{H}_{\mathrm{Q}} \otimes \mathscr{H}_{\mathrm{Q}} \otimes \mathscr{H}^{\dagger}_{\mathrm{Q}}$, there exists a preselection + postselection procedure, made by Alice and specified by $\rho_{\mathrm{AQ}} \in \mathscr{S}(\mathscr{H}_{\mathrm{A}} \otimes \mathscr{H}_{\mathrm{Q}})$ and $E_{\mathrm{post}} \in \mathscr{E}(\mathscr{H}_{\mathrm{A}} \otimes \mathscr{H}_{\mathrm{Q}})$, that provides $\Pr(\mu \,|\, E_{\mathrm{post}}) = (\eta \bullet K(\mu))/(\eta \bullet K)$. «Any proper TBS η is preparable».

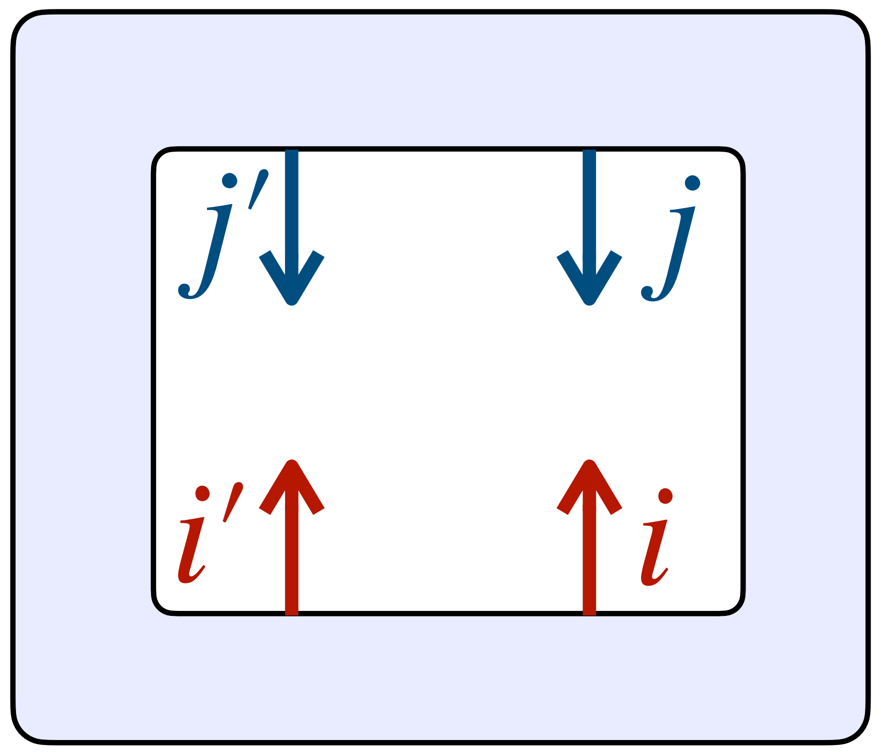
Claim 2.There exists a procedure with only pure states preselection and only projective measurements that prepares η. «TBS η can be purified»

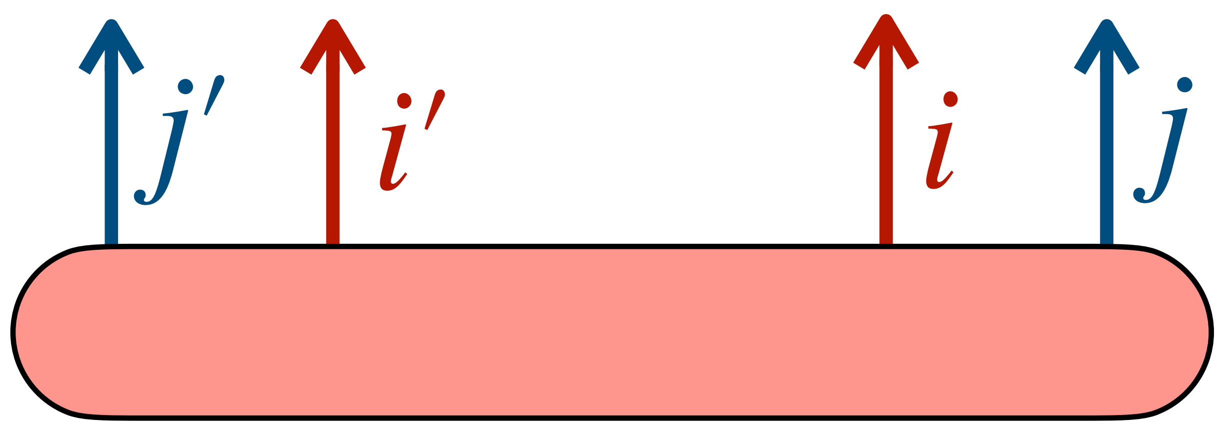
Idea of the proof: using the spectral decomposition via thinking about η as non-normalized state of two particles



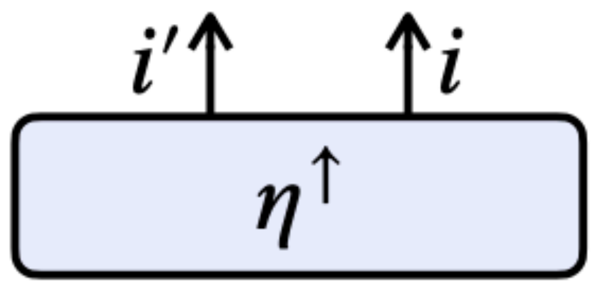
Time-bidirectional states and bipartite states

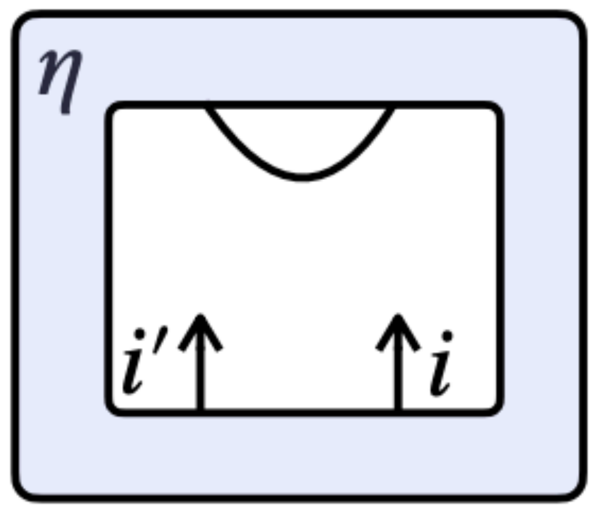
η is equivalent to a non-normalized mixed state of a bipartite system («two copies of a particle»)

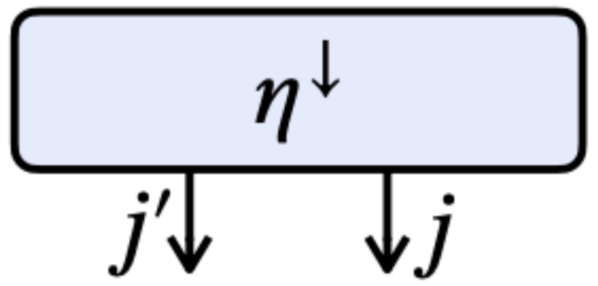
$\eta =$ 

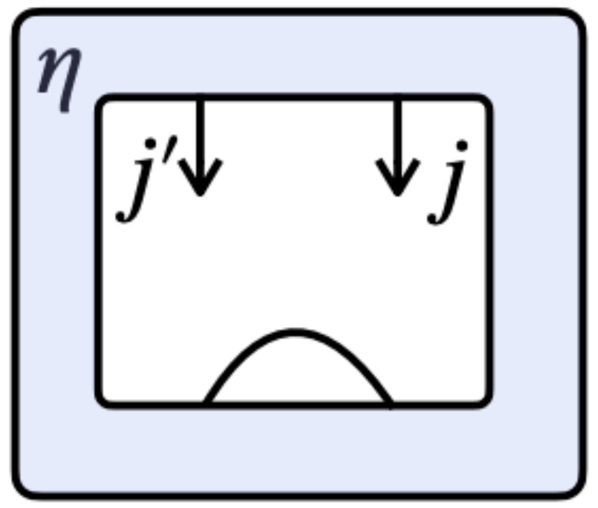
$\longleftrightarrow$ $\rho_{QQ'} =$ 

Reduced states of η, which correspond to forward and backward evolving parts can be obtained:



$=$ 



$=$ 

$(\eta^{\uparrow} = \mathrm{Tr}_{\downarrow}\eta)$

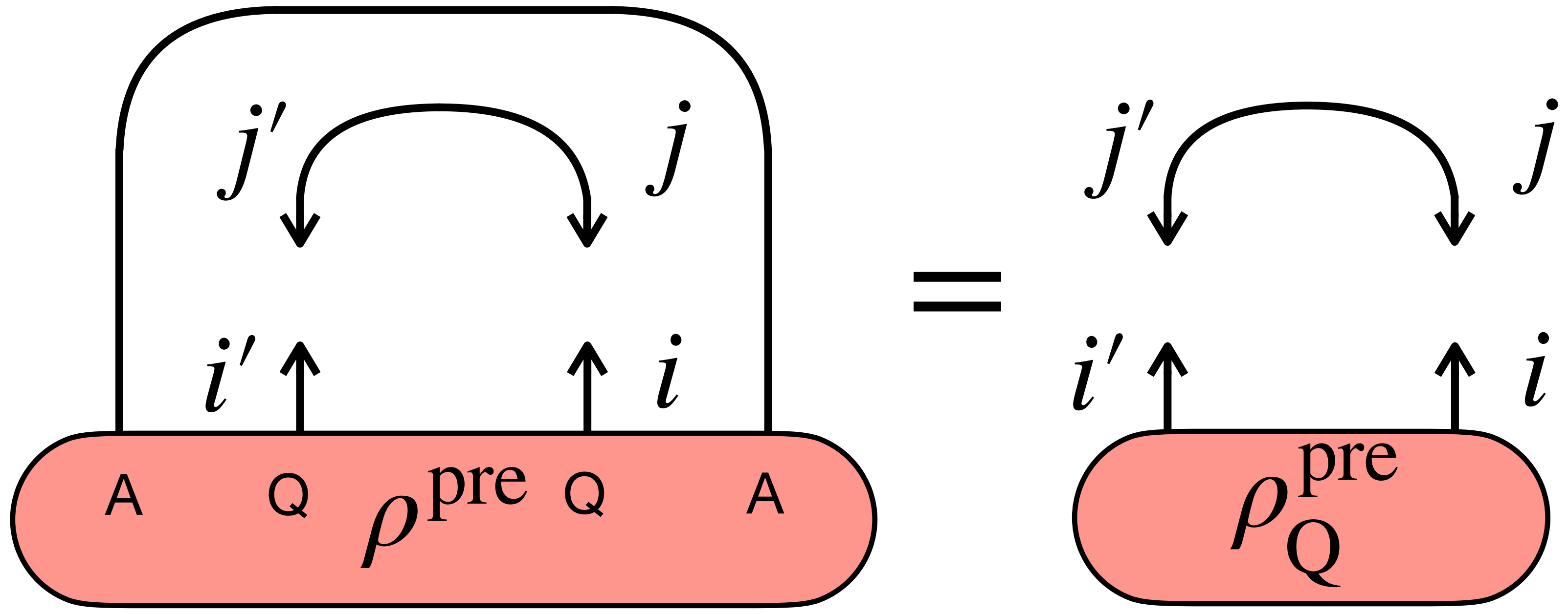
$(\eta_{\downarrow} = \mathrm{Tr}_{\uparrow}\eta)$

Special cases of time-bidirectional states

[1] Y. Aharonov, P. G. Bergmann, J. L. Lebowitz, Phys. Rev. **134**, B1410 (1964).
[2] L. Vaidman, A. Ben-Israel, J. Dziewior, et al. Phys. Rev. A **96**, 032114 (2017).
[3] Y. Aharonov, L. Vaidman, J. Phys. A Math. **24**, 2315 (1991).
[4] R. Silva, Y. Guryanova, N. Brunner, et al. Phys. Rev. A **89**, 012121 (2014).

Special cases of time-bidirectional states

No post-selection



$$= \rho_Q^{\mathrm{pre}} \otimes \mathbf{1}$$

[1] Y. Aharonov, P. G. Bergmann, J. L. Lebowitz, Phys. Rev. **134**, B1410 (1964).
[2] L. Vaidman, A. Ben-Israel, J. Dziewior, et al. Phys. Rev. A **96**, 032114 (2017).
[3] Y. Aharonov, L. Vaidman, J. Phys. A Math. **24**, 2315 (1991).
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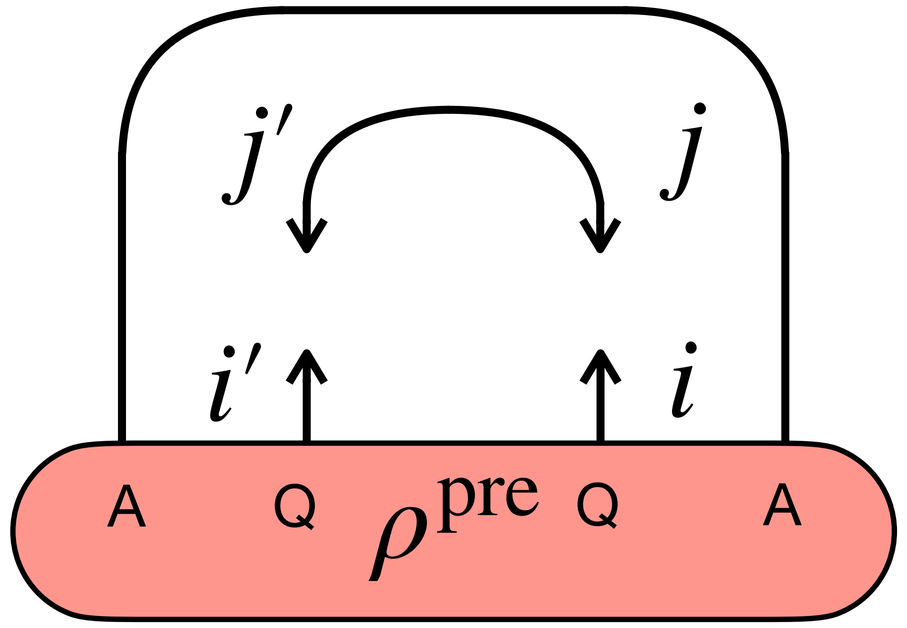
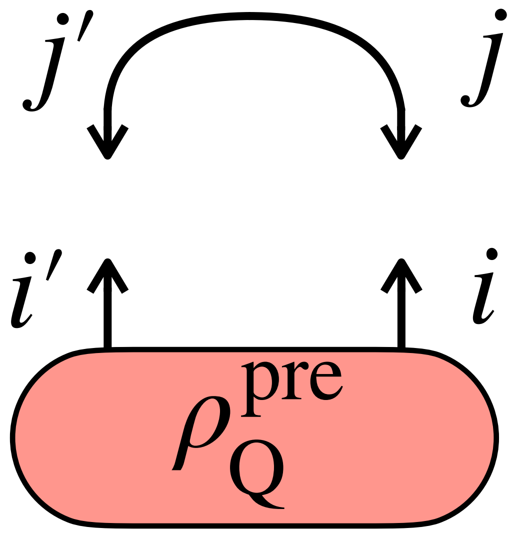
Special cases of time-bidirectional states

No post-selection

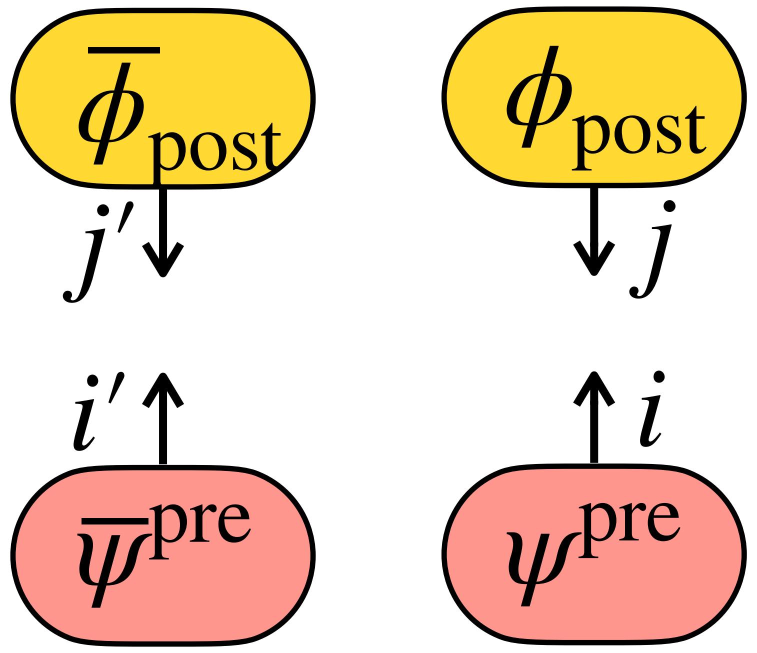


$=$



$= \rho_Q^{\mathrm{pre}} \otimes \mathbf{1}$

Two-state vector[1]

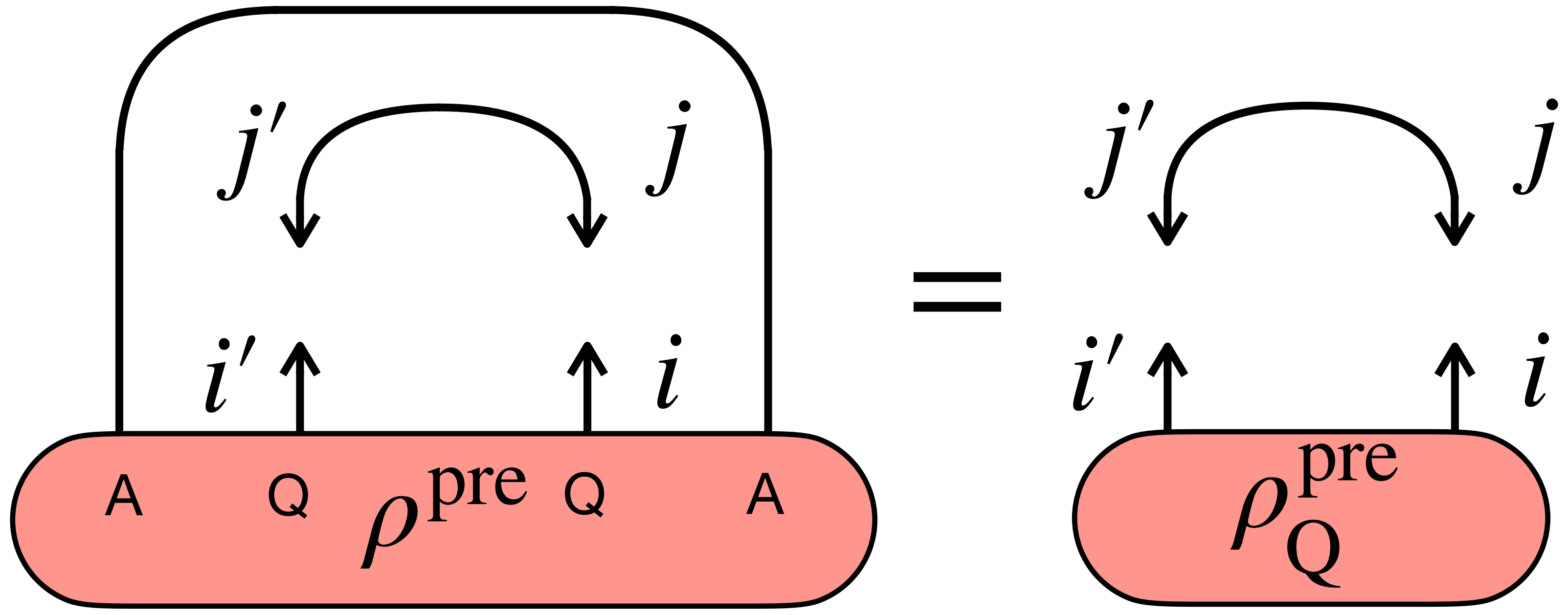


$= |\psi^{\mathrm{pre}}\rangle\langle\psi^{\mathrm{pre}}| \otimes |\phi_{\mathrm{post}}\rangle\langle\phi_{\mathrm{post}}|$

[1] Y. Aharonov, P. G. Bergmann, J. L. Lebowitz, Phys. Rev. **134**, B1410 (1964).
[2] L. Vaidman, A. Ben-Israel, J. Dziewior, et al. Phys. Rev. A **96**, 032114 (2017).
[3] Y. Aharonov, L. Vaidman, J. Phys. A Math. **24**, 2315 (1991).
[4] R. Silva, Y. Guryanova, N. Brunner, et al. Phys. Rev. A **89**, 012121 (2014).

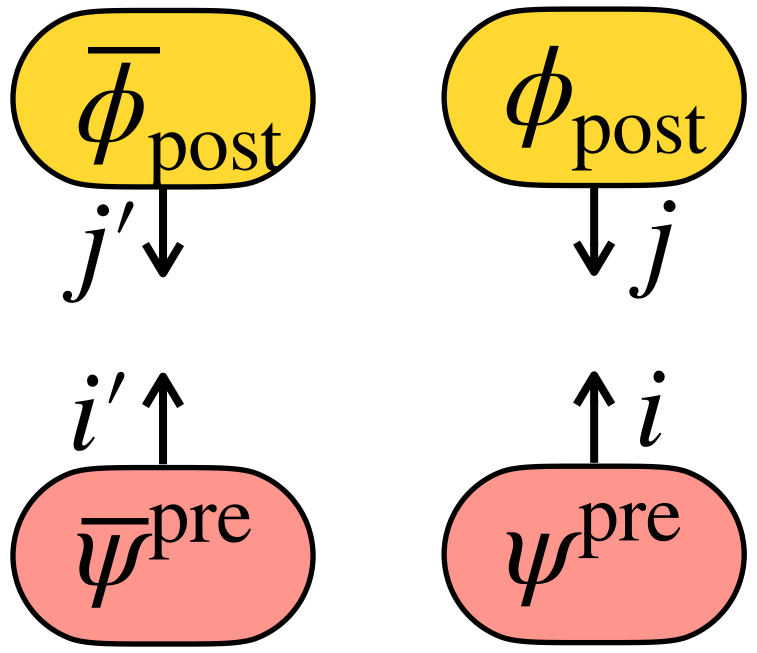
Special cases of time-bidirectional states

No post-selection



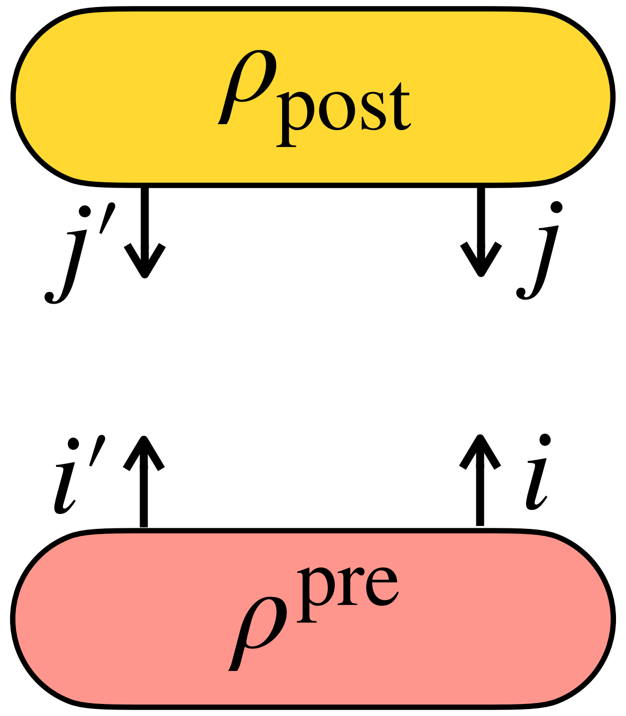
$= \rho_Q^{\mathrm{pre}} \otimes \mathbf{1}$

Two-state vector[1]



$= |\psi^{\mathrm{pre}}\rangle\langle\psi^{\mathrm{pre}}| \otimes |\phi_{\mathrm{post}}\rangle\langle\phi_{\mathrm{post}}|$

Mixed two-state vector[2]

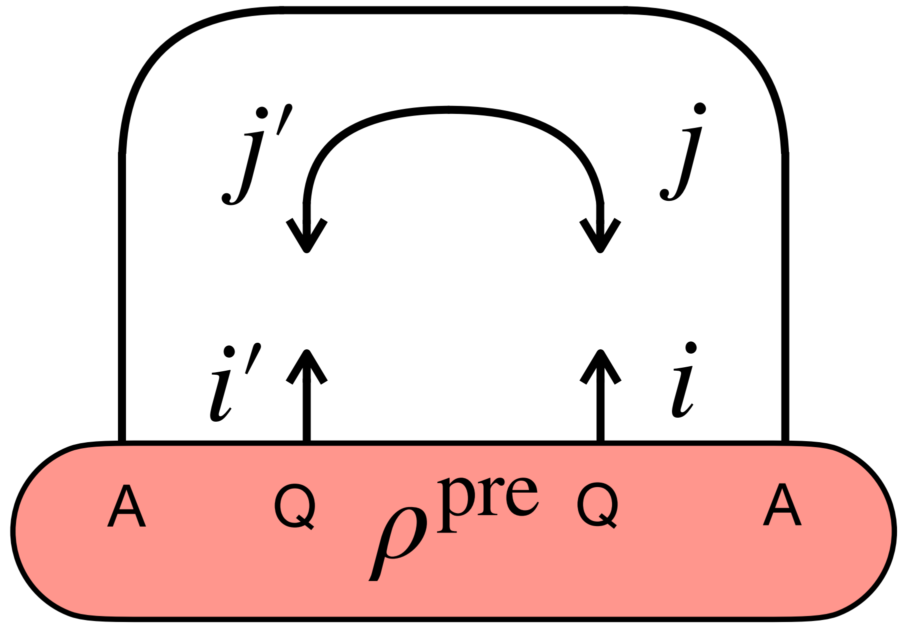


$= \rho^{\mathrm{pre}} \otimes \rho_{\mathrm{post}}$

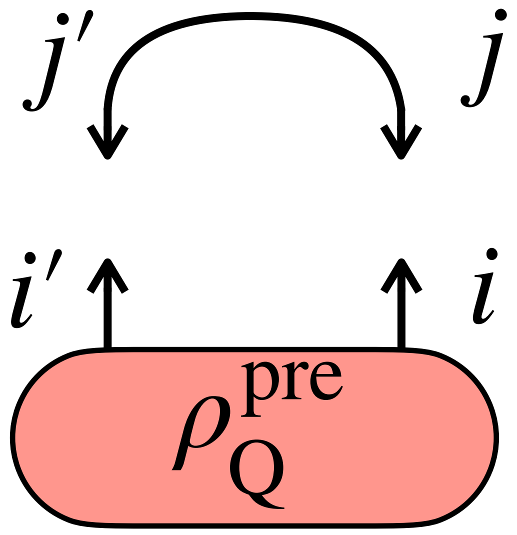
[1] Y. Aharonov, P. G. Bergmann, J. L. Lebowitz, Phys. Rev. **134**, B1410 (1964).
[2] L. Vaidman, A. Ben-Israel, J. Dziewior, et al. Phys. Rev. A **96**, 032114 (2017).
[3] Y. Aharonov, L. Vaidman, J. Phys. A Math. **24**, 2315 (1991).
[4] R. Silva, Y. Guryanova, N. Brunner, et al. Phys. Rev. A **89**, 012121 (2014).

Special cases of time-bidirectional states

No post-selection

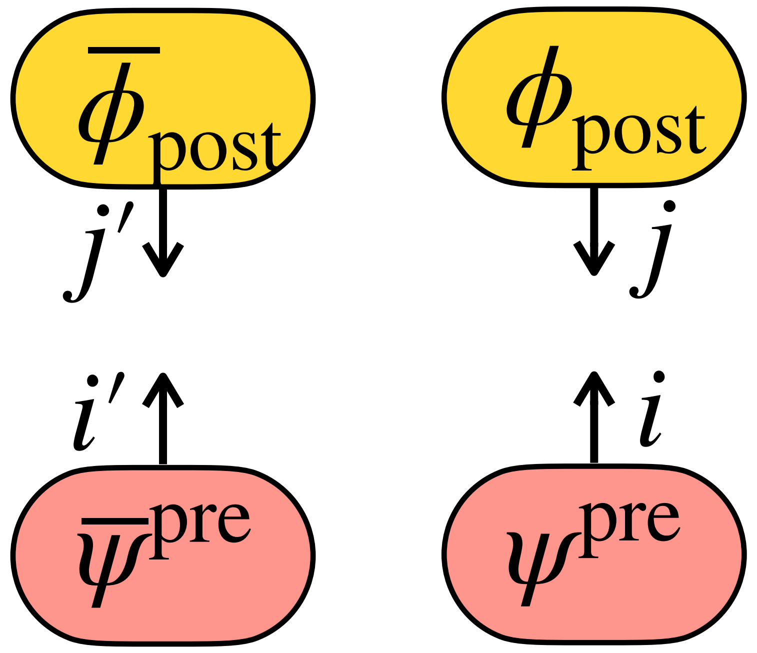


$=$



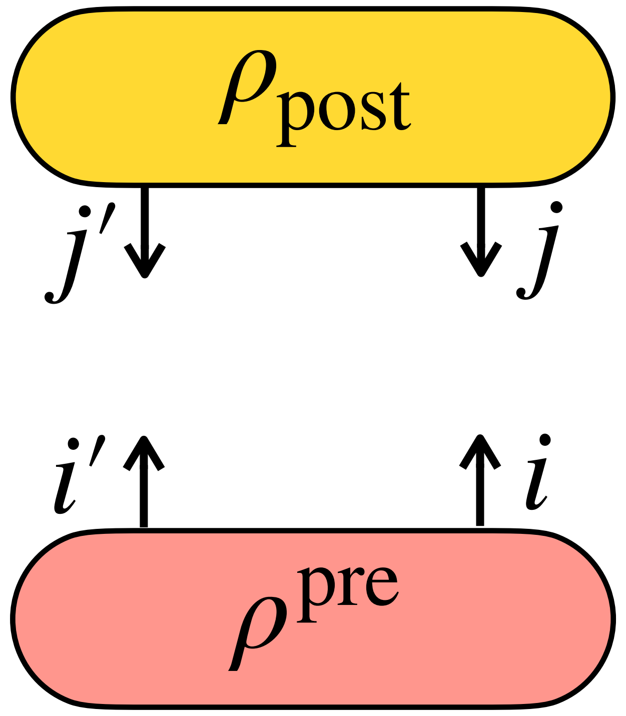
$= \rho_Q^{\mathrm{pre}} \otimes \mathbf{1}$

Two-state vector[1]



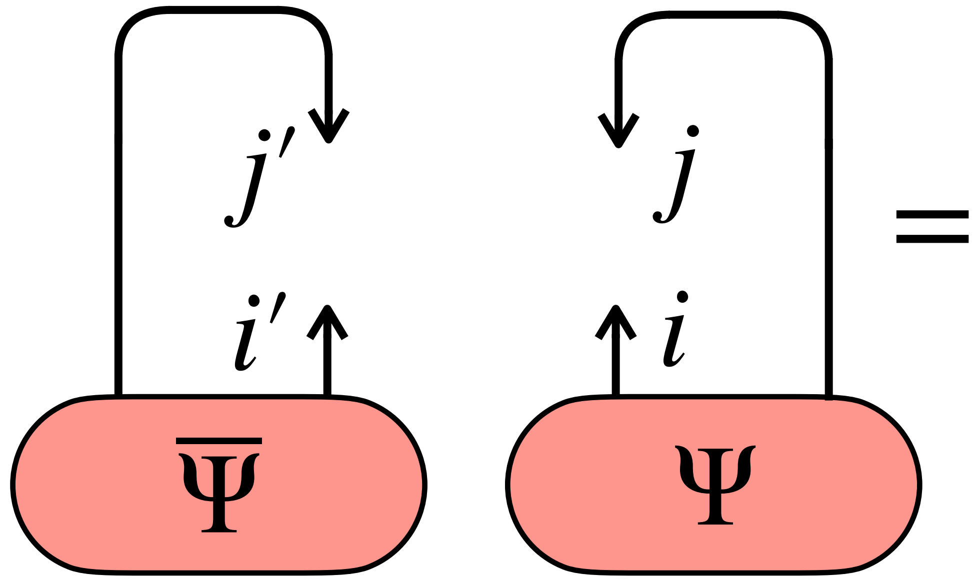
$= |\psi^{\mathrm{pre}}\rangle\langle\psi^{\mathrm{pre}}| \otimes |\phi_{\mathrm{post}}\rangle\langle\phi_{\mathrm{post}}|$

Mixed two-state vector[2]



$= \rho^{\mathrm{pre}} \otimes \rho_{\mathrm{post}}$

Generalized two-state vector[3]

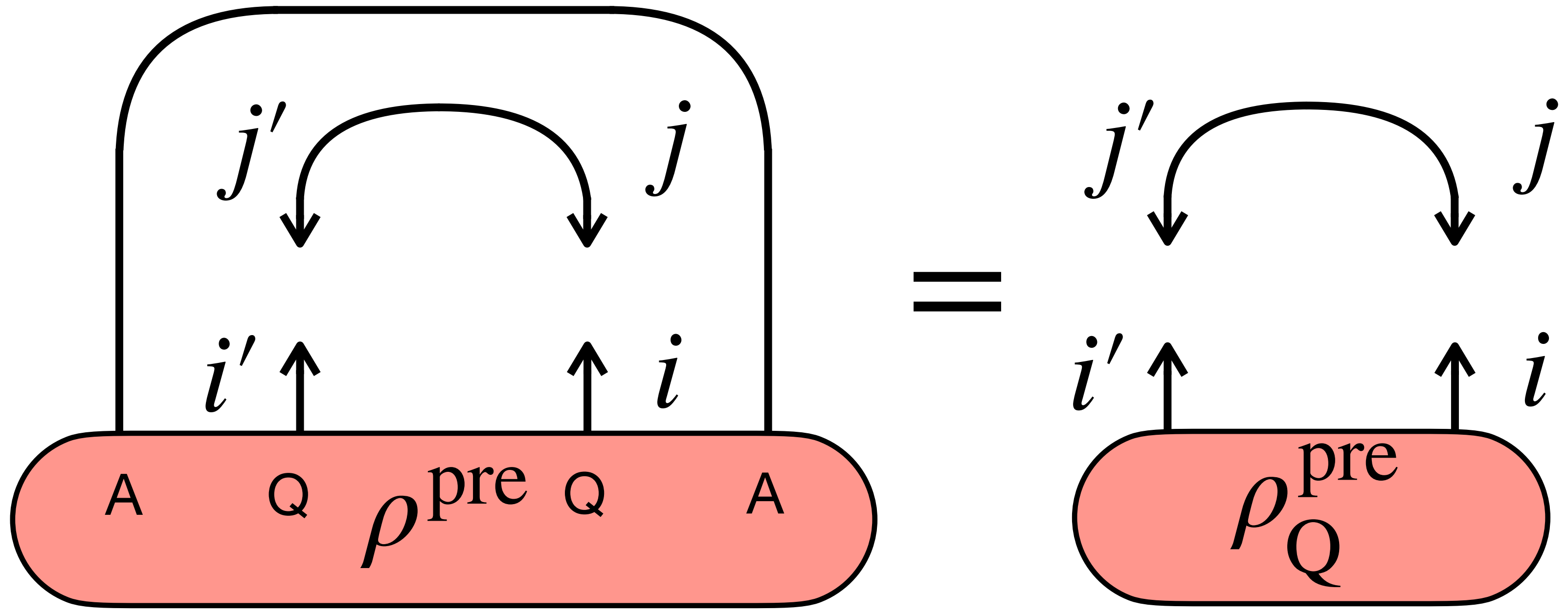


$= |\Psi\rangle\langle\Psi|$

[1] Y. Aharonov, P. G. Bergmann, J. L. Lebowitz, Phys. Rev. **134**, B1410 (1964).
[2] L. Vaidman, A. Ben-Israel, J. Dziewior, et al. Phys. Rev. A **96**, 032114 (2017).
[3] Y. Aharonov, L. Vaidman, J. Phys. A Math. **24**, 2315 (1991).
[4] R. Silva, Y. Guryanova, N. Brunner, et al. Phys. Rev. A **89**, 012121 (2014).

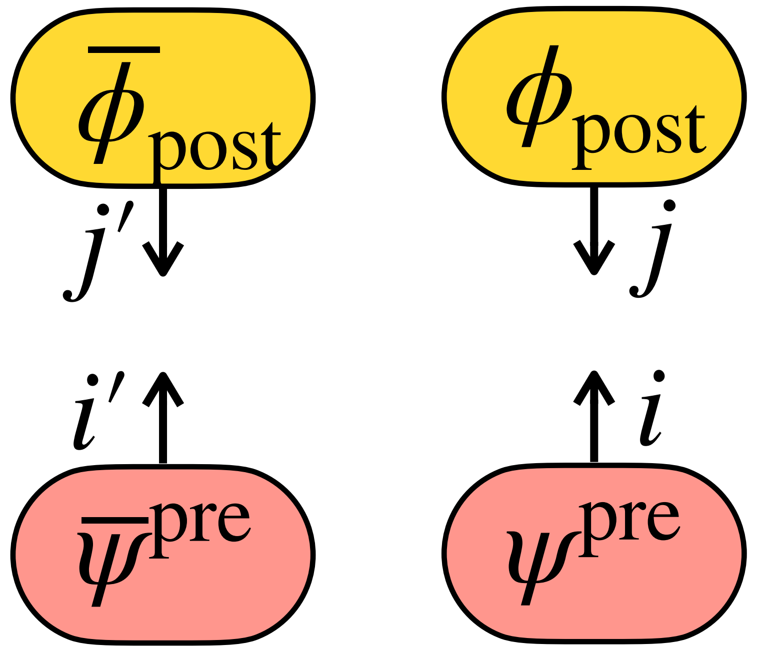
Special cases of time-bidirectional states

No post-selection



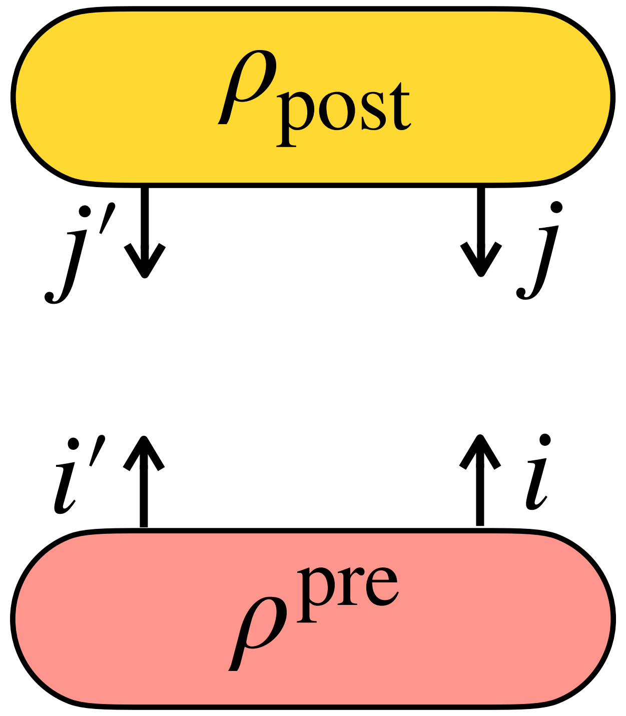
$$= \rho_Q^{pre} \otimes \mathbf{1}$$

Two-state vector[1]



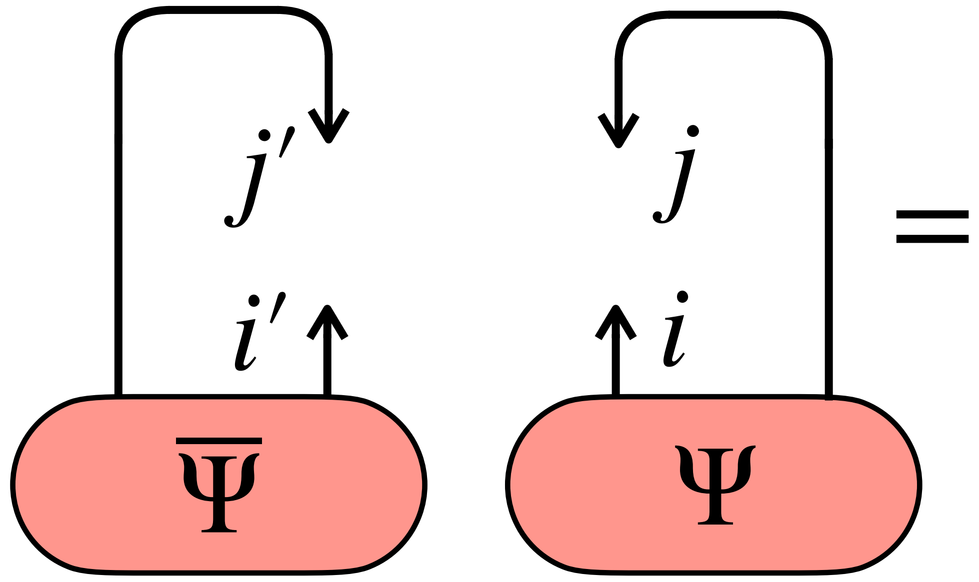
$$= |\psi^{pre}\rangle\langle\psi^{pre}| \otimes |\phi_{post}\rangle\langle\phi_{post}|$$

Mixed two-state vector[2]



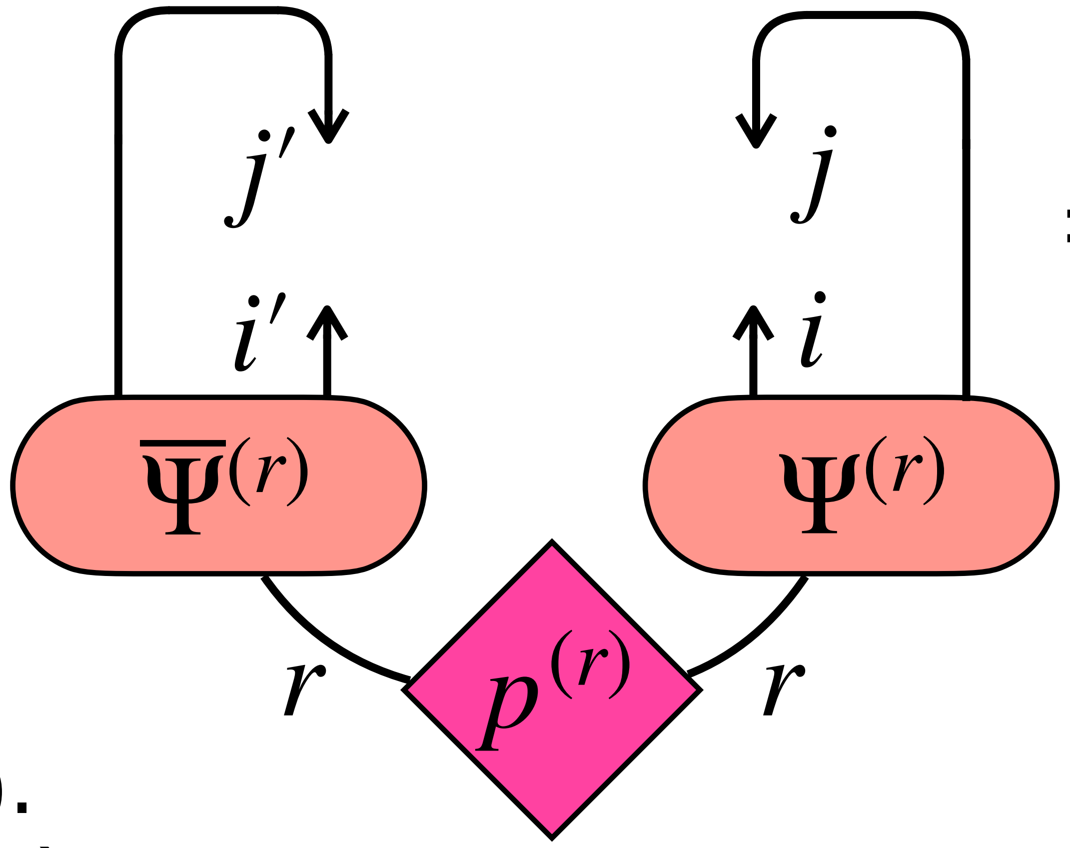
$$= \rho^{pre} \otimes \rho_{post}$$

Generalized two-state vector[3]



$$= |\Psi\rangle\langle\Psi|$$

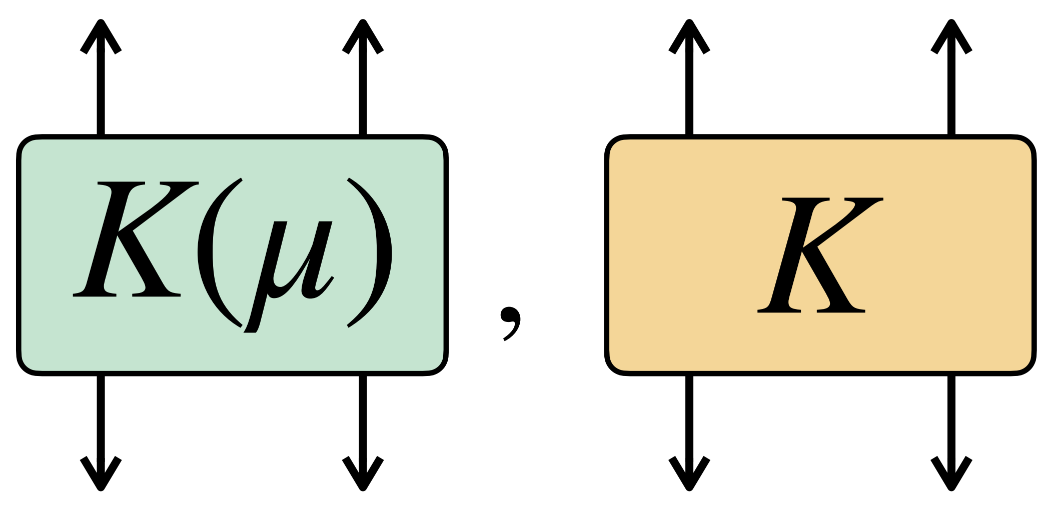
Two-state density vector[4]



$$= \sum_r p^{(r)} |\Psi^{(r)}\rangle\langle\Psi^{(r)}|$$

[1] Y. Aharonov, P. G. Bergmann, J. L. Lebowitz, Phys. Rev. **134**, B1410 (1964).
[2] L. Vaidman, A. Ben-Israel, J. Dziewior, et al. Phys. Rev. A **96**, 032114 (2017).
[3] Y. Aharonov, L. Vaidman, J. Phys. A Math. **24**, 2315 (1991).
[4] R. Silva, Y. Guryanova, N. Brunner, et al. Phys. Rev. A **89**, 012121 (2014).

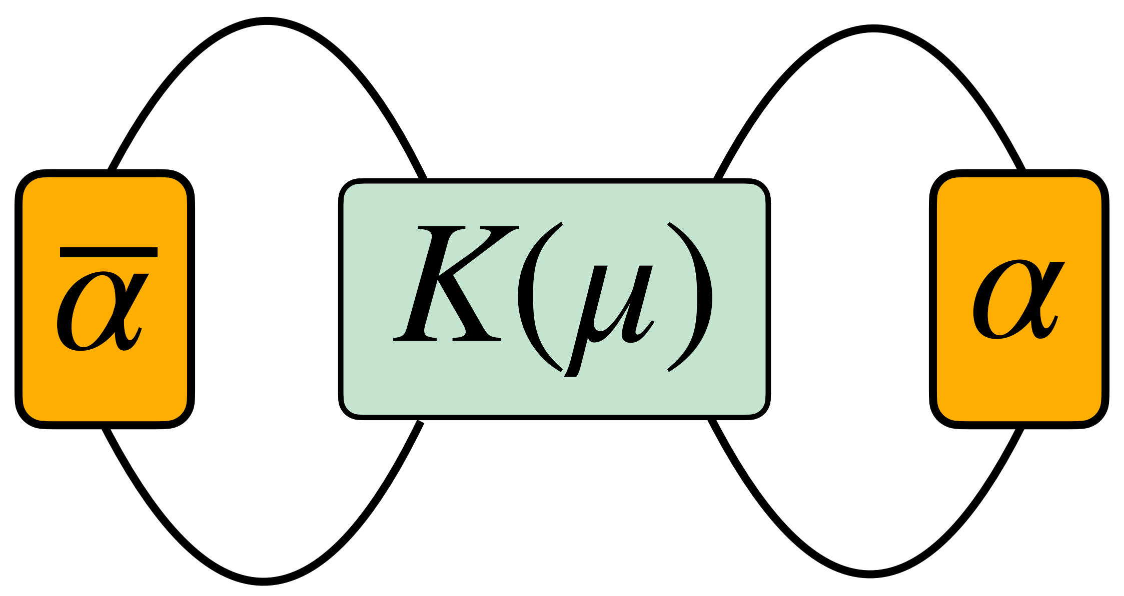
Some facts about operation outcome and operation tensors



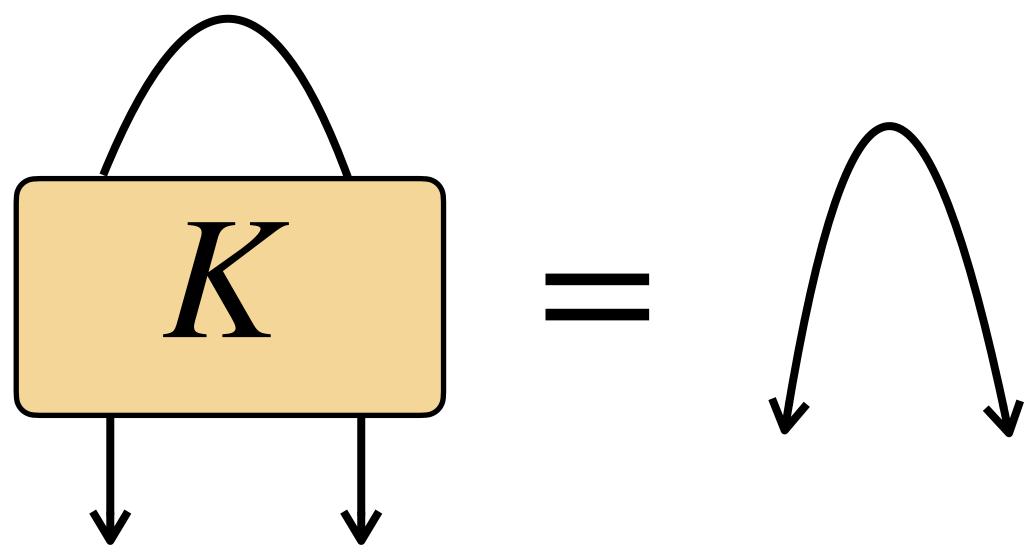
$\in \mathscr{H}_{\mathrm{Q}}^{\dagger} \otimes \mathscr{H}_{\mathrm{Q}} \otimes \mathscr{H}_{\mathrm{Q}} \otimes \mathscr{H}_{\mathrm{Q}}^{\dagger}$

K is a completely-positive trace-preserving (CPTP) map $\mathscr{L}(\mathscr{H}_{\mathrm{Q}}) \to \mathscr{L}(\mathscr{H}_{\mathrm{Q}})$.

$\forall \alpha \in \mathscr{H}_{\mathrm{Q}}^{\dagger} \otimes \mathscr{H}_{\mathrm{Q}}$:

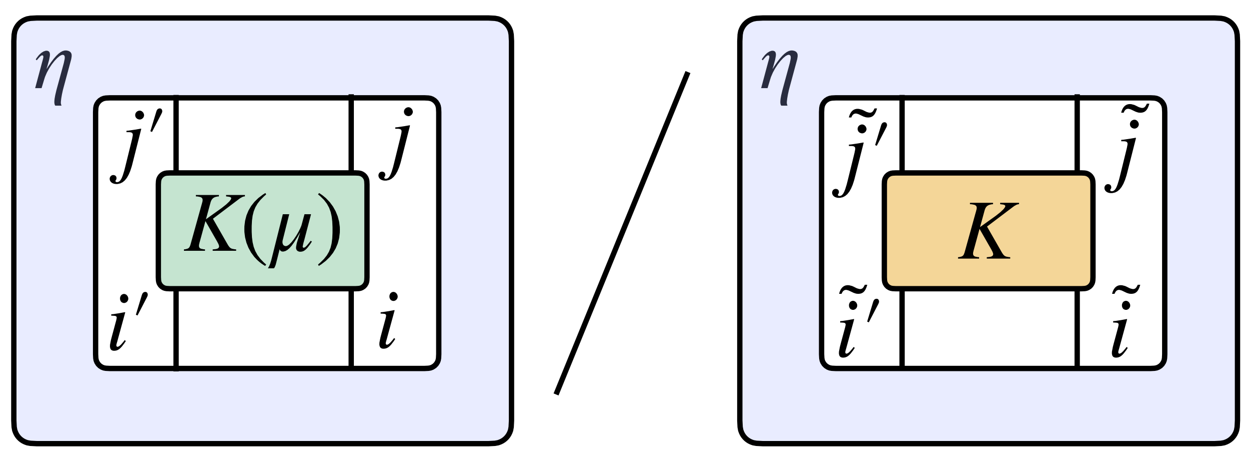


≥ 0 «CP»

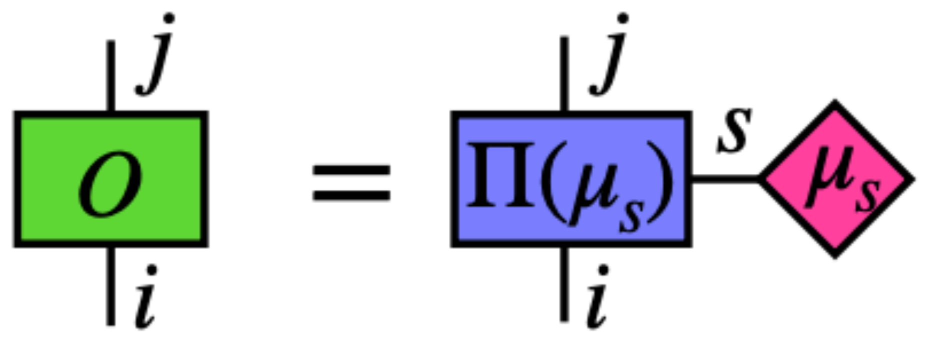


«TP»

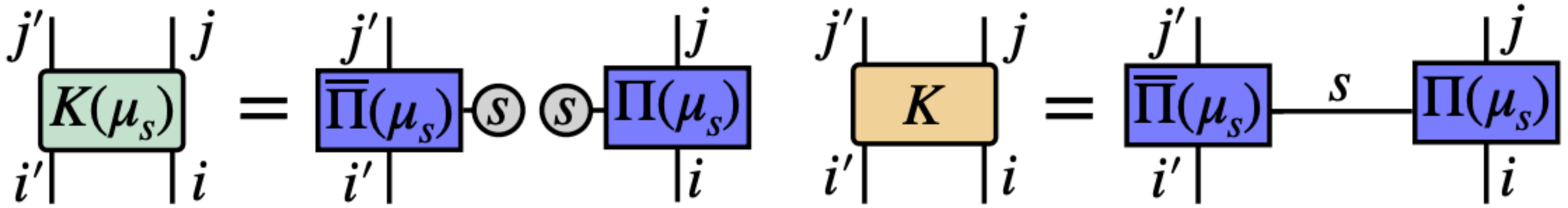
$P(\mu) =$



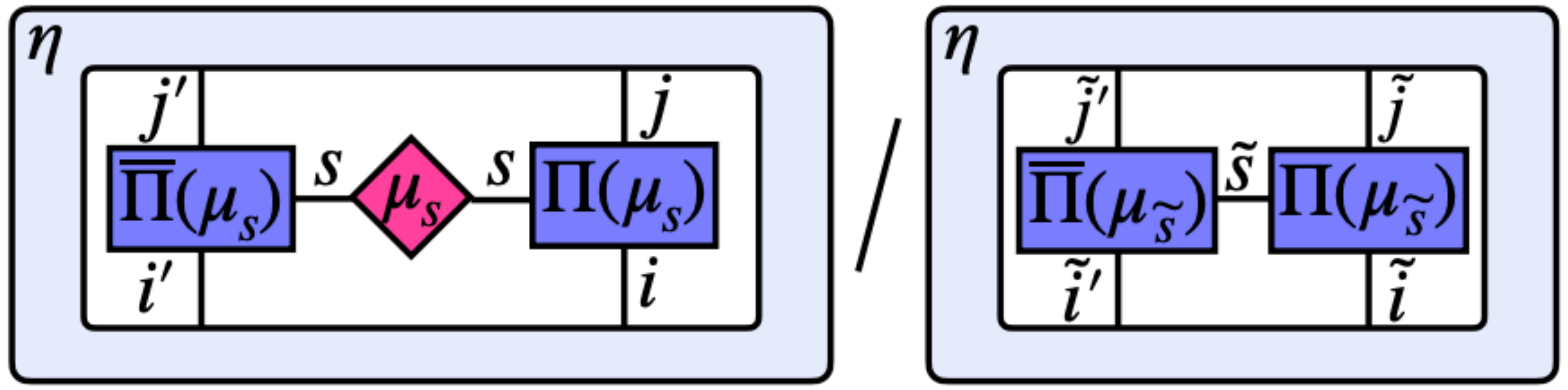
Measuring Hermitian observables



spectral decomposition of Hermitian observable $O = O^\dagger$

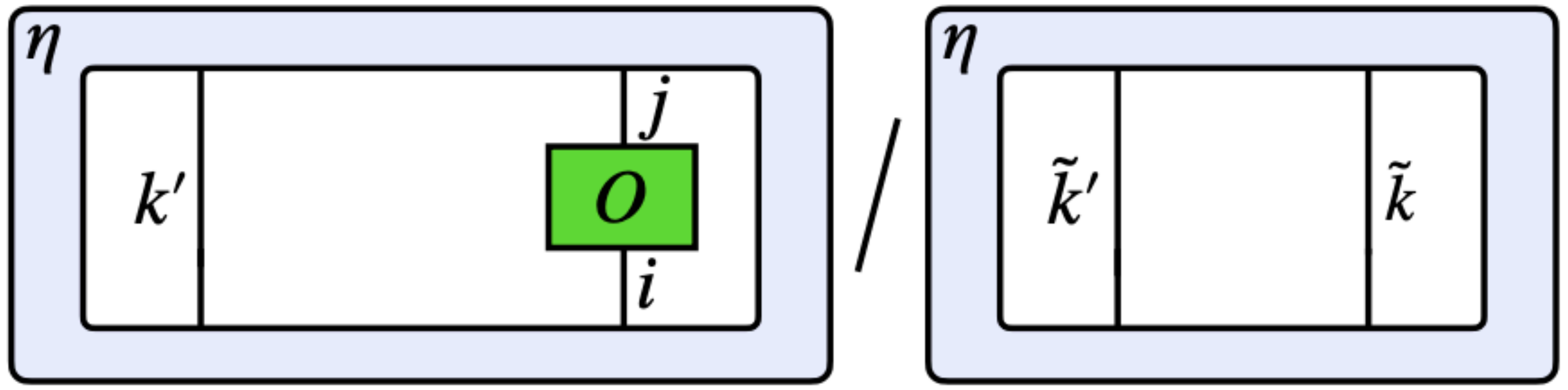


$\langle O\rangle_\eta =$



$= \mathrm{Tr}(\rho O)$ for $\eta = \rho \otimes \mathbf{1}$

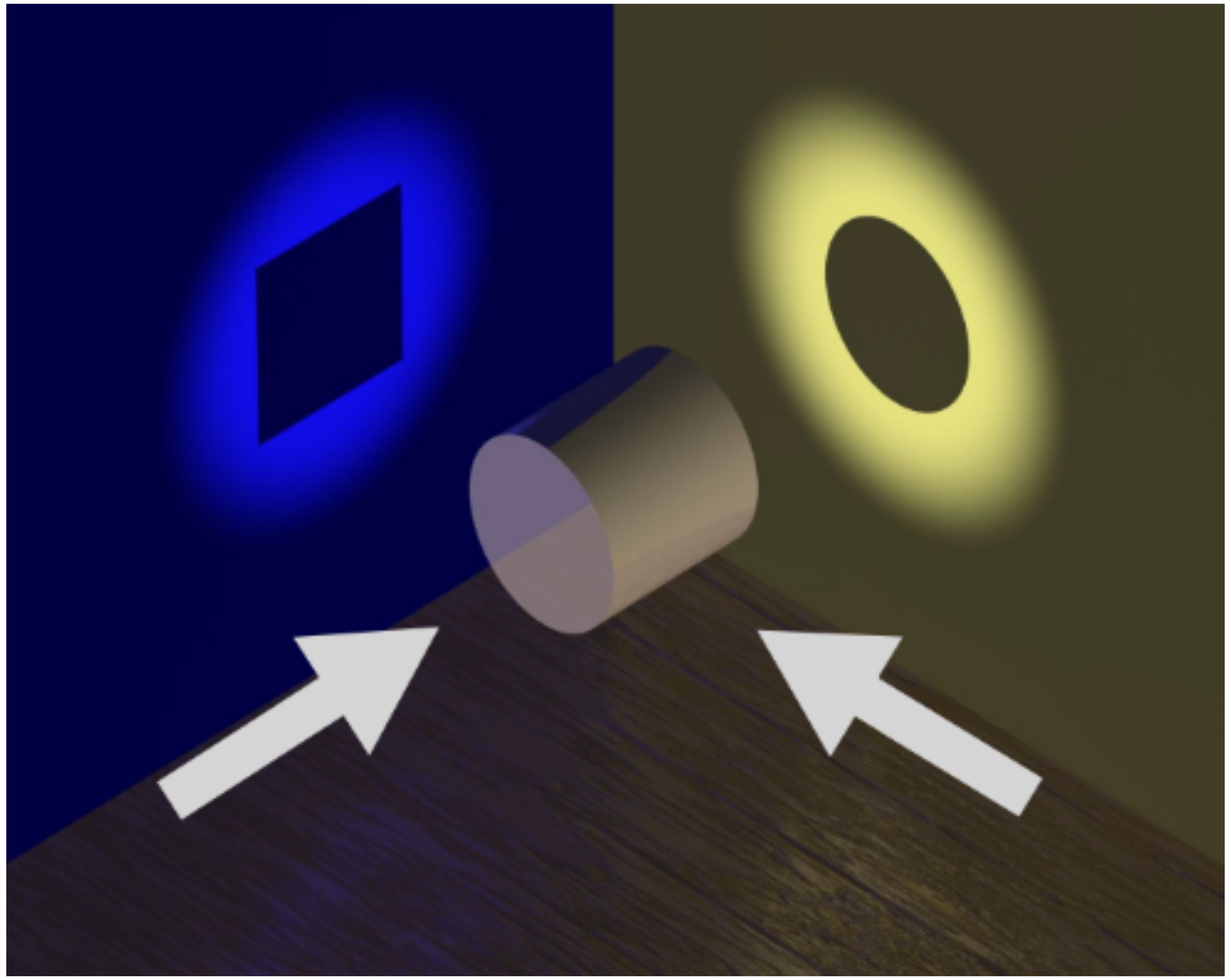
$O^{\mathrm{weak}}_\eta =$



$= \dfrac{\langle \phi | O | \psi \rangle}{\langle \phi | \psi \rangle}$ for $\eta = |\psi\rangle\langle\psi| \otimes |\phi\rangle\langle\phi|$

J. Dressel, M. Malik, F. M. Miatto, A. N. Jordan, R. W. Boyd, "Colloquium: Understanding quantum weak values: Basics and applications," Rev. Mod. Phys. 86, 307 (2014).

Tomographical reconstruction of a TBS



https://hongyehu.github.io/Hamiltonian-driven-shadow-tomography-page

Tomography of single-qubit TBS

Accumulating measurement data $\Rightarrow$ Postselecting «good» outcomes $\Rightarrow$ Processing remaining data $\Rightarrow$ η

Informationally complete set of measurement: $\{\mathbf{K}^{(r)}\}_r$, $\mathbf{K}^{(r)} = \{K^{(r)}(\mu')\}_{\mu'}$

Updated Born rule: $\Pr^{(r)}(\mu\,|\,\mathrm{post}) = \dfrac{K^{(r)}(\mu) \bullet \eta}{K^{(r)} \bullet \eta}$

Problem: extract η from $\{\Pr^{(r)}(\mu\,|\,\mathrm{post})\}_{r,\mu}$

optimization over Riemannian manifolds can be applied

A. Pechen et al., “Control landscapes for two-level open quantum systems,” J. Phys. A **41**, 045205 (2008).
I. A. Luchnikov et al. “Qgopt: Riemannian optimization for quantum technologies,” SciPost Phys. **10**, 079 (2021).

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Problem: extract η from $\{\overset{\text{Freq}}{\cancel{\Pr}}{}^{(r)}(\mu\,|\,\text{post})\}_{r,\mu}$

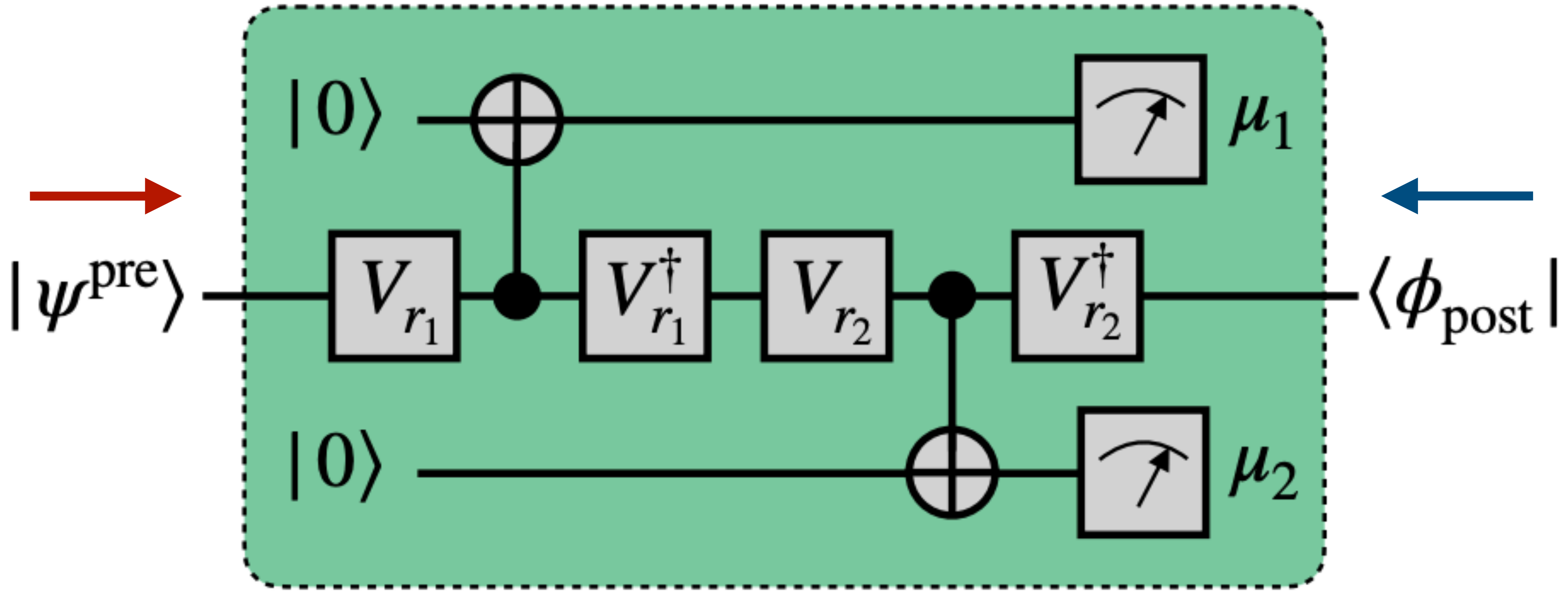
$\boxed{\text{optimization over Riemannian manifolds can be applied}}$

A. Pechen et al., “Control landscapes for two-level open quantum systems,” J. Phys. A **41**, 045205 (2008).
I. A. Luchnikov et al. “Qgopt: Riemannian optimization for quantum technologies,” SciPost Phys. **10**, 079 (2021).

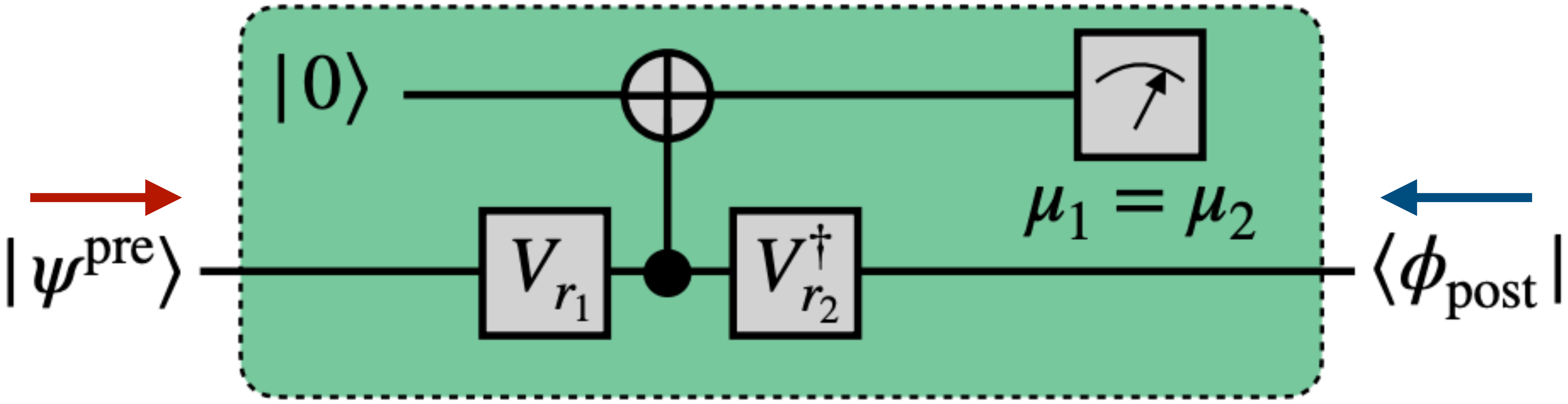
Single-qubit MUB-based approach

(mutually unbiased bases)

$r_1 \neq r_2$



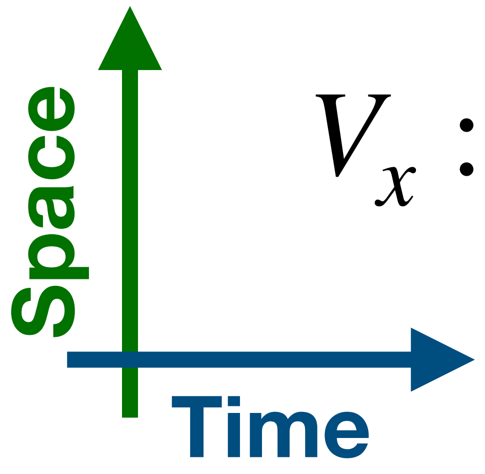
$r_1 = r_2$



$$r_1, r_2 \in \{x, y, z\}$$

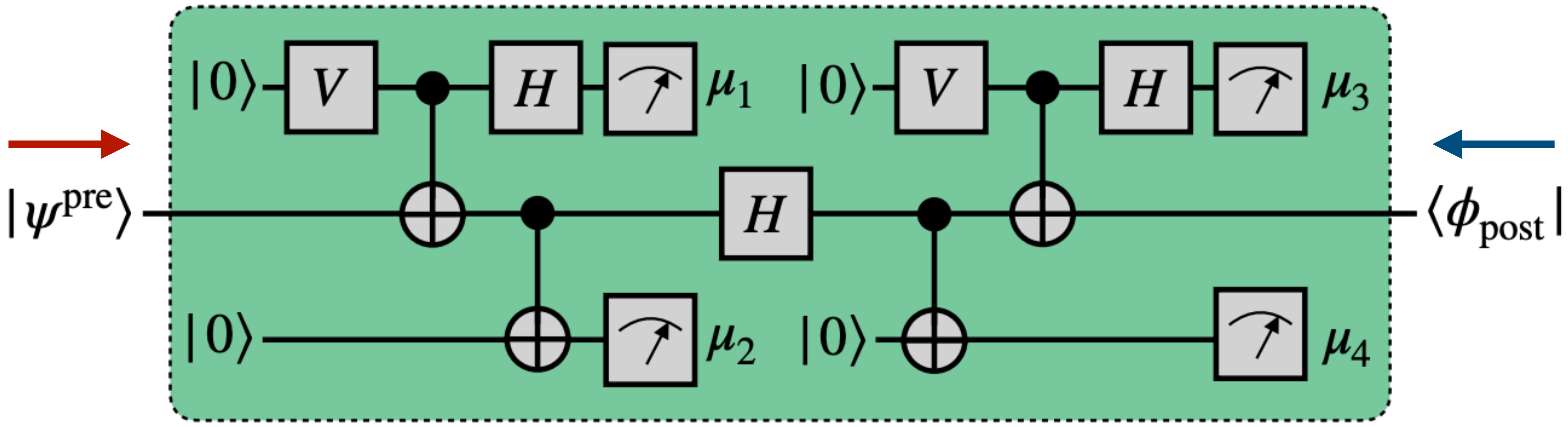
Idea: measure ↑ and ↓ qubits in x,y,z basis (9 measurement configurations are required)

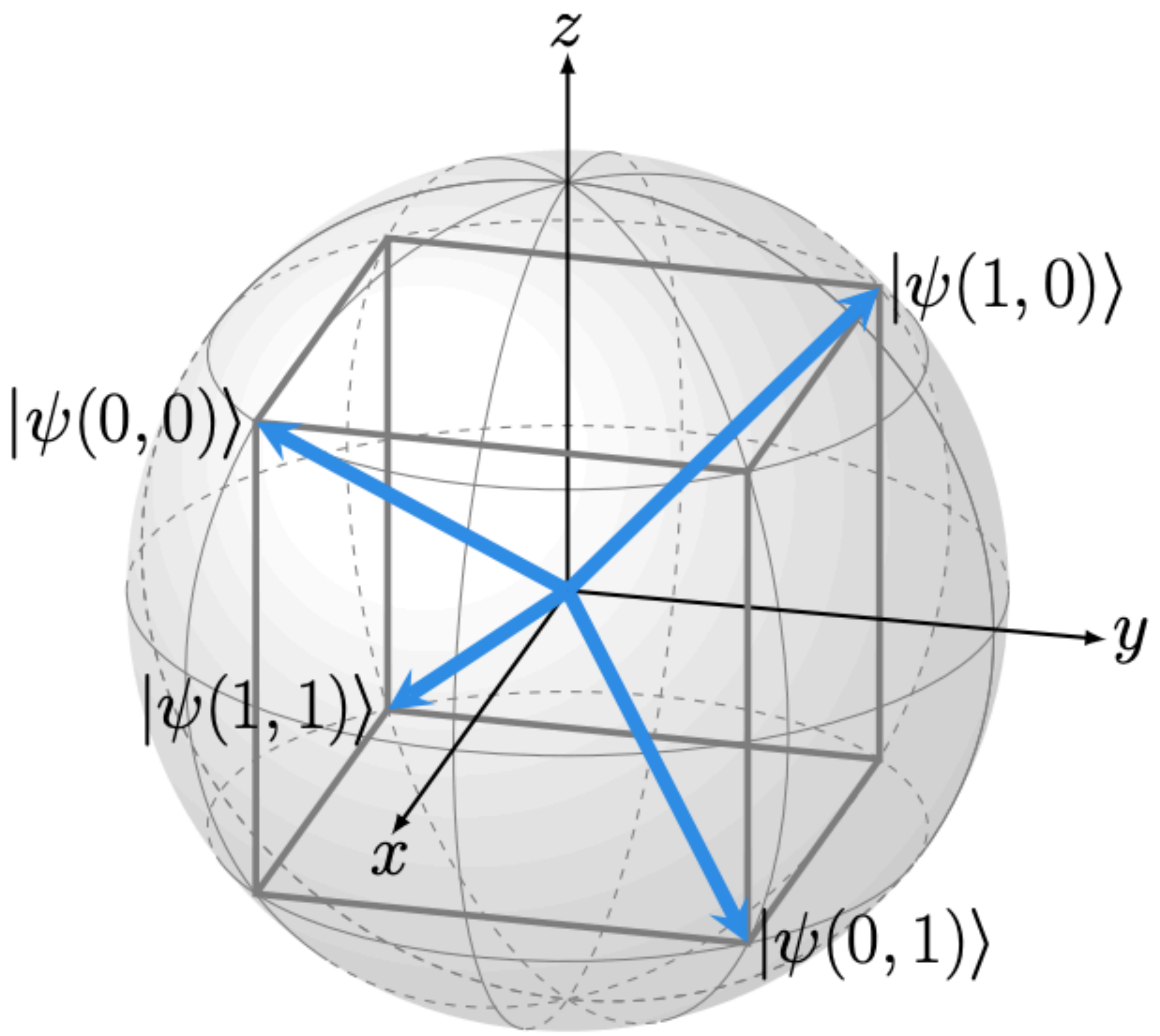
$$V_x := R_y(\pi/2), \quad V_y := R_x(\pi/2) \quad V_z := \mathbf{I}$$



Single-qubit SIC-POVM-based approach

(symmetric informationally complete)



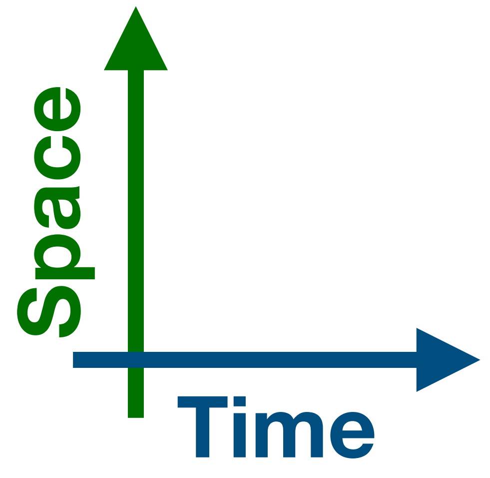


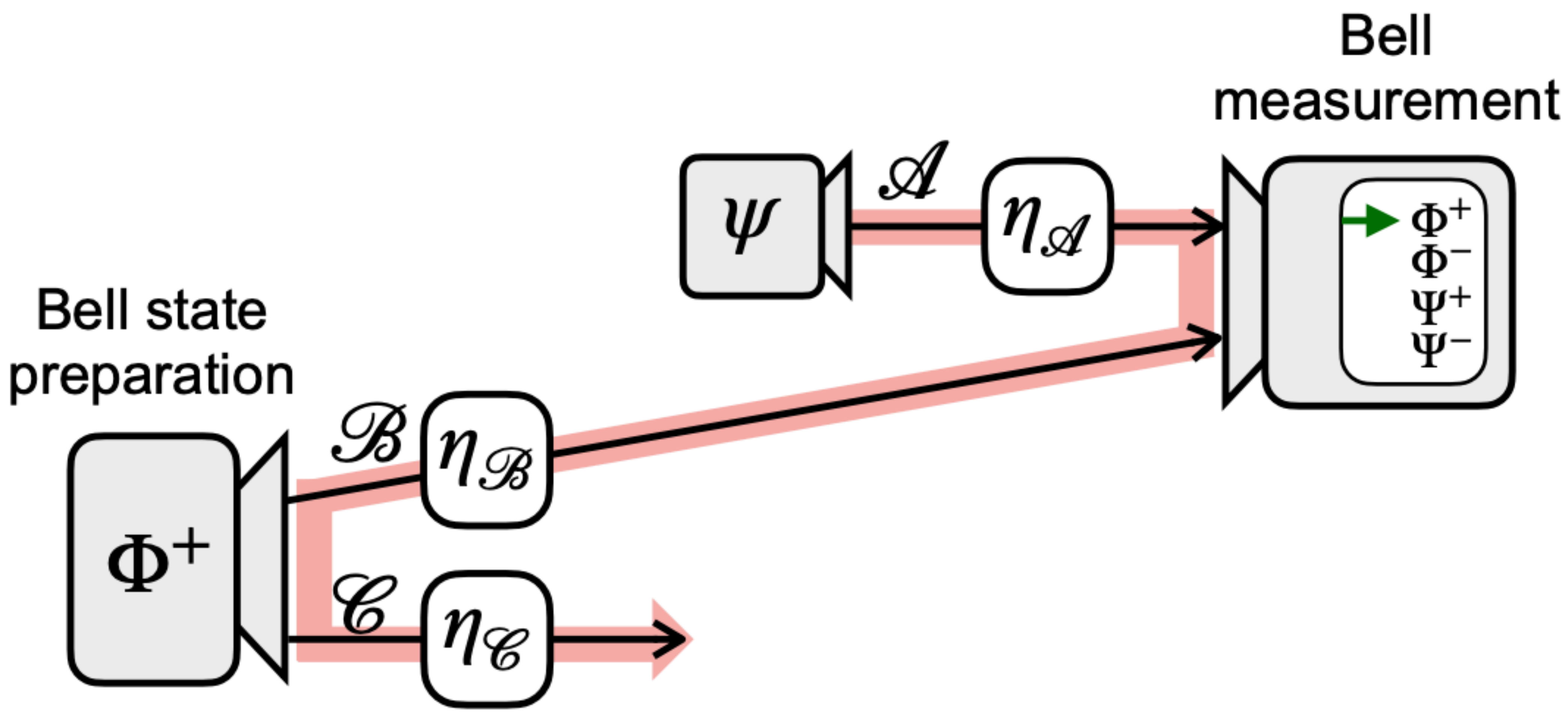
Idea: measure both ↑ and ↓ qubits
with SIC-POVMs
(1 fixed measurement configuration is required)

Experimental demonstration of employing TBS formalism

H. H. Genç, S. Aydın and H. Erdal, "A Novel Approach to the Implementation of Cloud-Based Quantum Programming Platforms in VR Environment," 2020 Innovations in Intelligent Systems and Applications Conference (ASYU), Istanbul, Turkey, 2020, pp. 1-6

Time-reversal in quantum teleportation





$$|0\rangle = \begin{pmatrix}1\\0\end{pmatrix} \quad |1\rangle = \begin{pmatrix}0\\1\end{pmatrix}$$

$$|\Phi^{\pm}\rangle \propto |00\rangle \pm |11\rangle$$

$$|\Psi^{\pm}\rangle \propto |01\rangle \pm |10\rangle$$

$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$$

J. Preskill, "Lecture Notes for Ph219/CS219" (2021)
B. Coecke, Contemp. Phys. **51**, 59–83 (2010).
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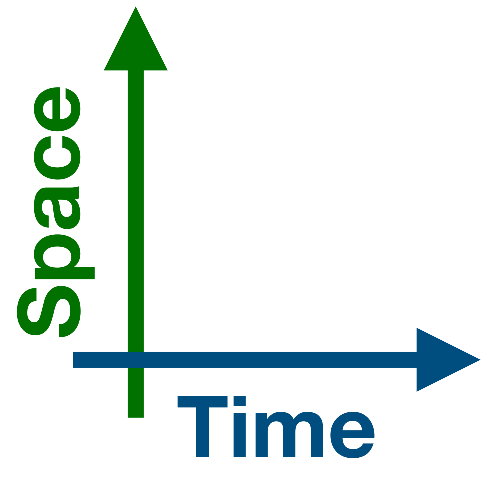

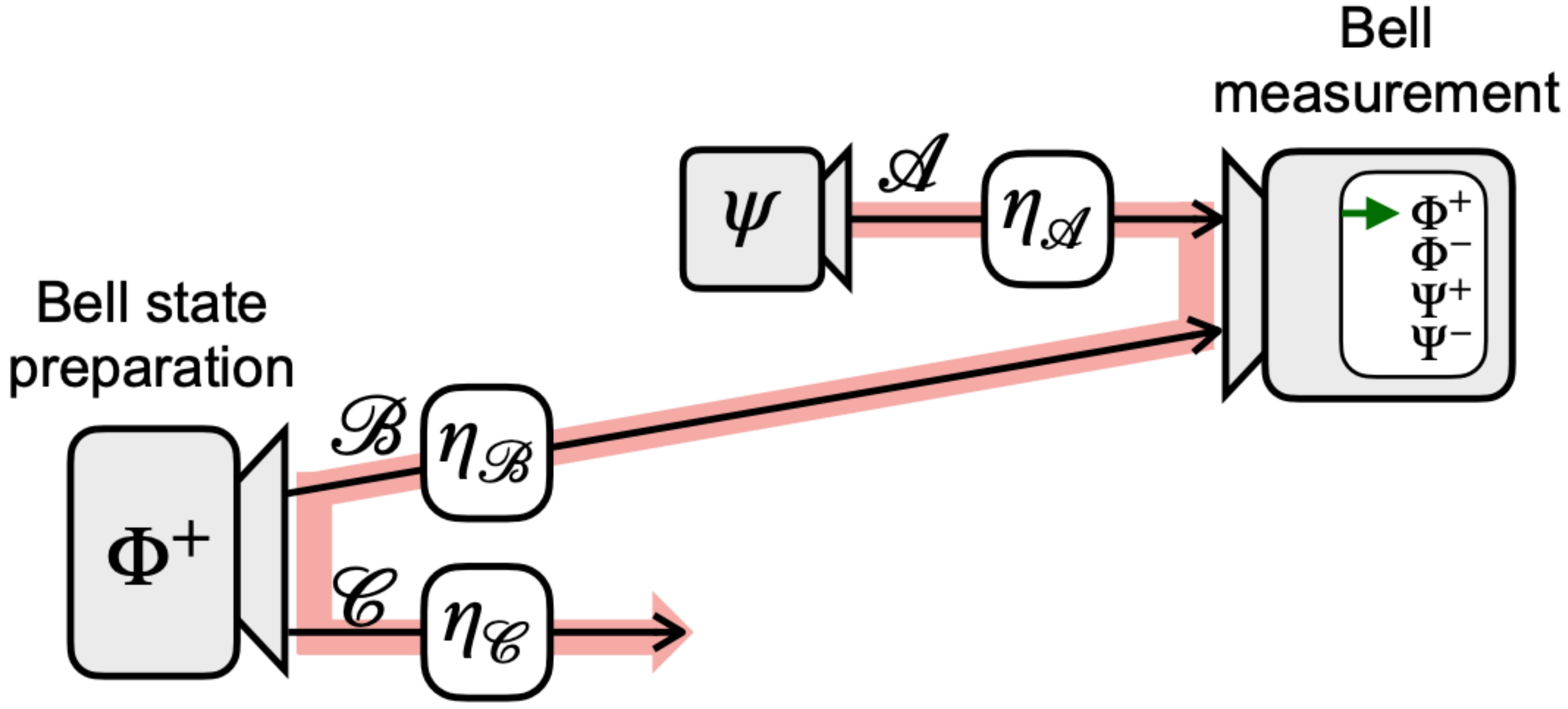

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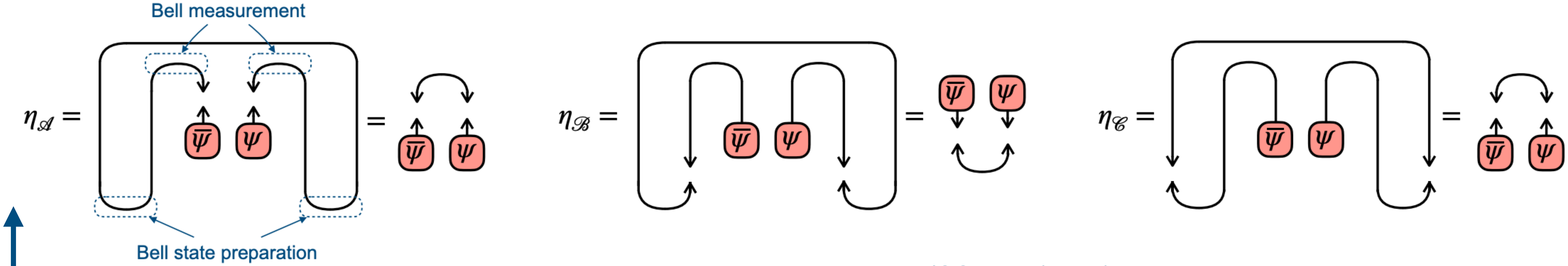

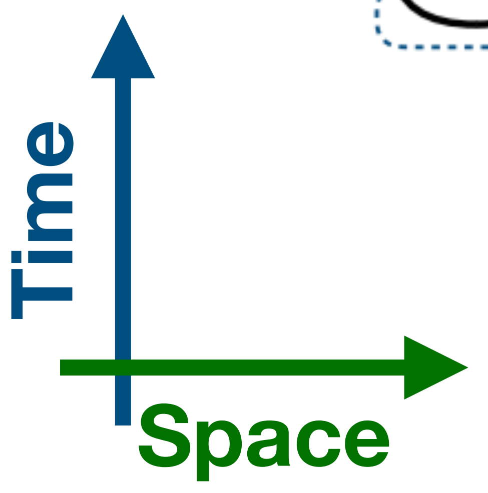

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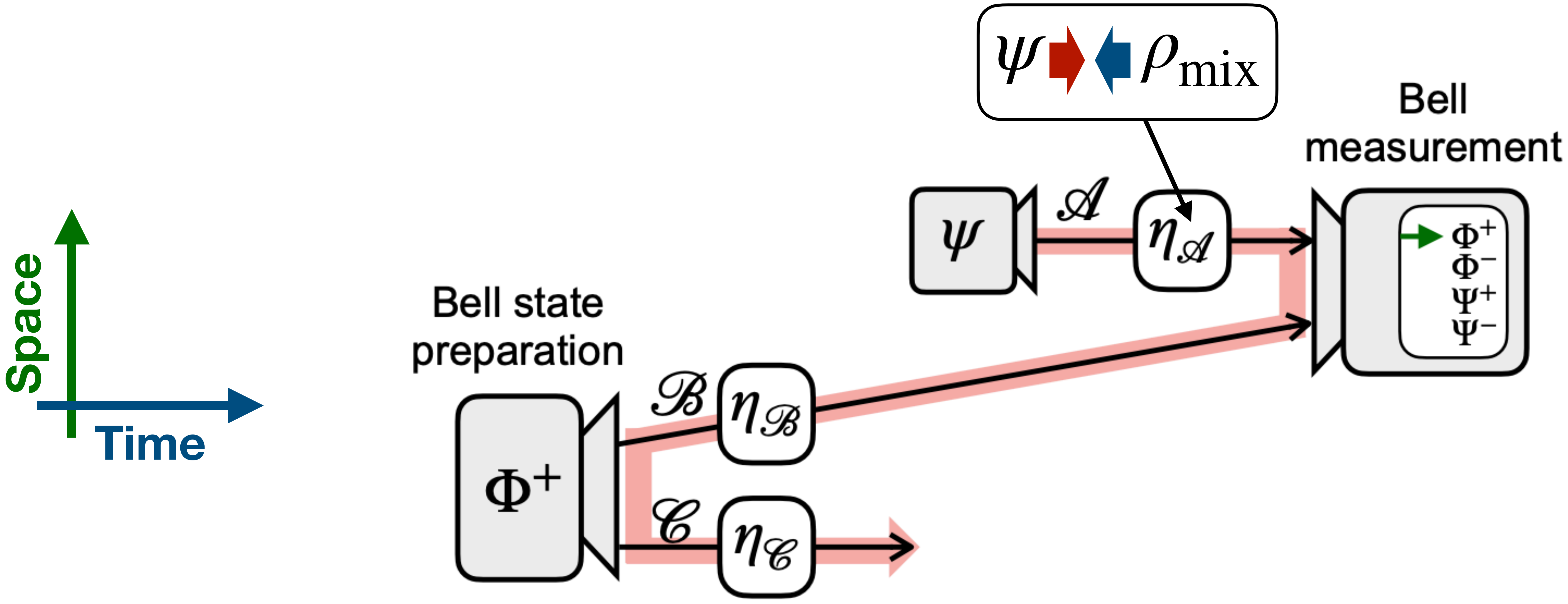
Time-reversal in quantum teleportation



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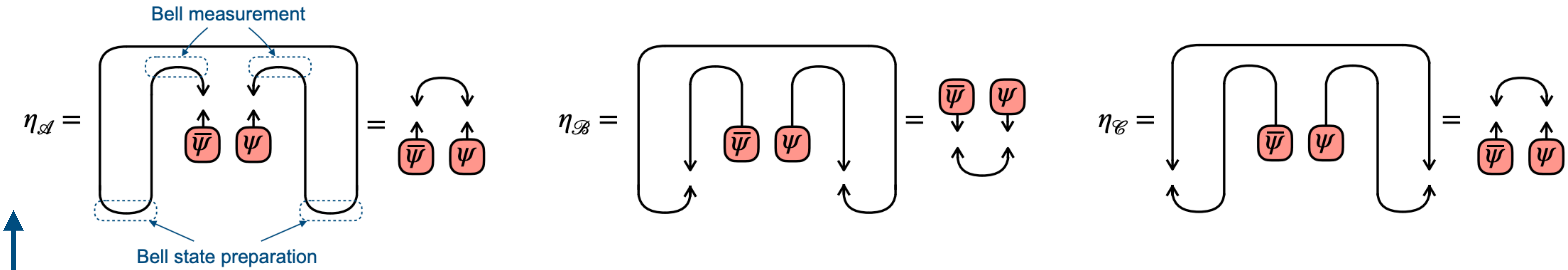

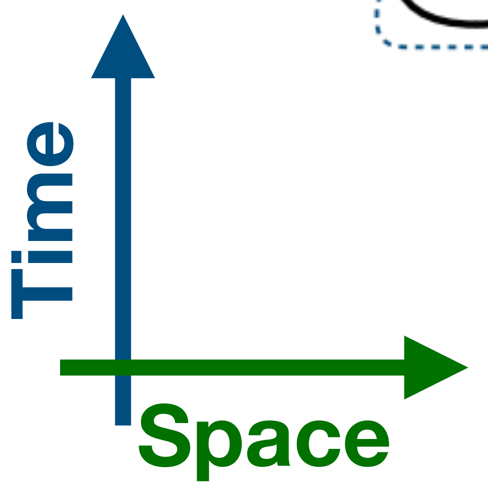

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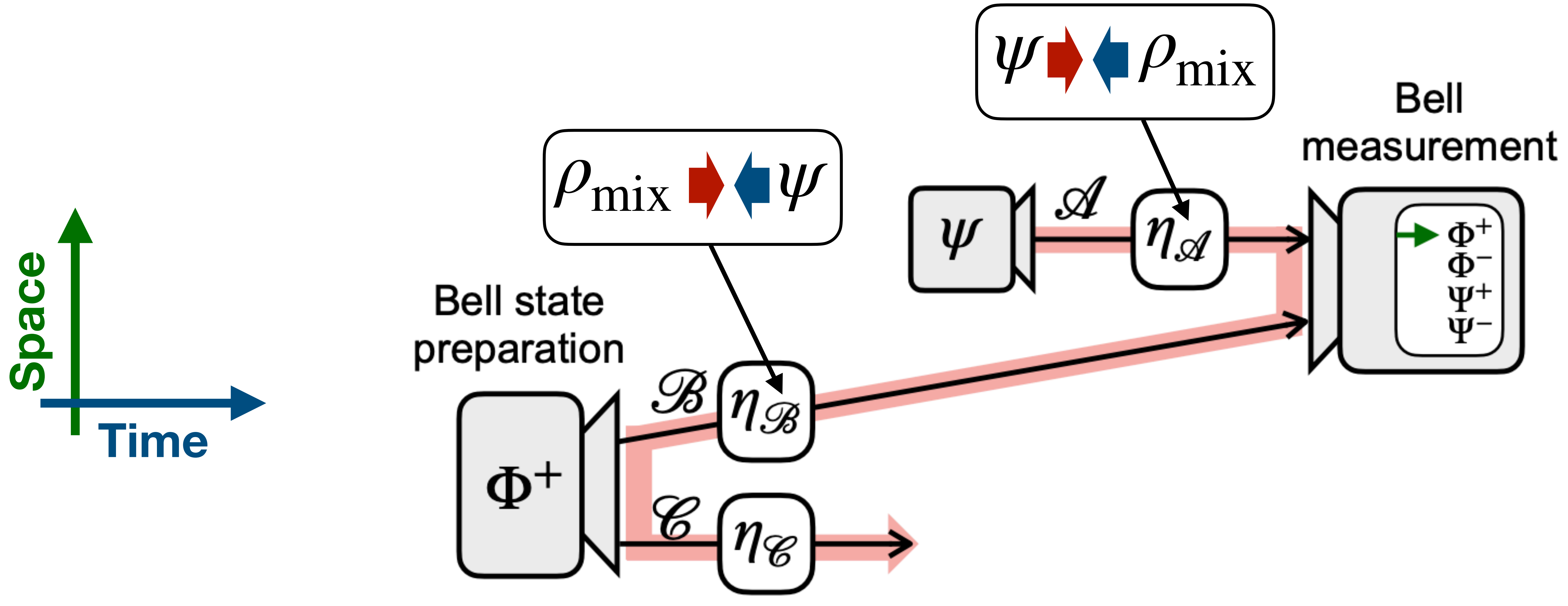

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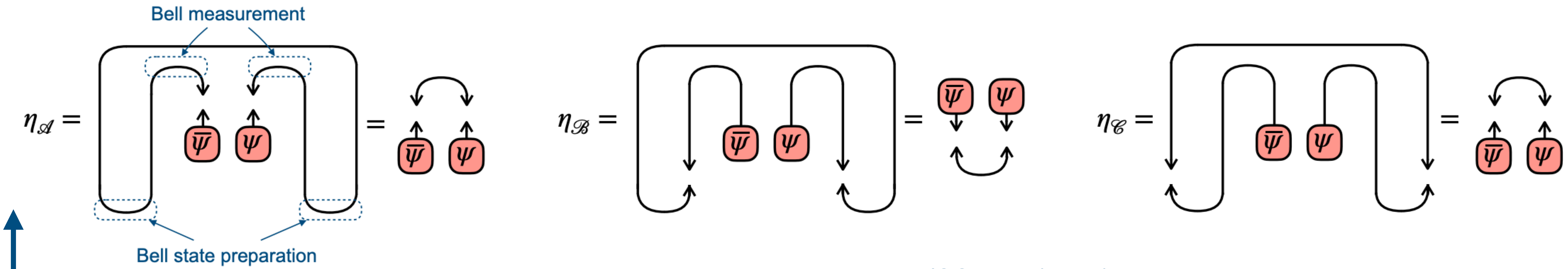

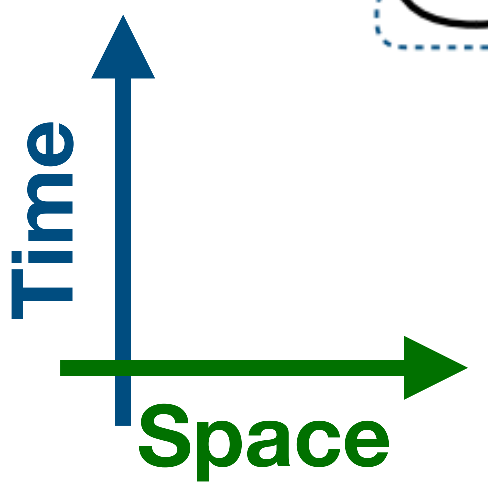

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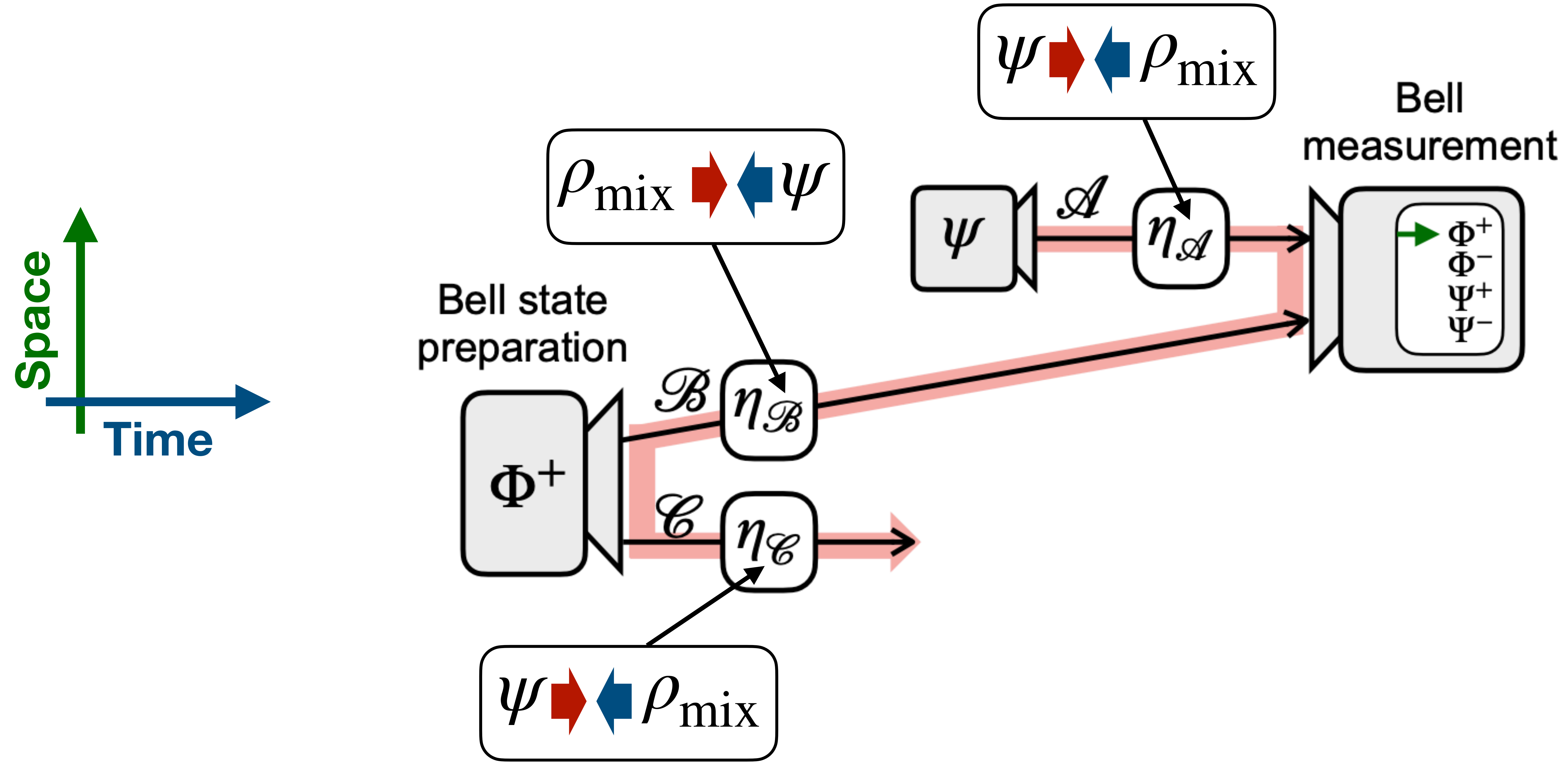
Time-reversal in quantum teleportation



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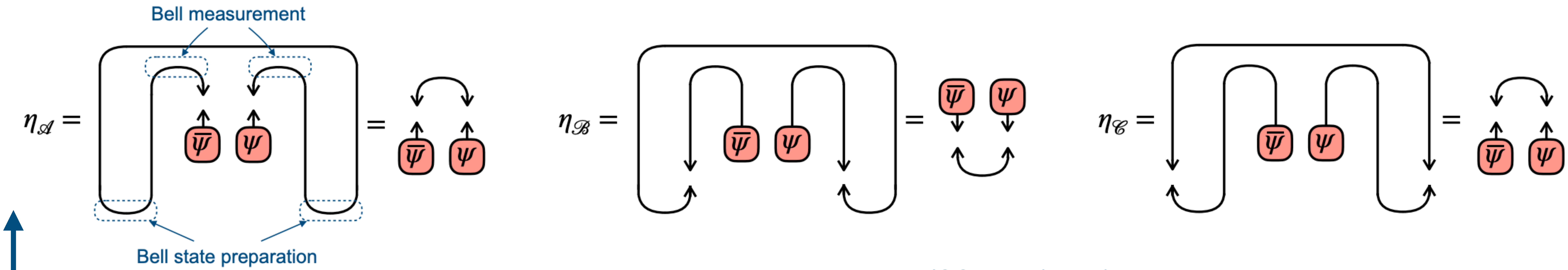

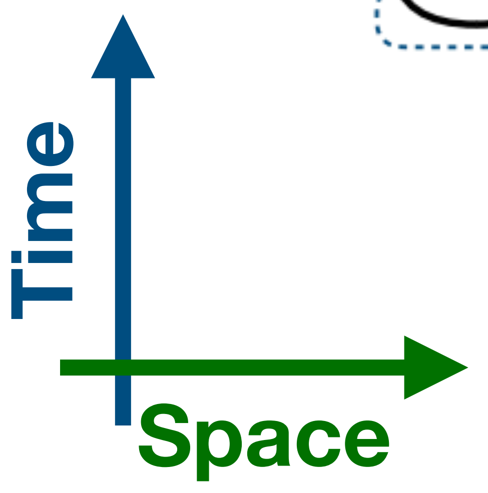

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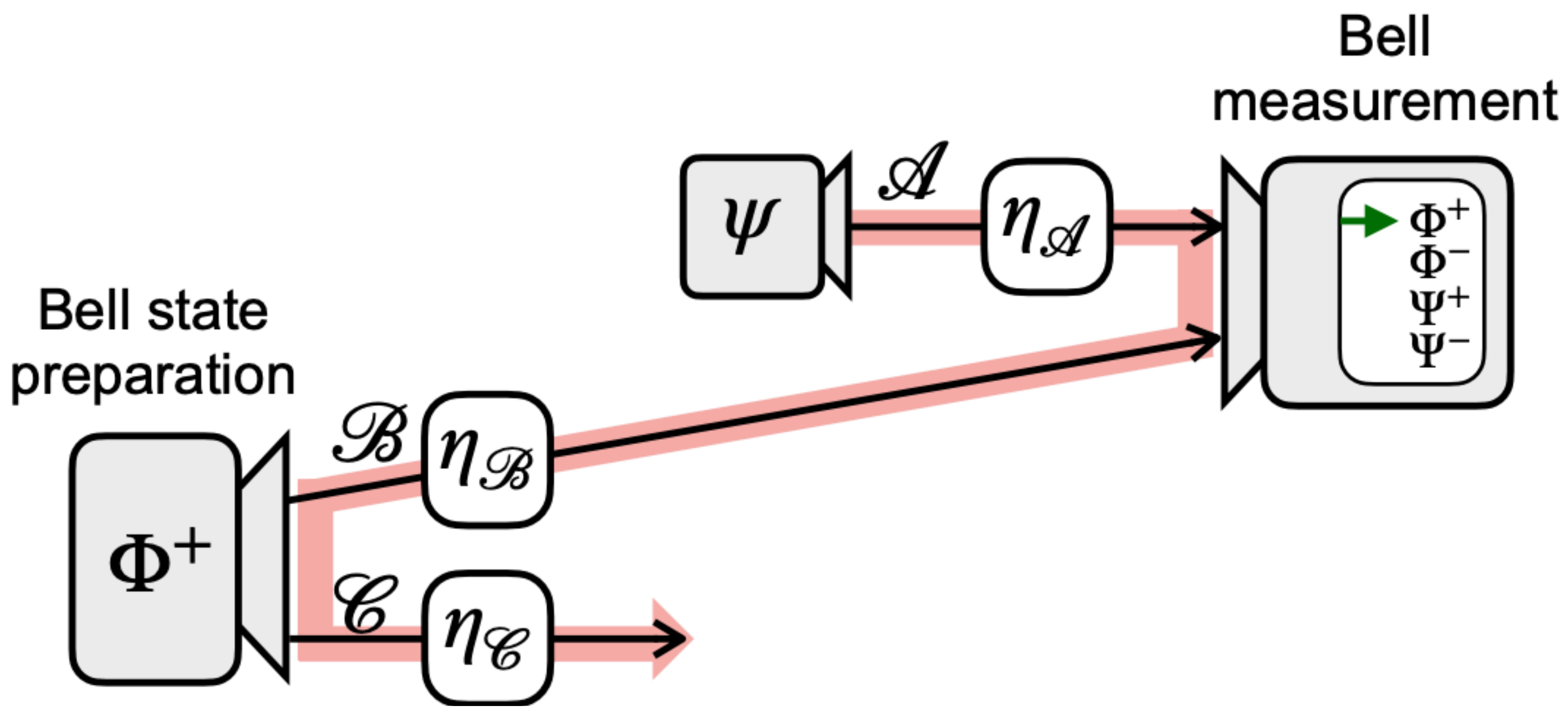
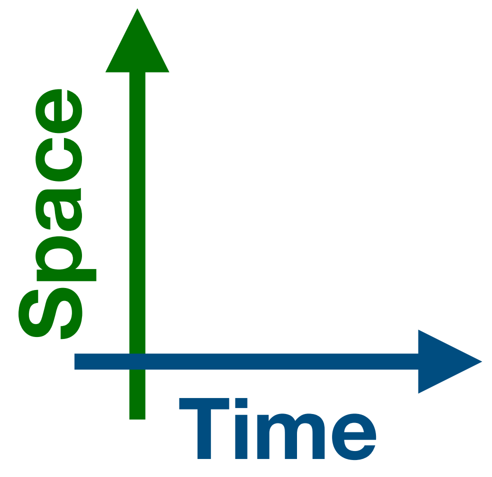
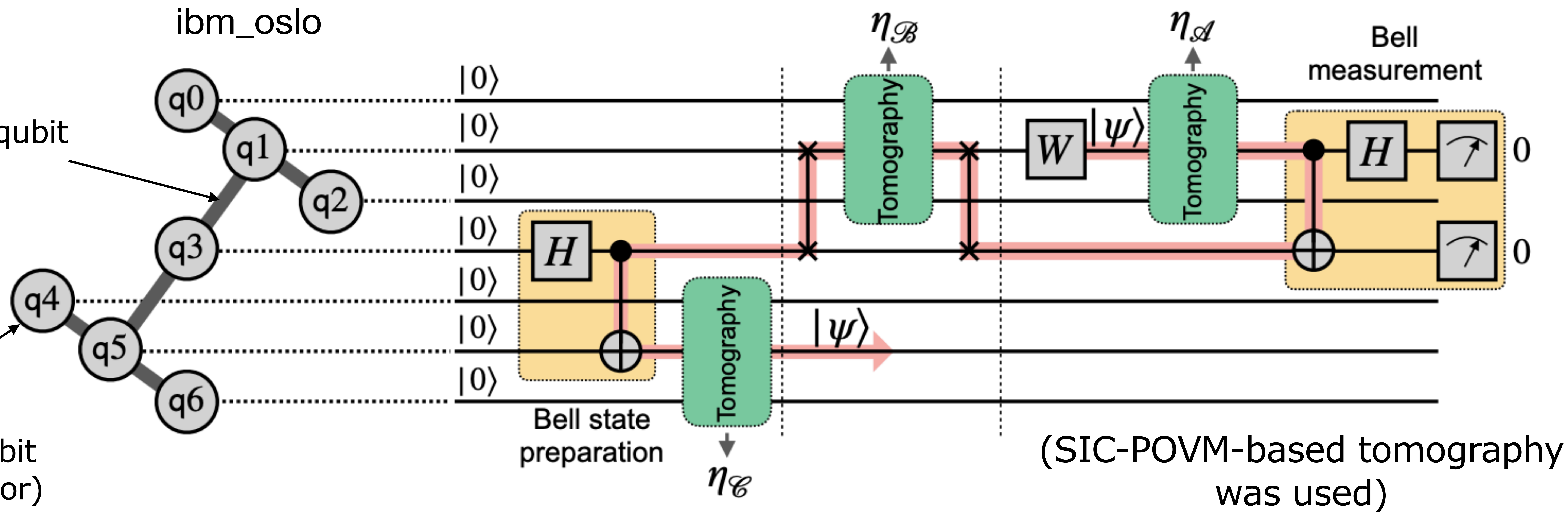
Tracking time-reversal quantum state propagation with 7-qubit superconducting quantum processor





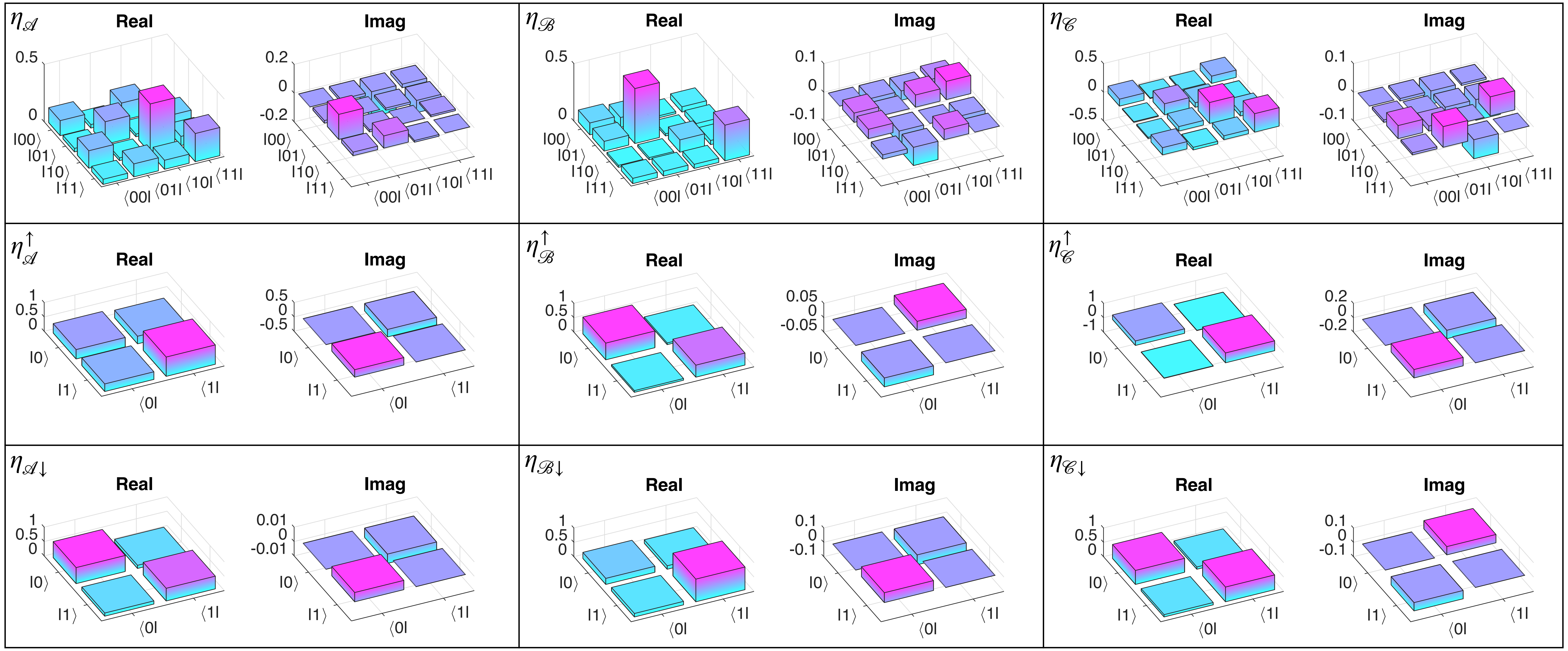
ability to make two-qubit entangling gate



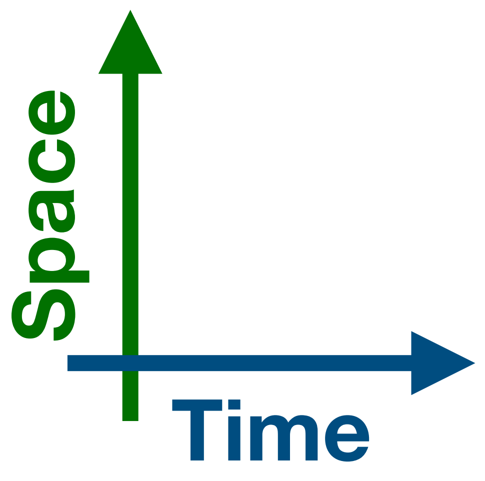
superconducting qubit (unharmonic oscillator)

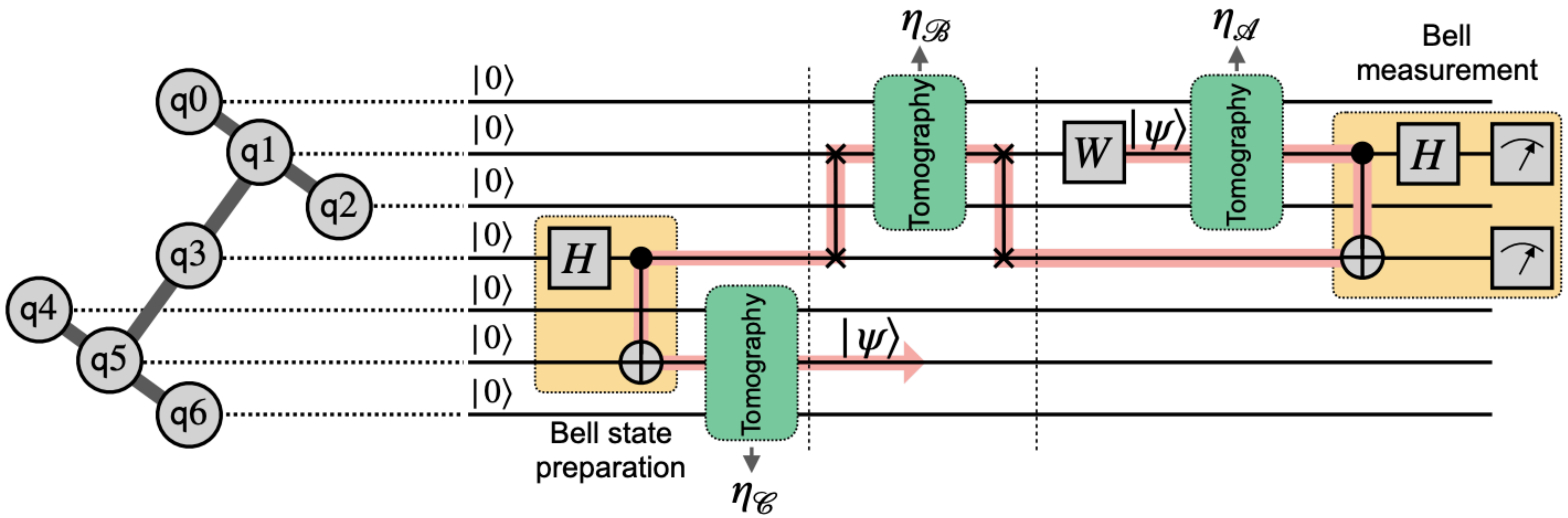
(SIC-POVM-based tomography was used)

Experimental results



	$\langle\psi\vert\eta_{\bullet}\vert\psi\rangle$	$1-\mathrm{Tr}(\eta_{\bullet}^2)$
State	Fidelity	Linear entropy
η_A	0.86	0.54
$\eta^{\uparrow}_A$	**0.89**	**0.20**
$\eta_{A\downarrow}$	0.97	0.45
η_B	0.76	0.57
$\eta^{\uparrow}_B$	0.98	0.47
$\eta_{B\downarrow}$	**0.79**	**0.25**
η_C	0.72	0.62
$\eta^{\uparrow}_C$	**0.74**	**0.37**
$\eta_{C\downarrow}$	0.99	0.48





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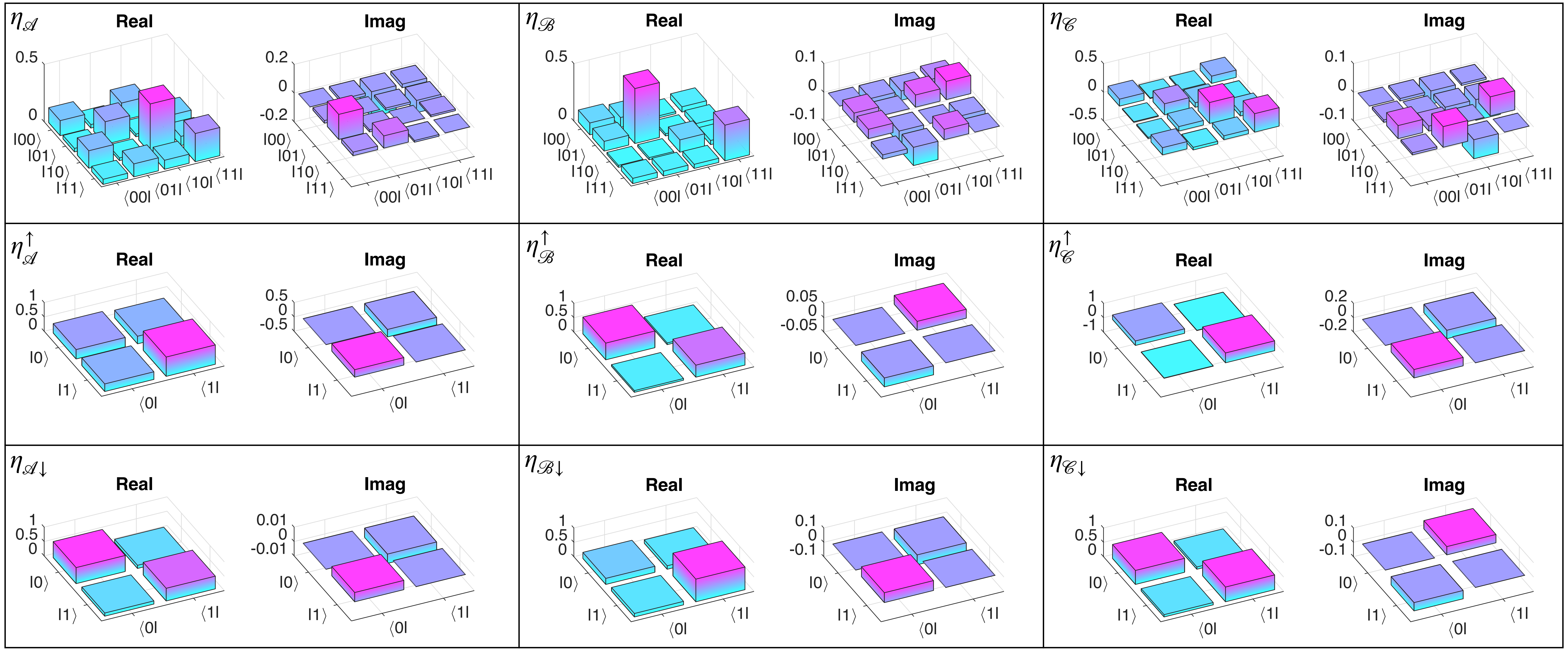

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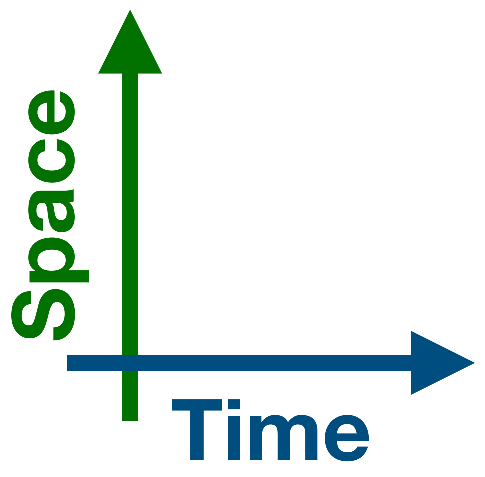
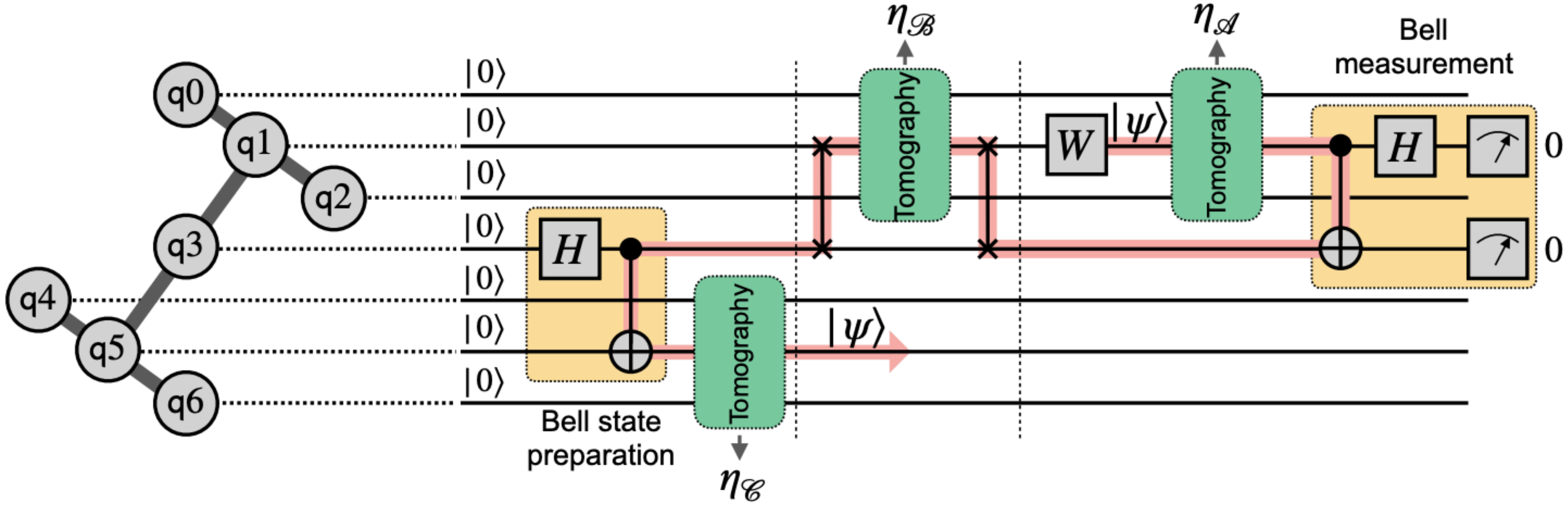
The state decays during its time-reversal propagation



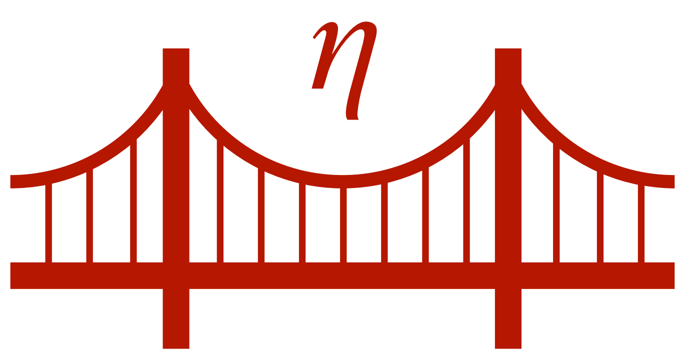


Conclusion

1) Generalisation of two-state vector formalism (TSVF) to mixed states and arbitrary measurements (POVMs) has been obtained. This bridges the gap between the standard formalism and TSVF.

no postselection

$$\eta = \rho \otimes \mathbf{1}$$



projective postselection

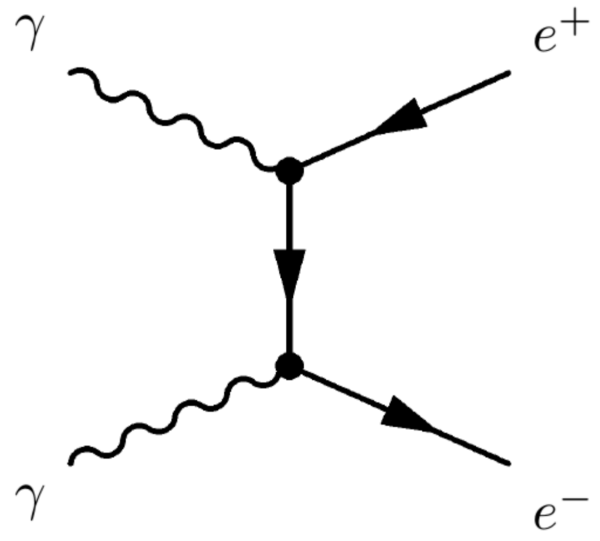
$$\eta = |\psi\rangle\langle\psi| \otimes |\phi\rangle\langle\phi|$$

2) Possibility of purification for TBS is shown.
3) Expressions for mean and weak values for arbitrary TBS have been derived.
4) MUB-based and SIC-POVM-based single-qubit TBS tomography schemes have been developed.
5) Applicability for studying time-reversal effects in noisy postselected quantum teleportation has been demonstrated.

See more details in Phys. Rev. A **107**, 032419 (2023) | arXiv:2210.01583.

Open questions:

1) Can the doubling of the Hilbert space be used in quantum information processing?
2) What are «dynamics» equations for TBS?
3) How this behaviour of quantum information relates to particle physics?



Thank you for attention!

e.kiktenko@rqc.ru

The author acknowledges use of the IBM Q Experience for this work.
The views expressed are those of the author and do not reflect the official policy or position of IBM or the IBM Q Experience team.
The work was supported by the Russian Federation represented by the Ministry of Science and Higher Education (Grant No. 075-15-2020-788).

Properties of time-bidirectional states

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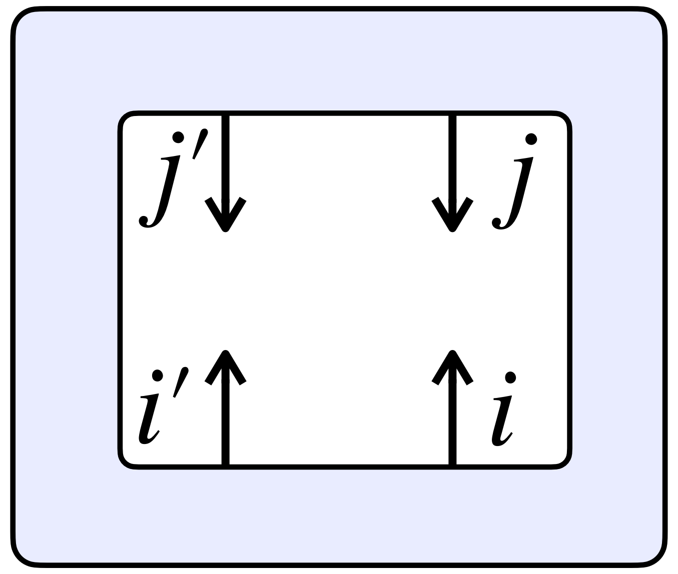
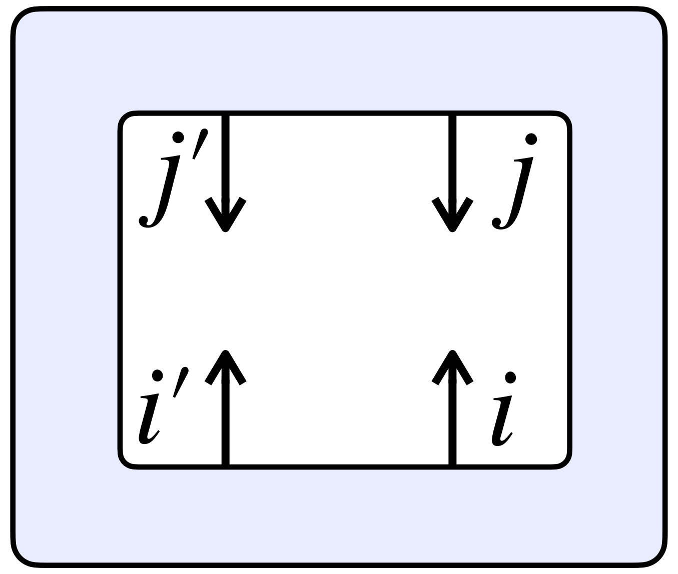
$\eta =$ 

$\in \mathscr{L}(\mathscr{H}^{\otimes 2}) = \mathscr{H} \otimes \mathscr{H}^* \otimes \mathscr{H} \otimes \mathscr{H}^*$

Properties of time-bidirectional states

$\eta =$



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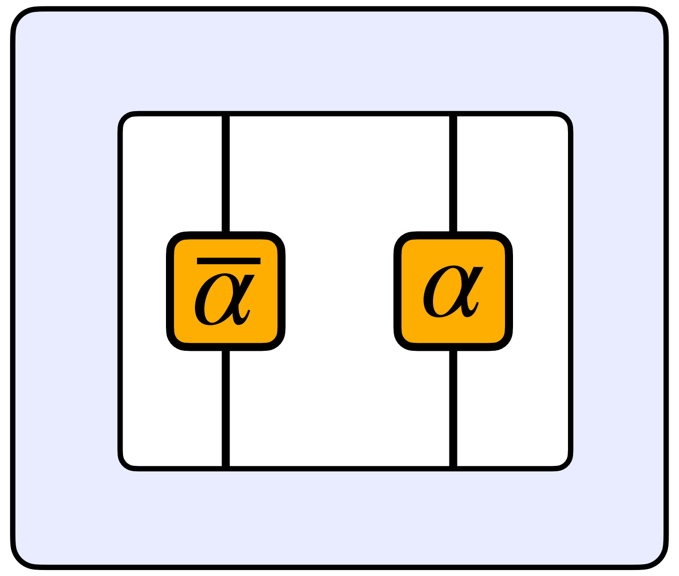
$\forall \alpha : \langle \alpha | \eta | \alpha \rangle =$



≥ 0 semi-positivity condition

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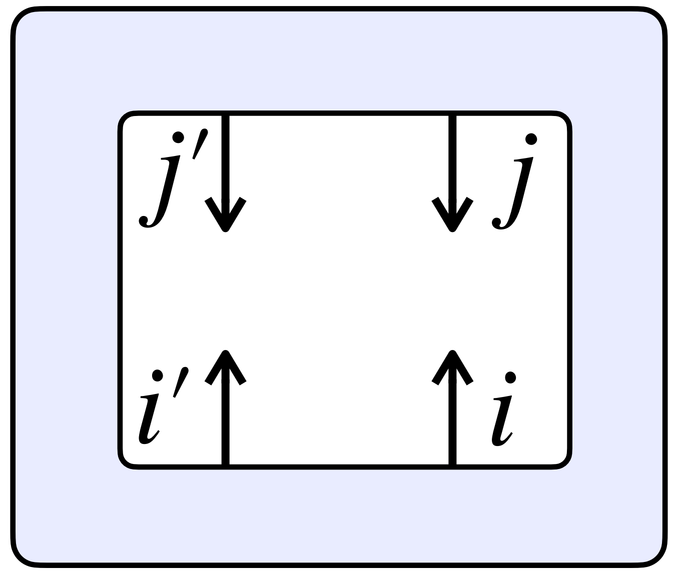
$\eta =$ 

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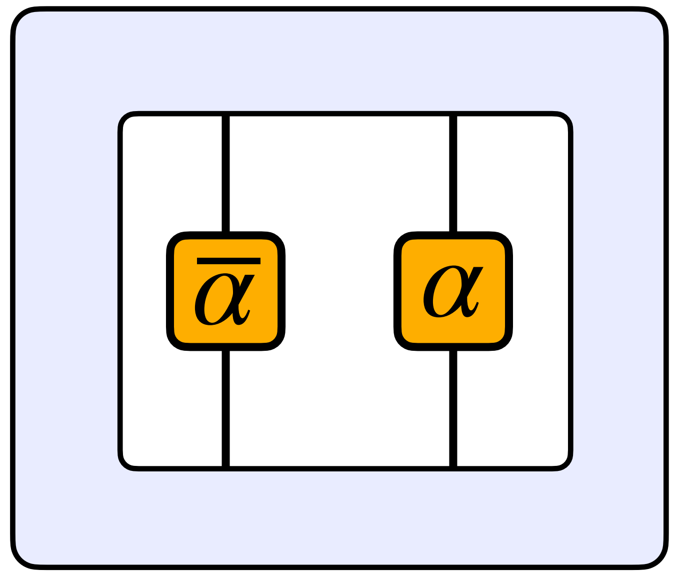
$\forall \alpha : \langle \alpha | \eta | \alpha \rangle =$ 

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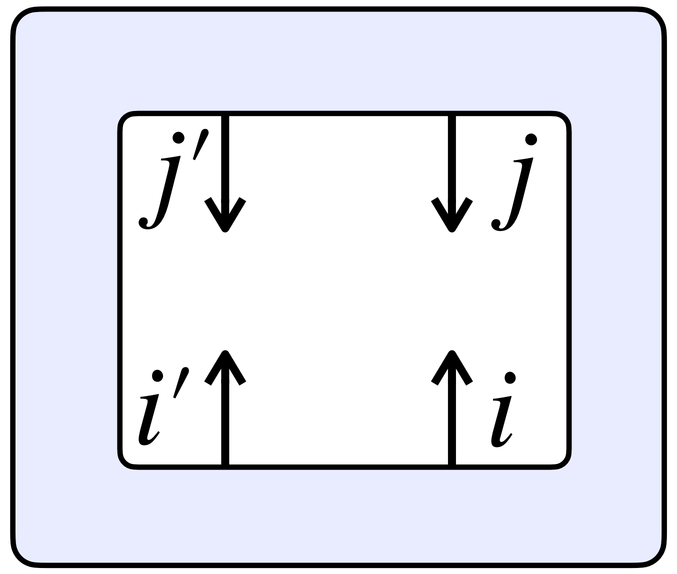
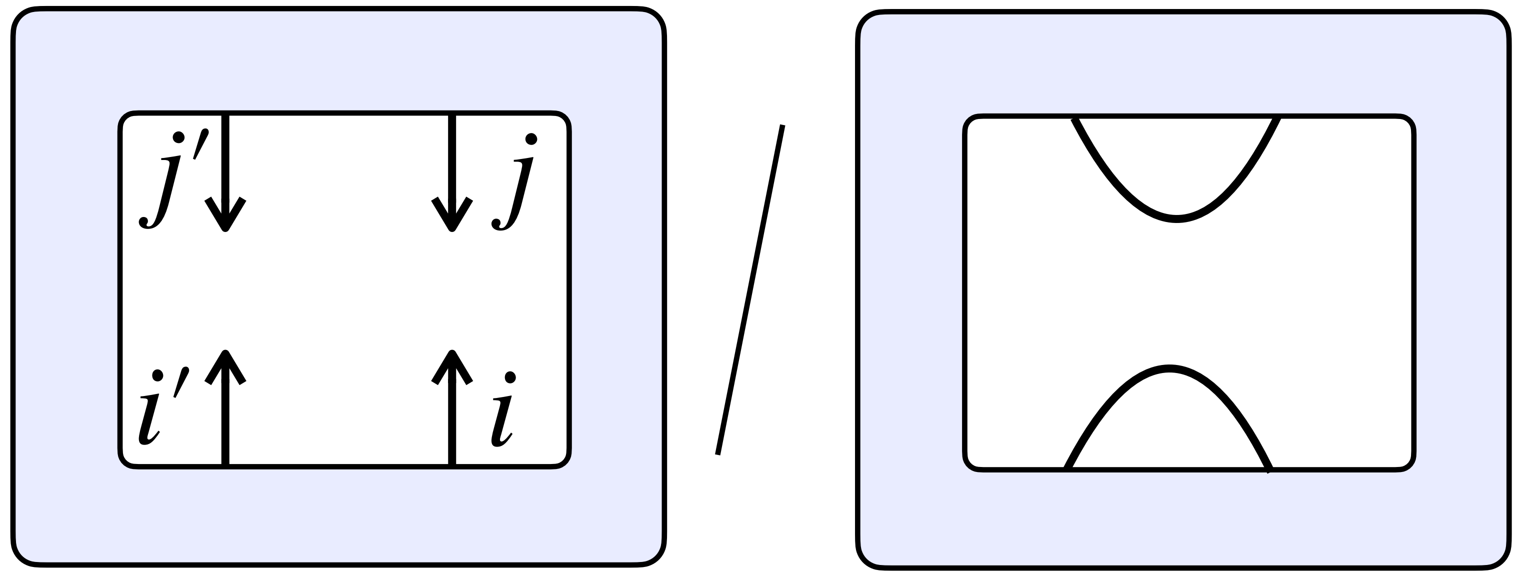
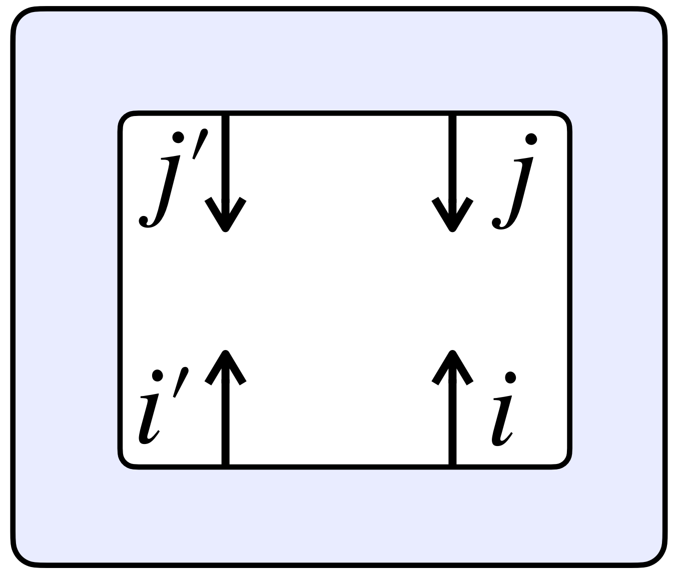


$\rightarrow$ 

normalization

Properties of time-bidirectional states

$\eta =$



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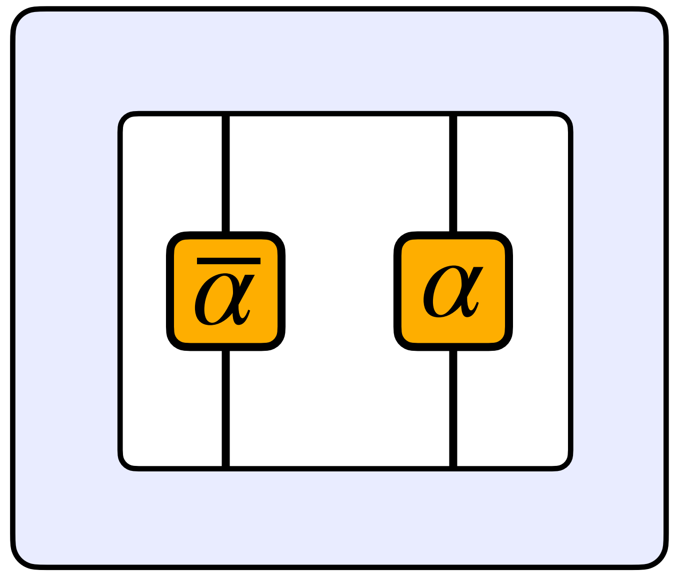

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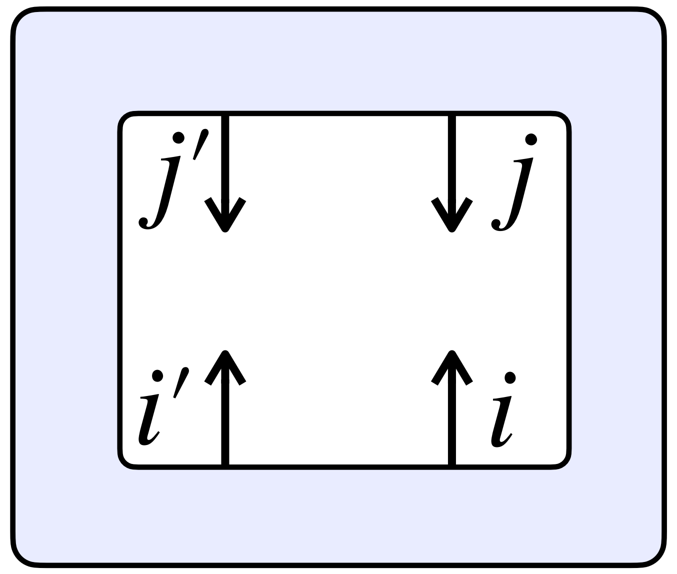

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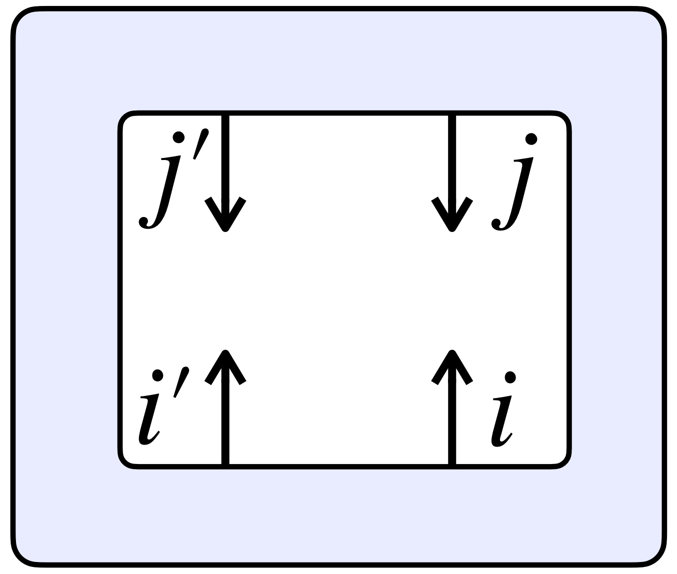

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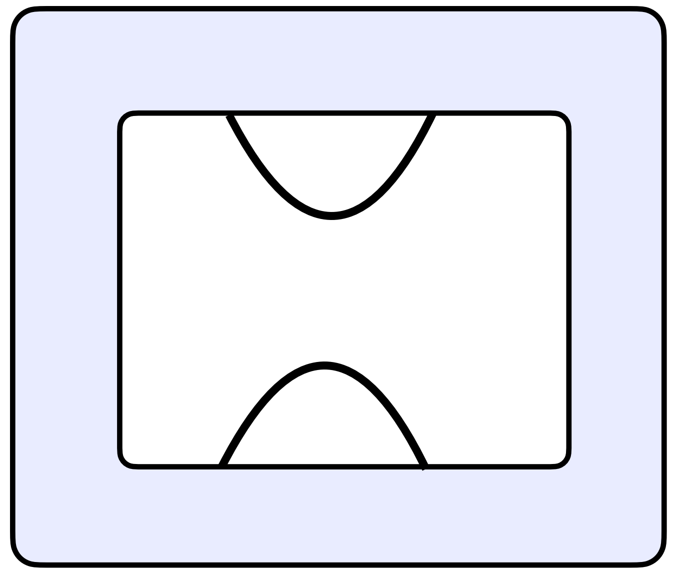

normalization

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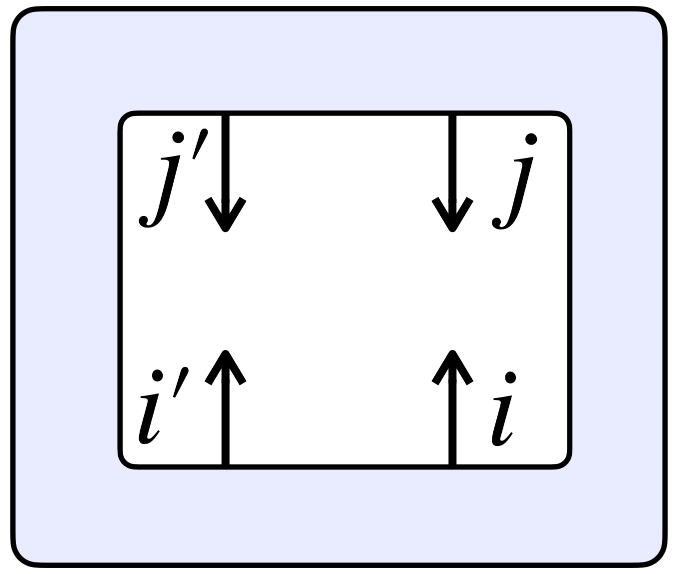

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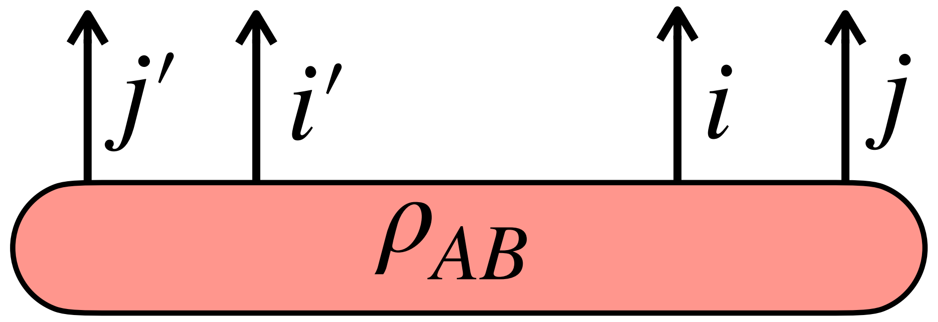
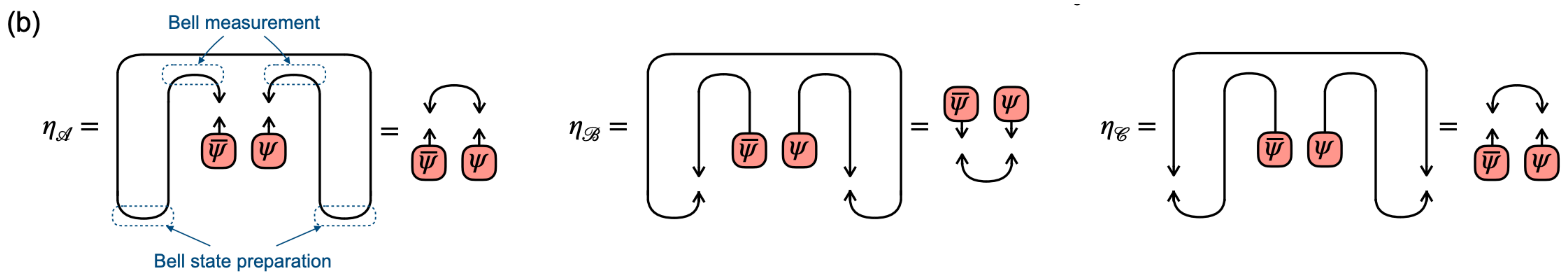


TBS is equivalent to a density matrix of two particles

Modelling noise with depolarizing channel



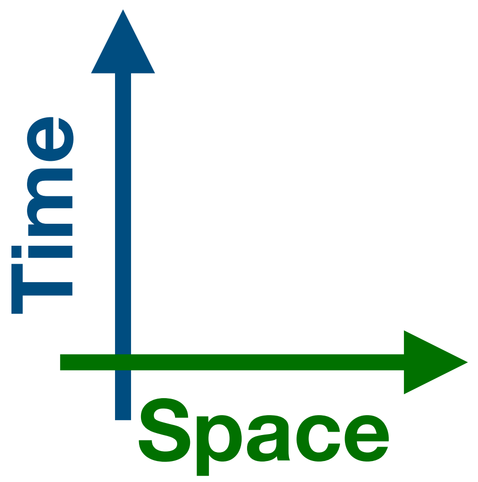
$\eta_{\mathscr{A}} = |\psi\rangle\langle\psi| \otimes \rho_{\mathrm{mix}}$ $\eta_{\mathscr{B}} = \rho_{\mathrm{mix}} \otimes |\psi\rangle\langle\psi|$ $\eta_{\mathscr{C}} = |\psi\rangle\langle\psi| \otimes \rho_{\mathrm{mix}}$

Noisy preparation: $|\Phi^+\rangle\langle\Phi^+| \to f_{\mathrm{pr}}|\Phi^+\rangle\langle\Phi^+| + (1-f_{\mathrm{pr}})\rho_{\mathrm{mix}} \otimes \rho_{\mathrm{mix}}$

Noisy measurement: $|\Phi^+\rangle\langle\Phi^+| \to f_{\mathrm{ms}}|\Phi^+\rangle\langle\Phi^+| + (1-f_{\mathrm{ms}})\rho_{\mathrm{mix}} \otimes \rho_{\mathrm{mix}}$

$\eta_{\mathscr{A}} = |\psi\rangle\langle\psi| \otimes \rho_{\mathrm{mix}}$ $\eta_{\mathscr{B}} = \rho_{\mathrm{mix}} \otimes [f_{\mathrm{pr}}|\psi\rangle\langle\psi| + (1-f_{\mathrm{pr}})\rho_{\mathrm{mix}}]$ $\eta_{\mathscr{C}} = [f_{\mathrm{pr}}f_{\mathrm{ms}}|\psi\rangle\langle\psi| + (1-f_{\mathrm{pr}}f_{\mathrm{ms}})\rho_{\mathrm{mix}}] \otimes \rho_{\mathrm{mix}}$

Postselection-induced time travel



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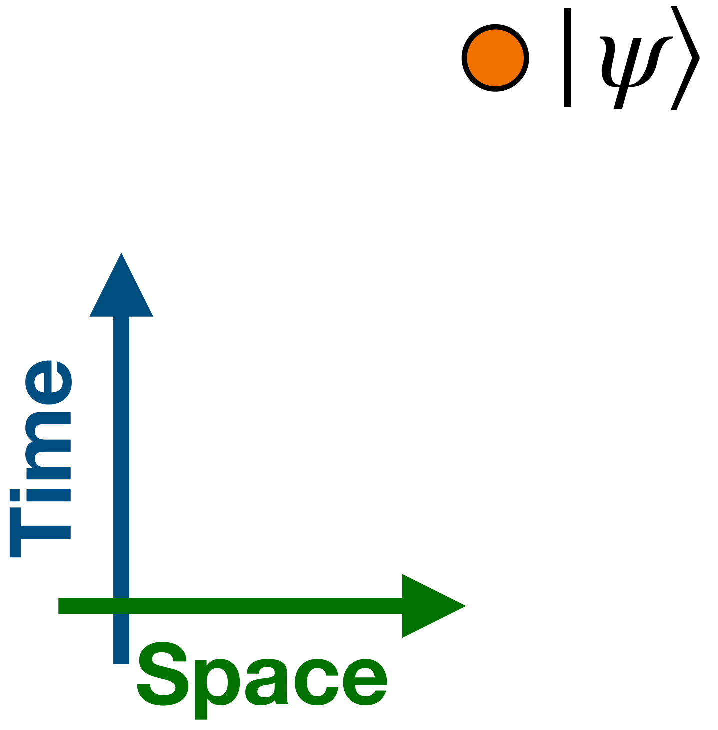

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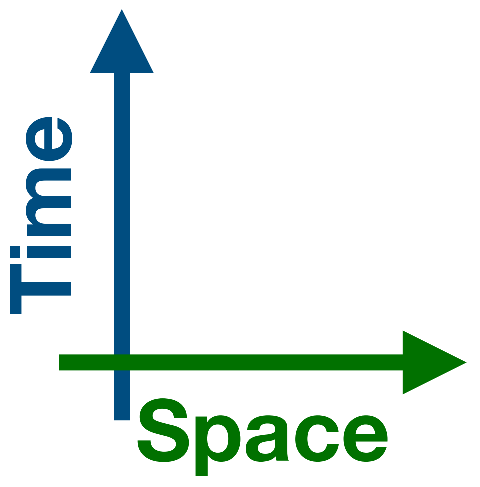
Postselection-induced time travel

$|\psi\rangle$

$|\Phi\rangle$

maximally entangled



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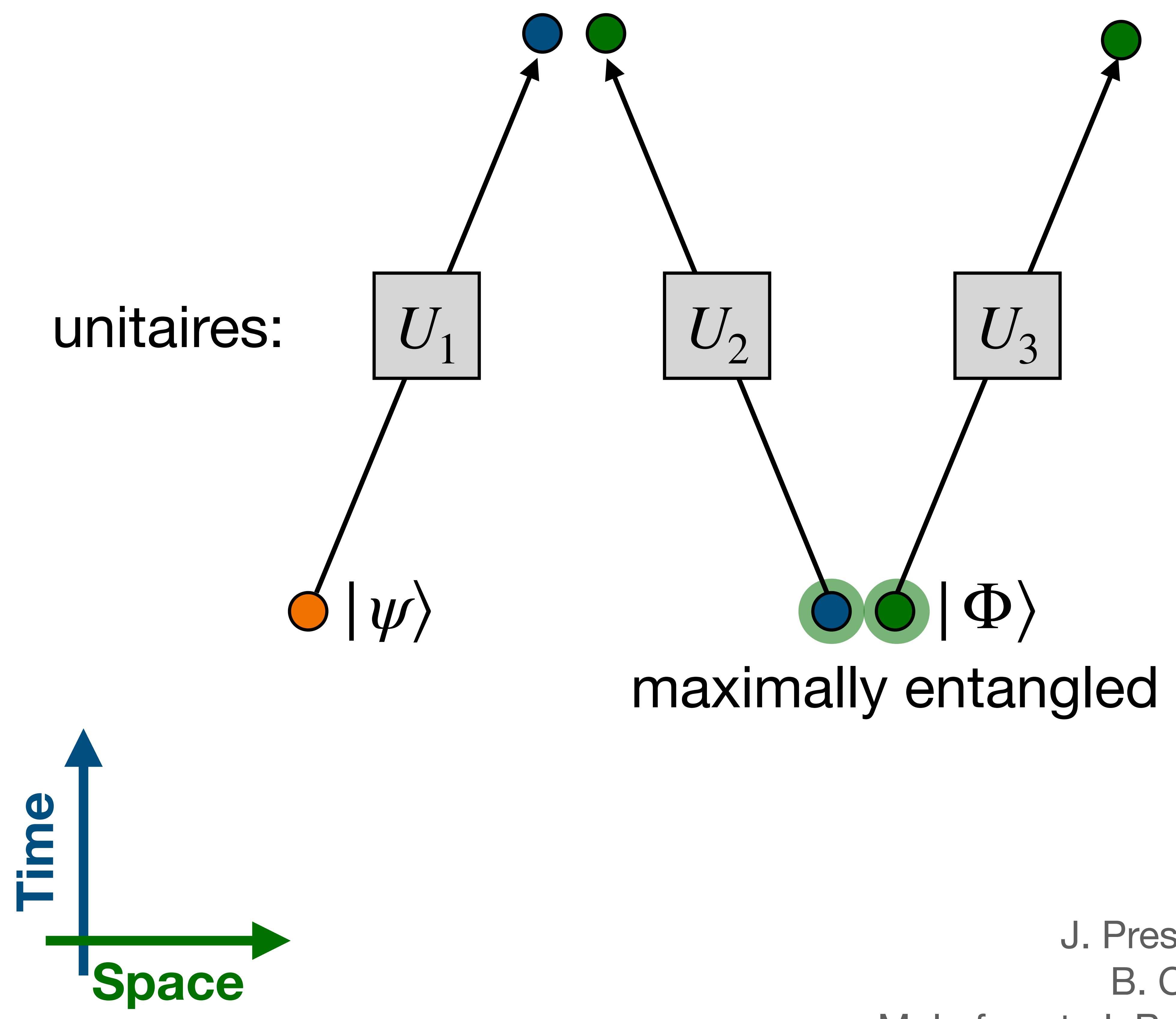

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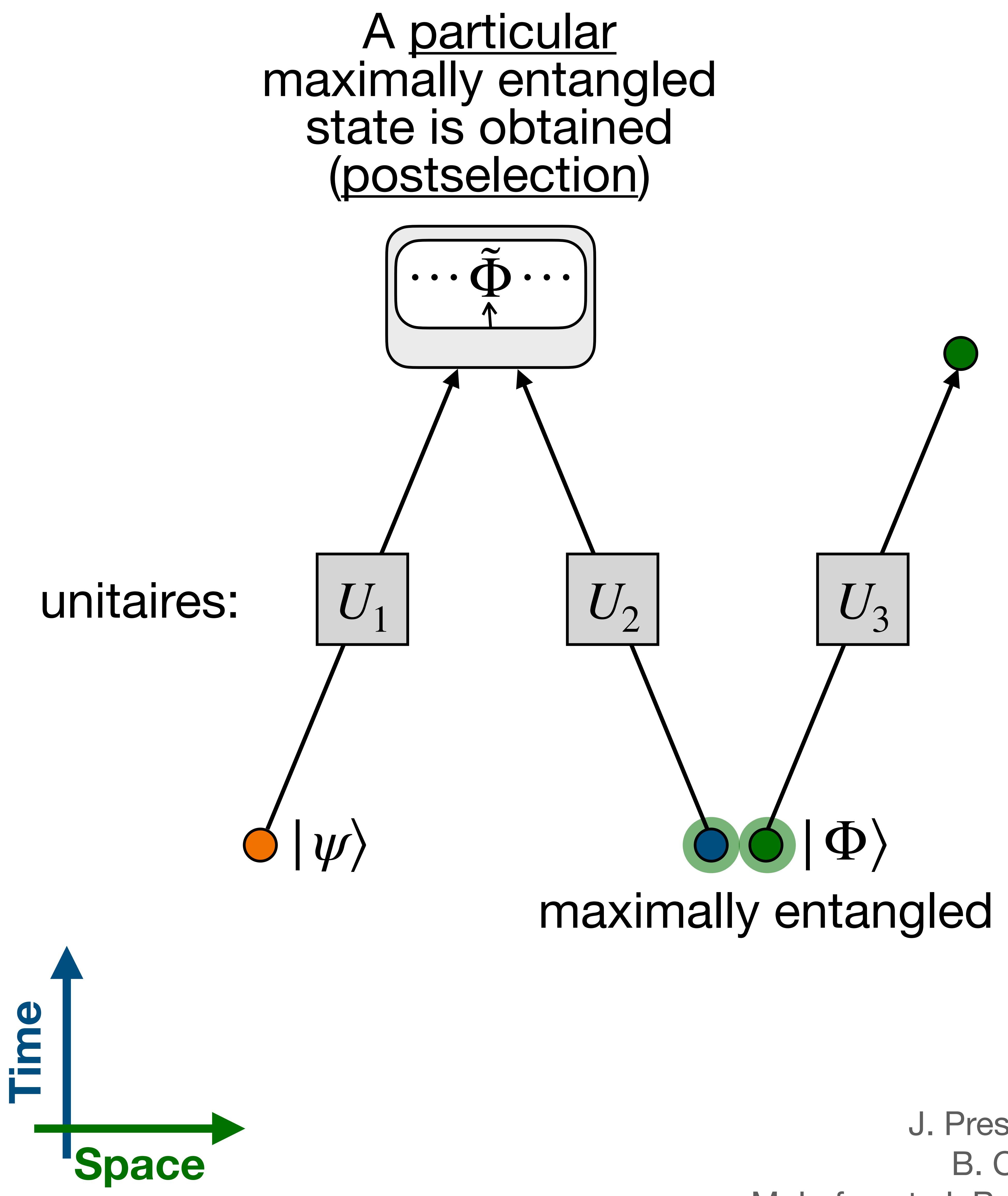

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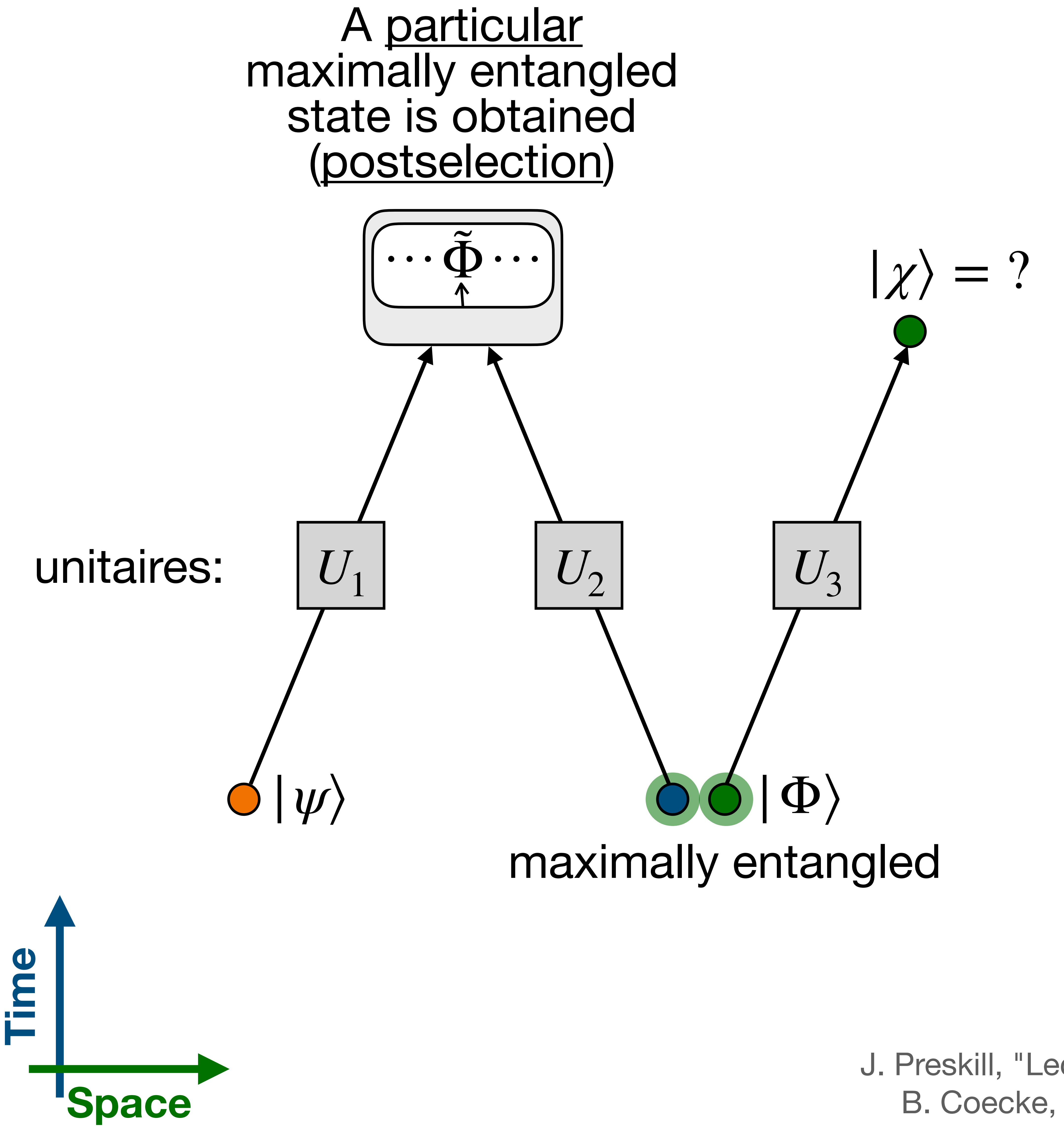
Postselection-induced time travel



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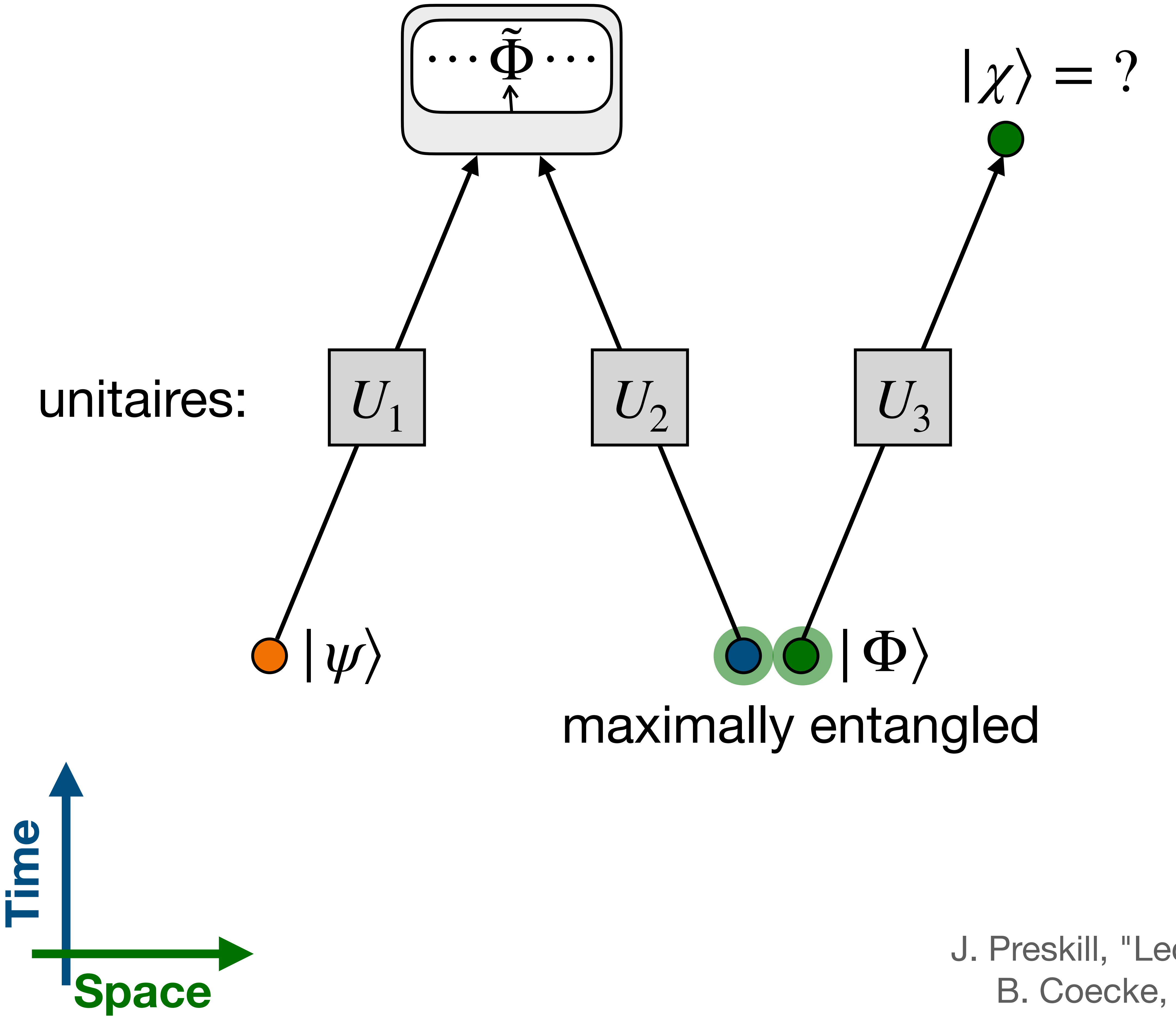
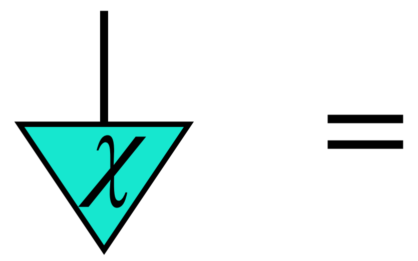
Postselection-induced time travel

A particular maximally entangled state is obtained (postselection)



Calculations with tensor networks



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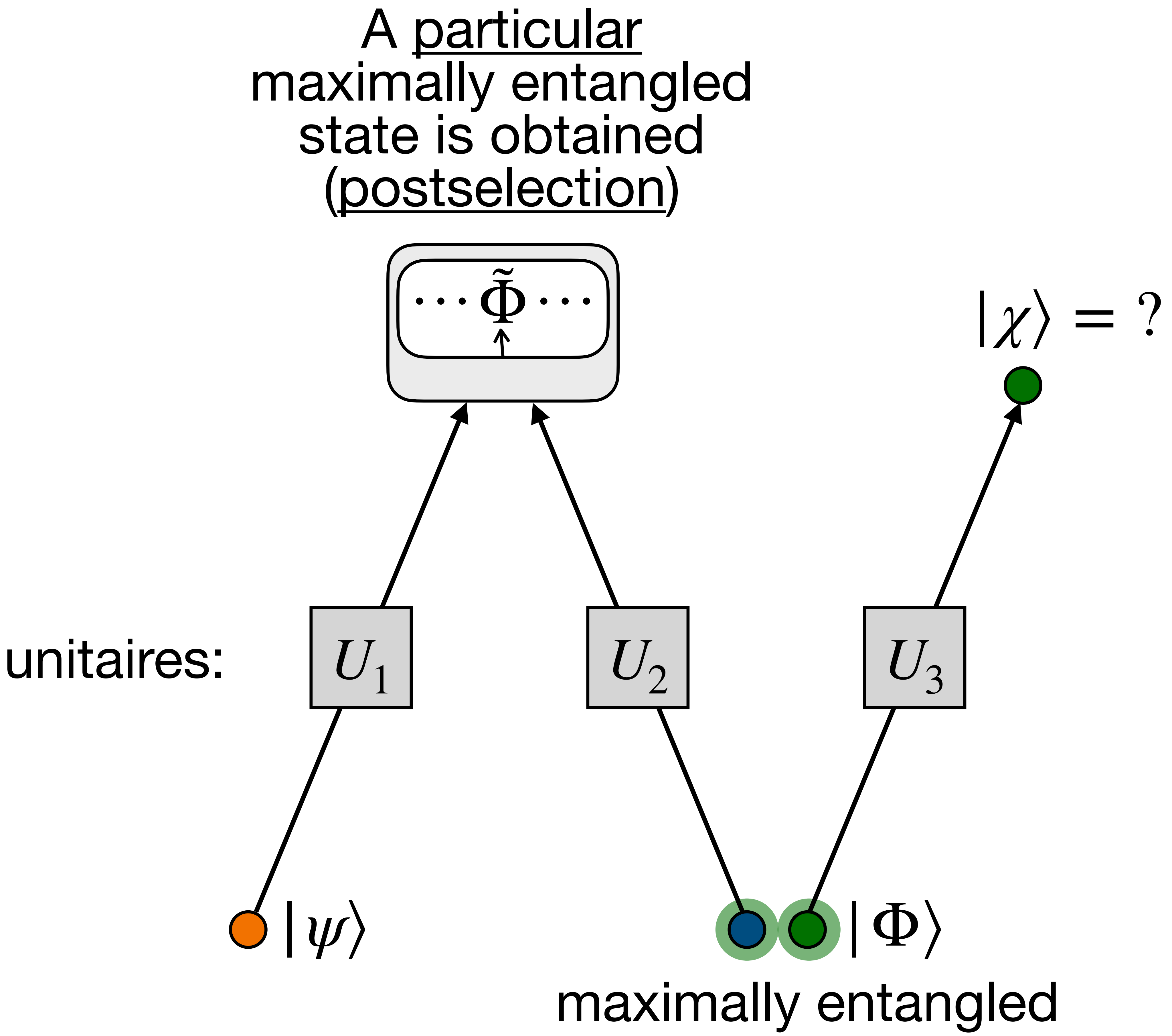
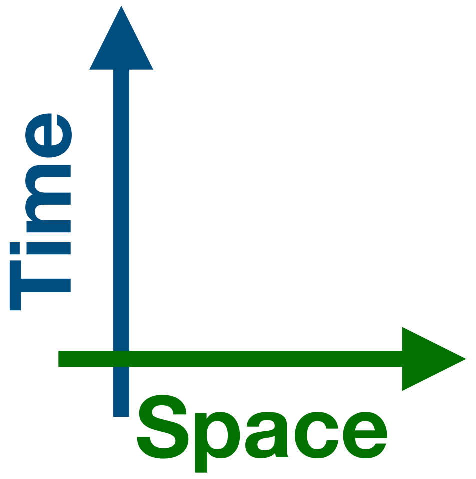
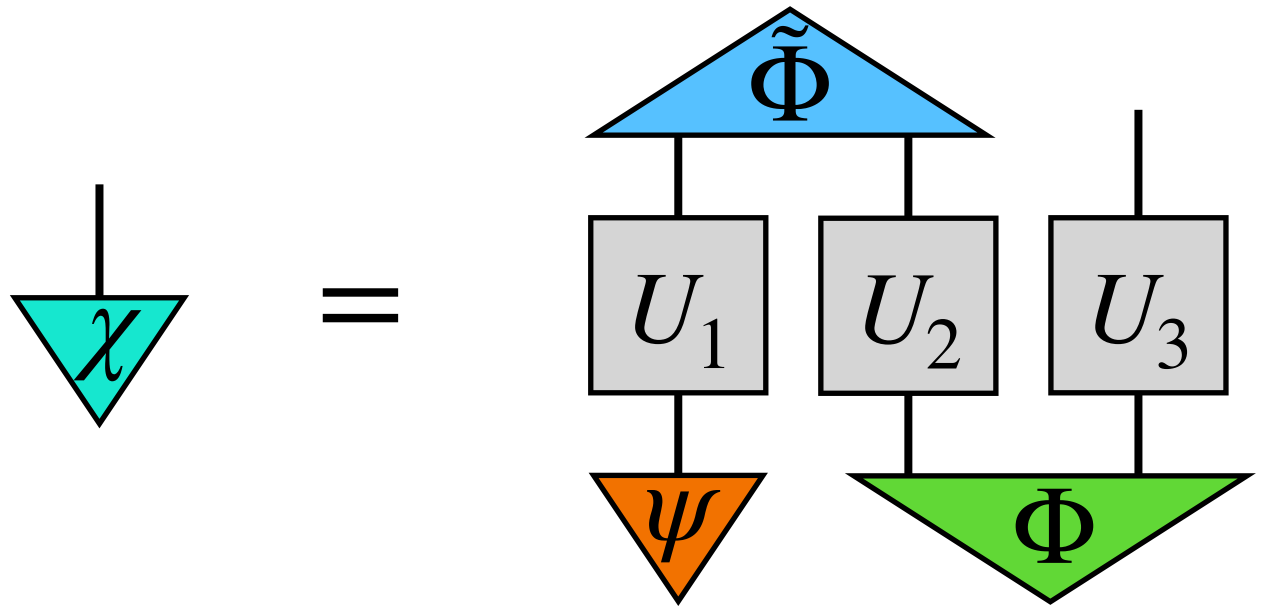




Calculations with tensor networks



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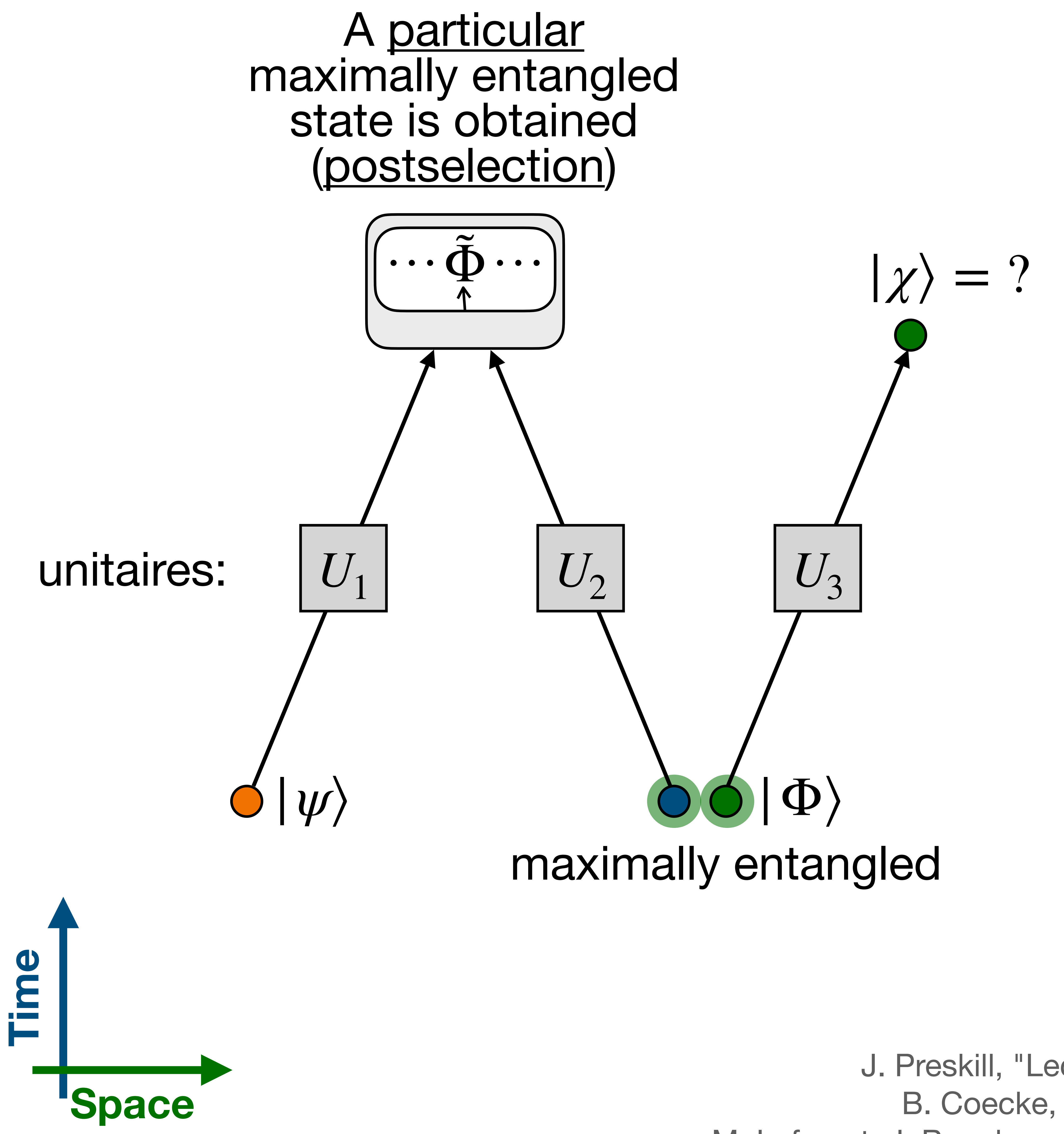

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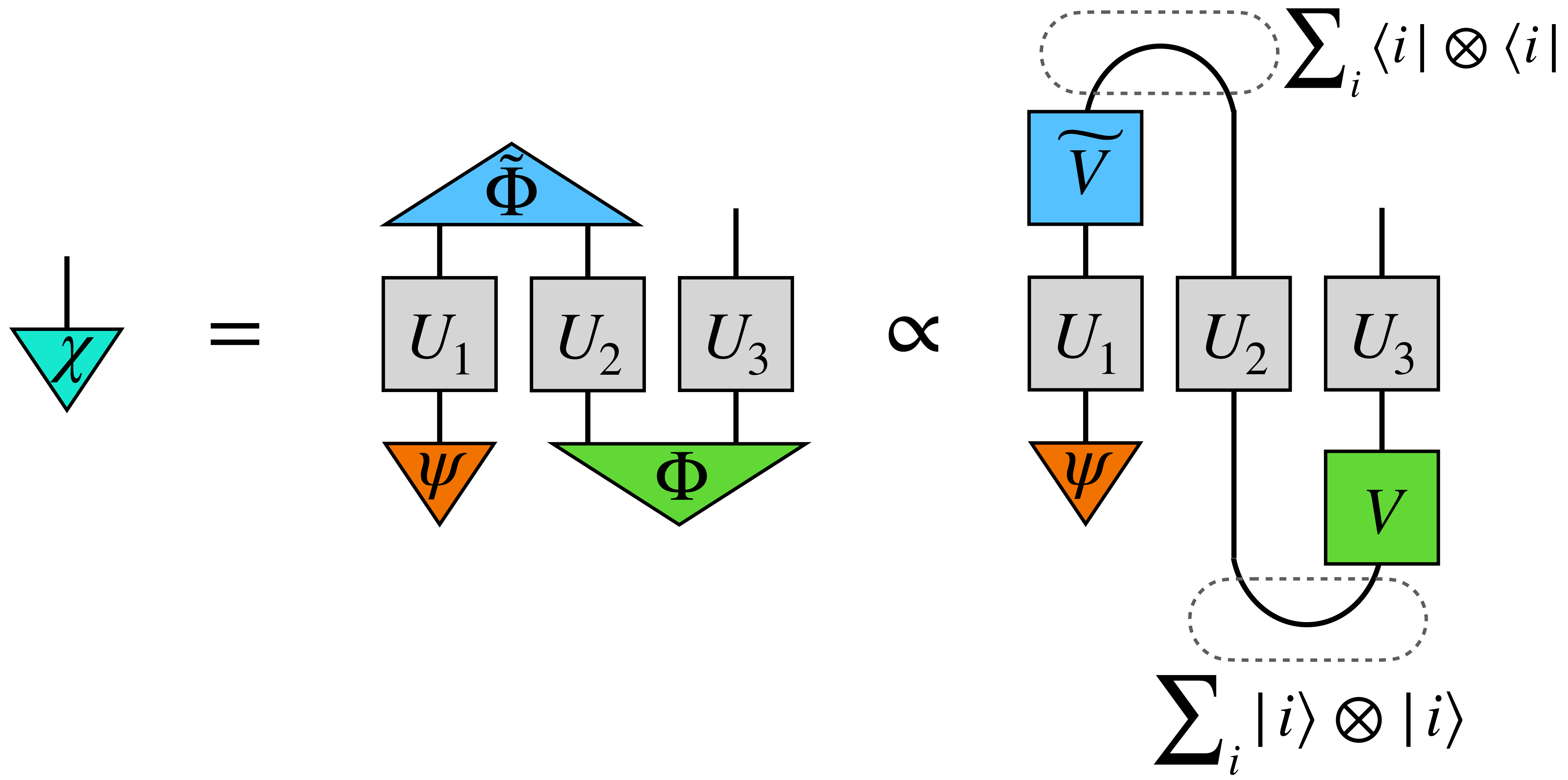

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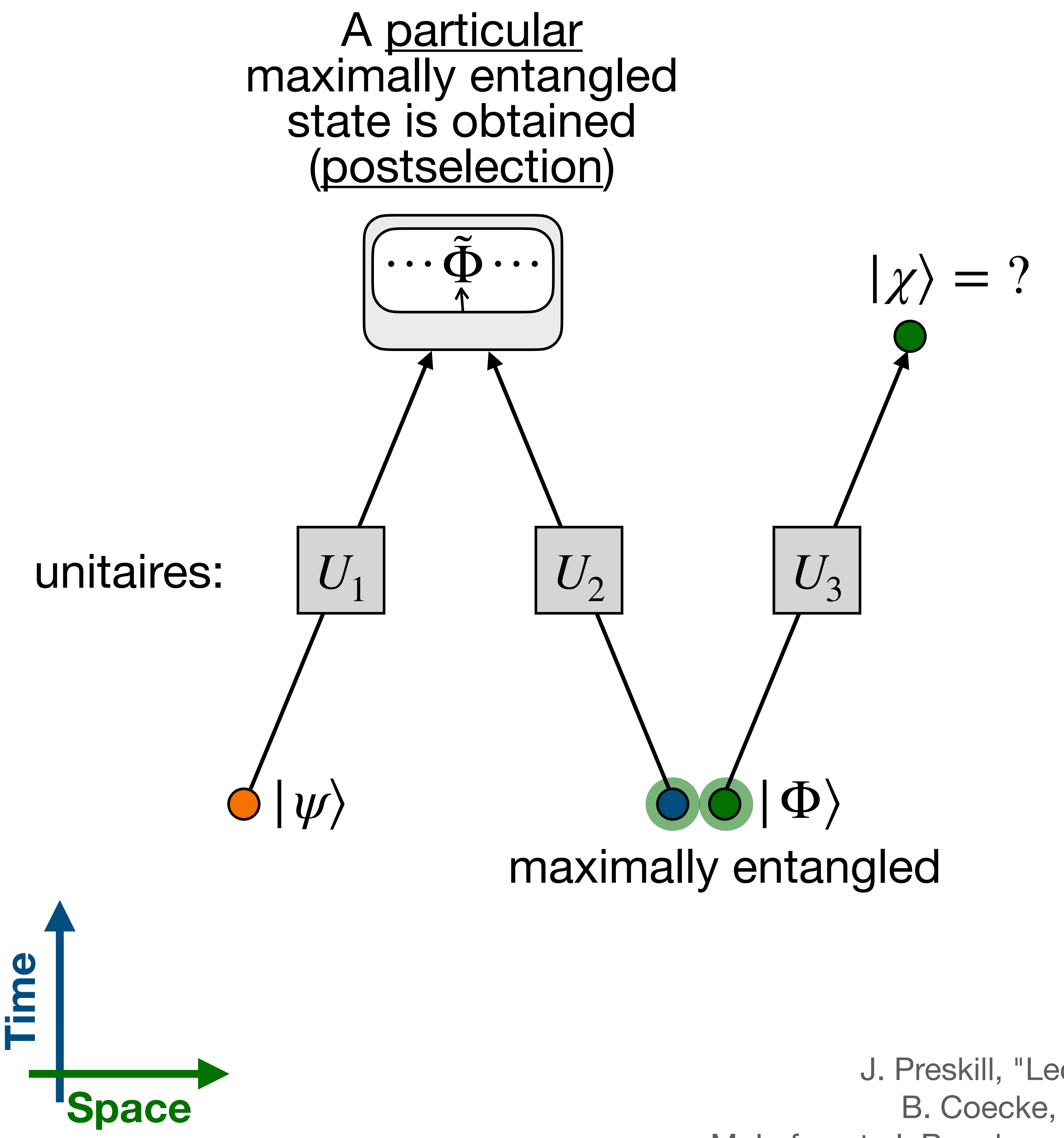
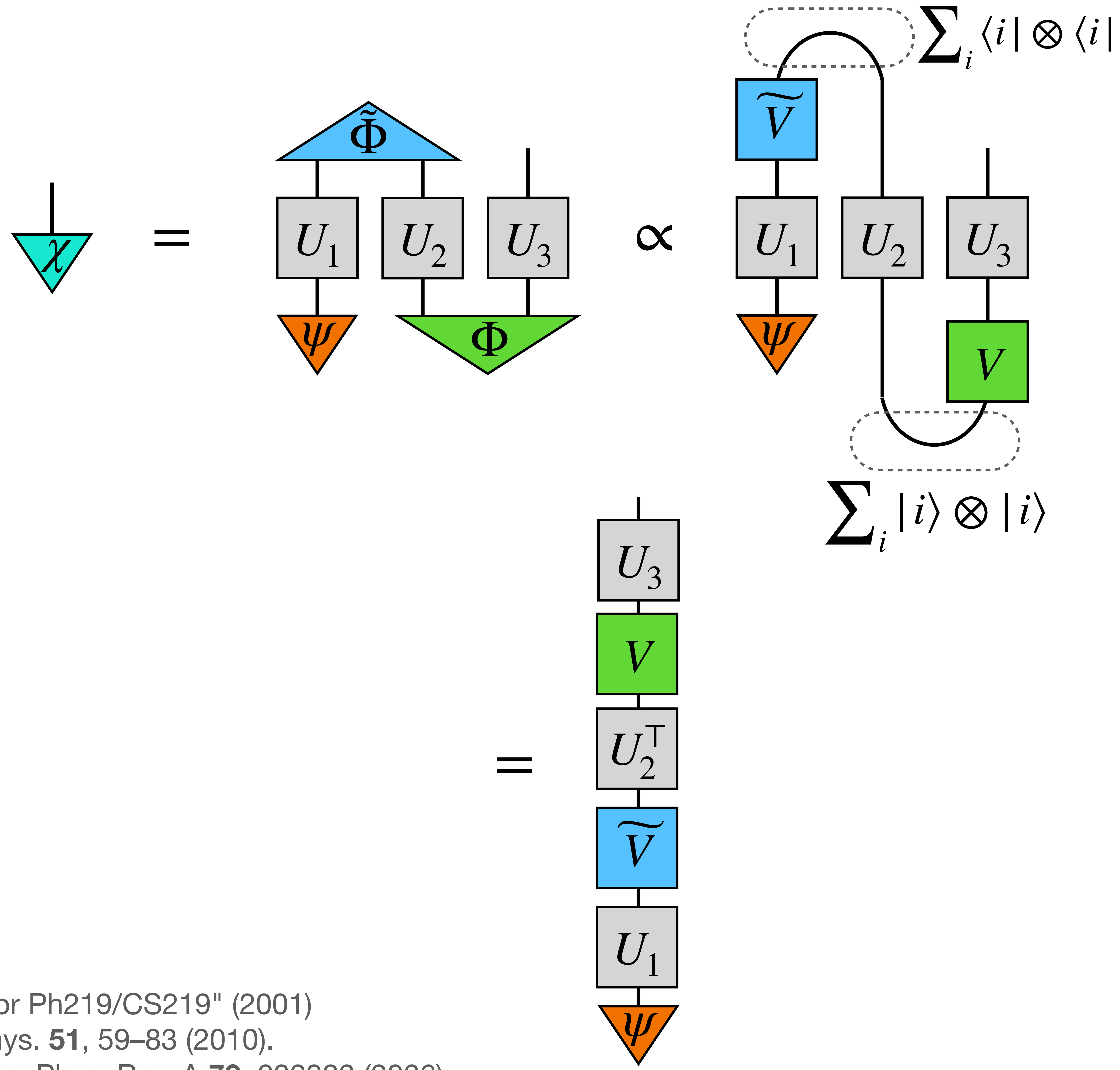


Calculations with tensor networks



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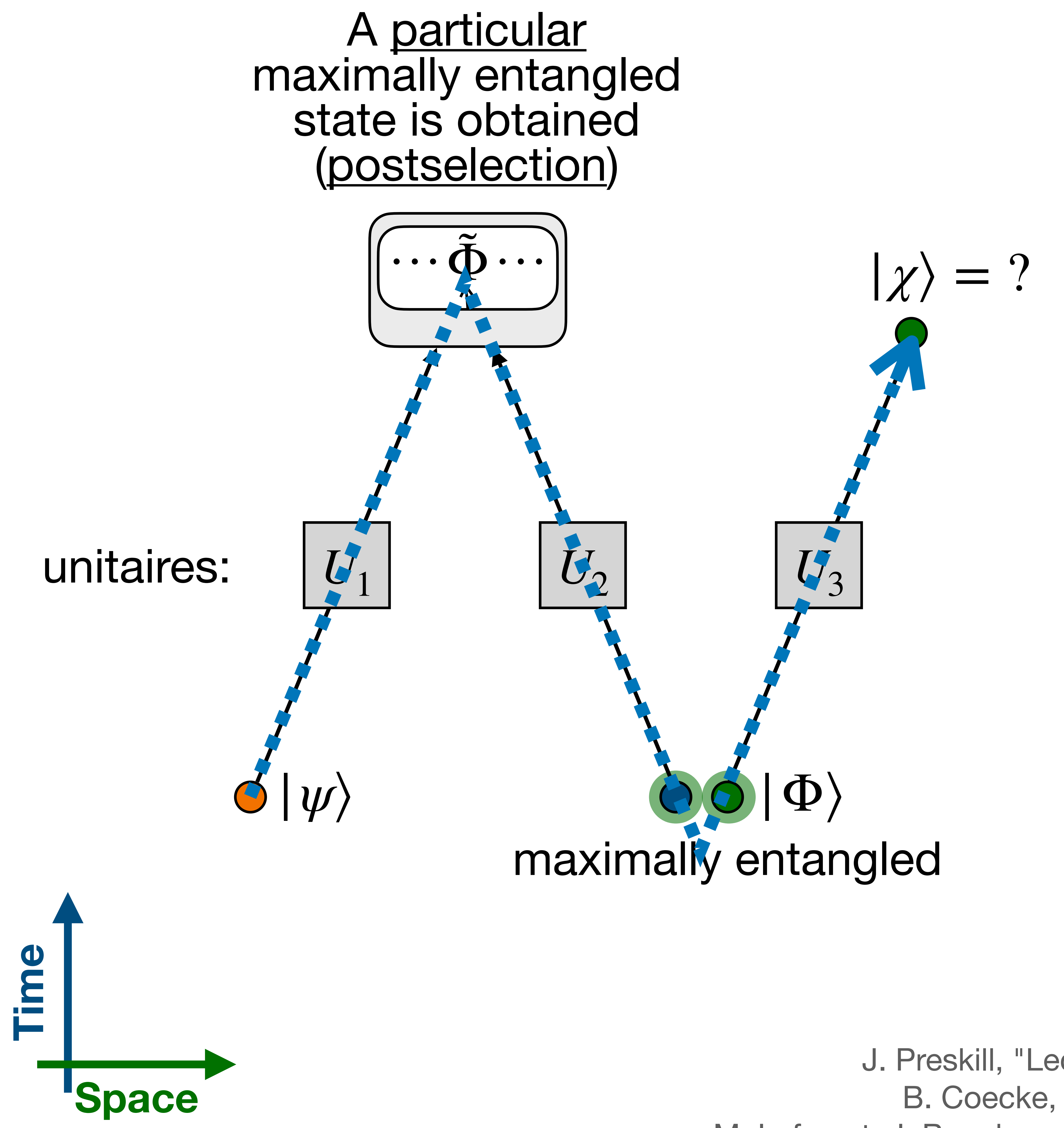
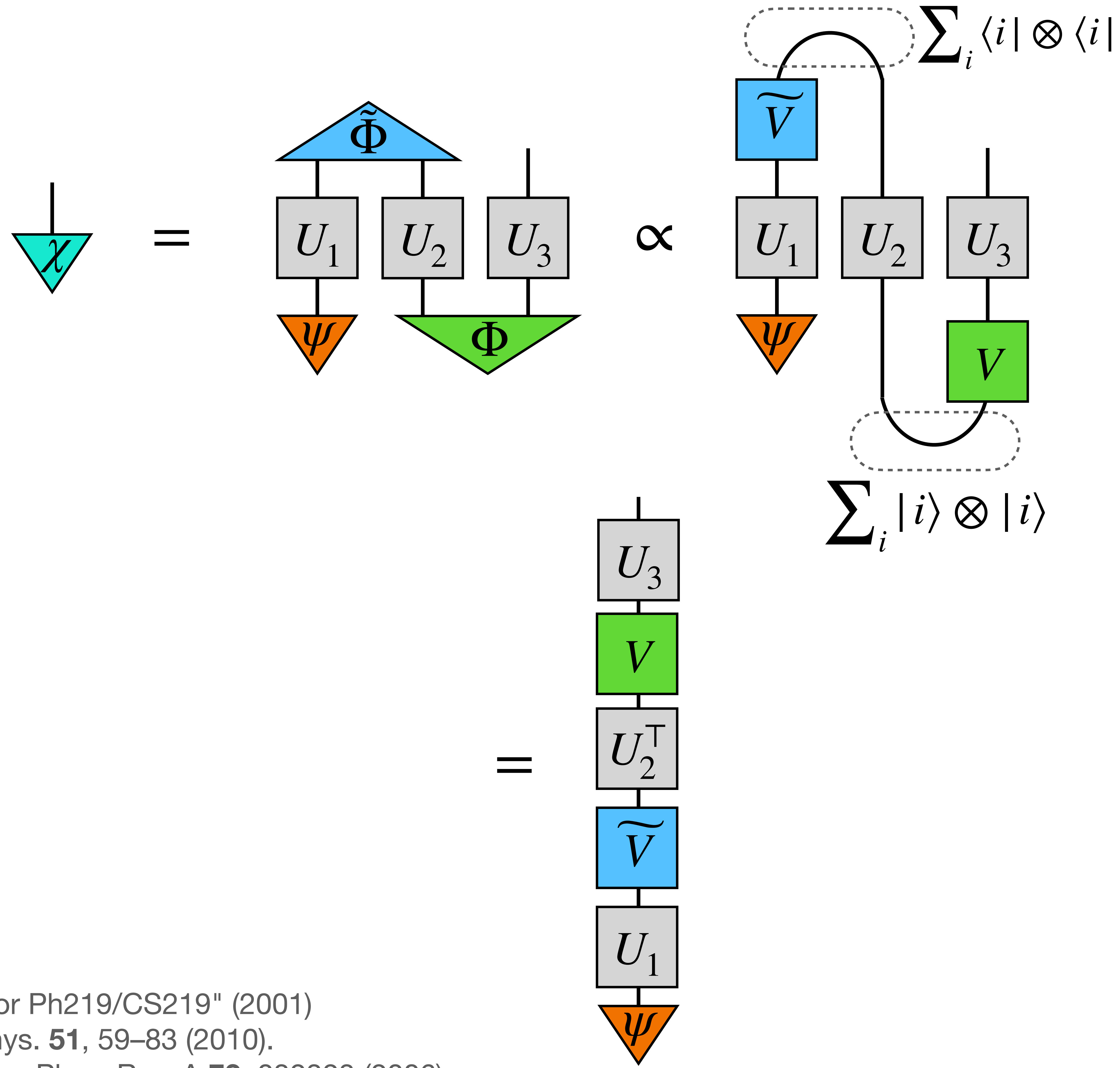


Calculations with tensor networks



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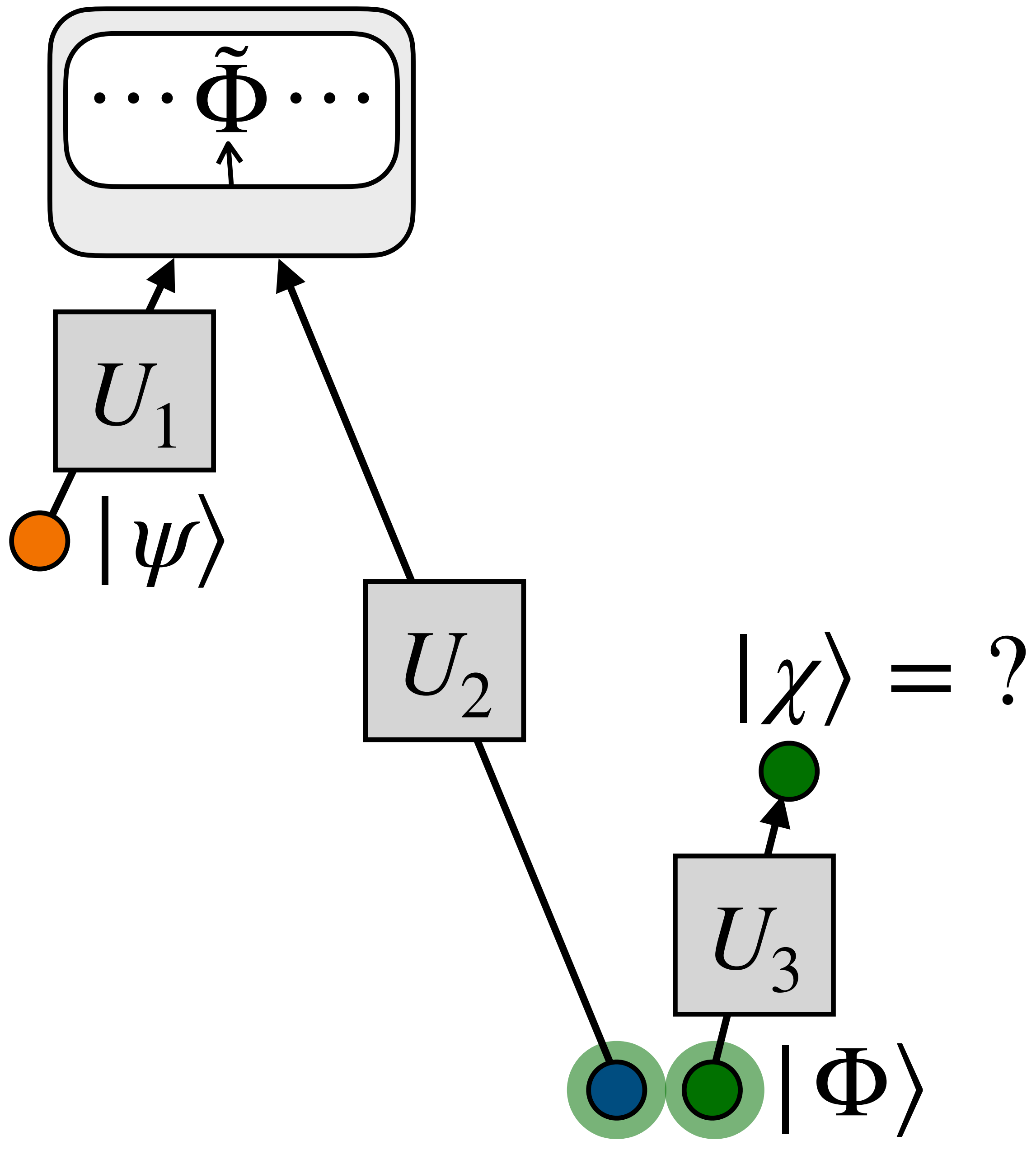
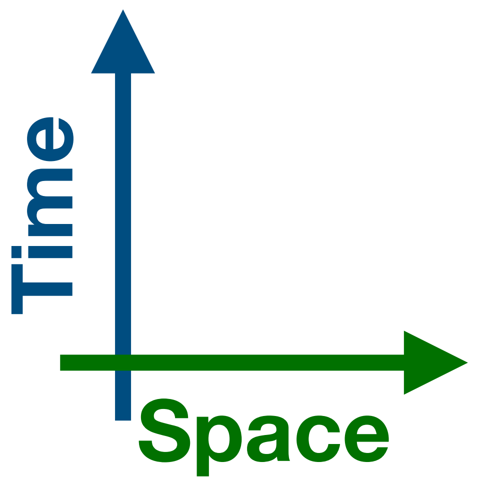
Postselection-induced time travel

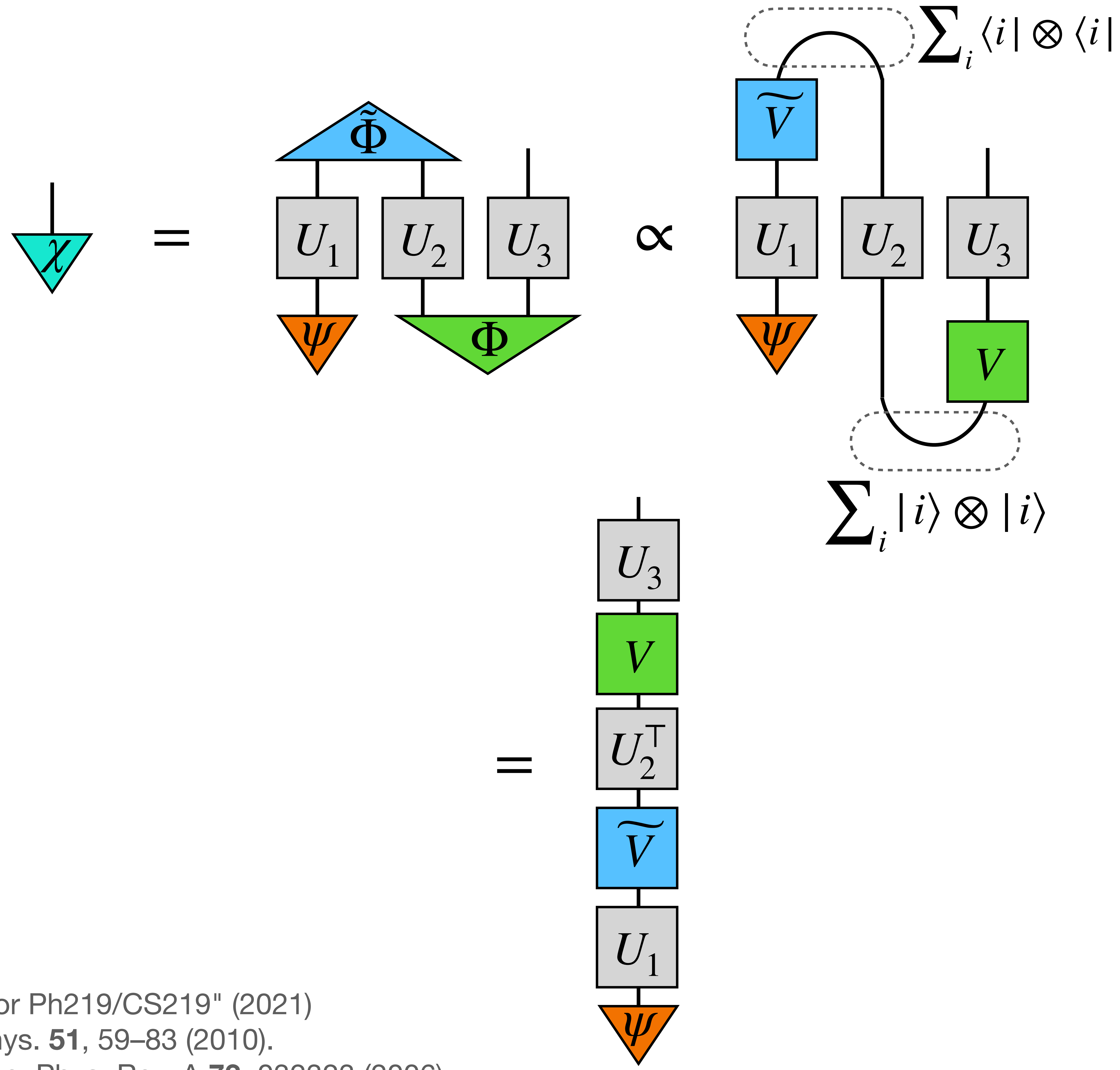
A particular maximally entangled state measured (postselection)



maximally entangled



Calculations with tensor networks



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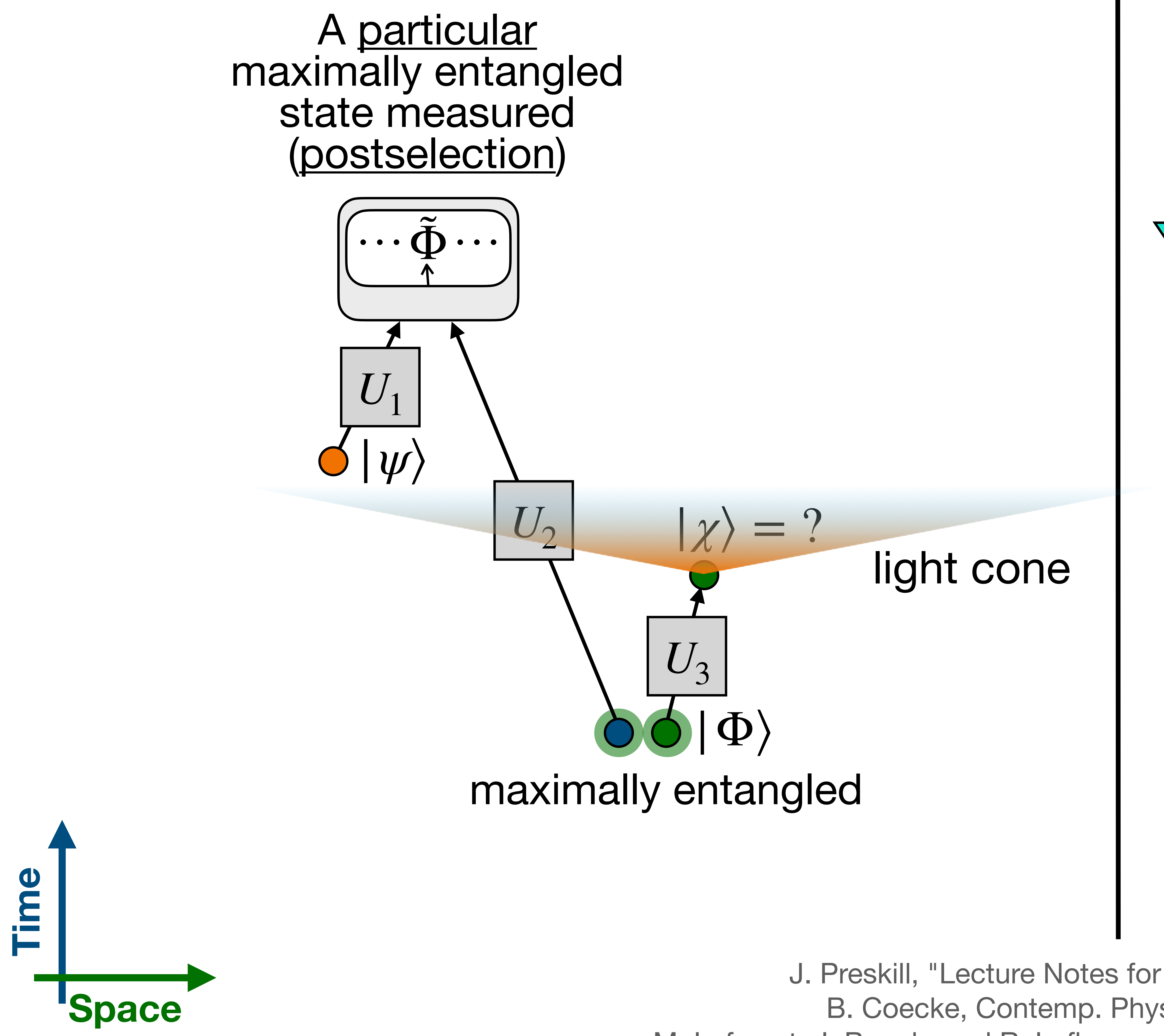
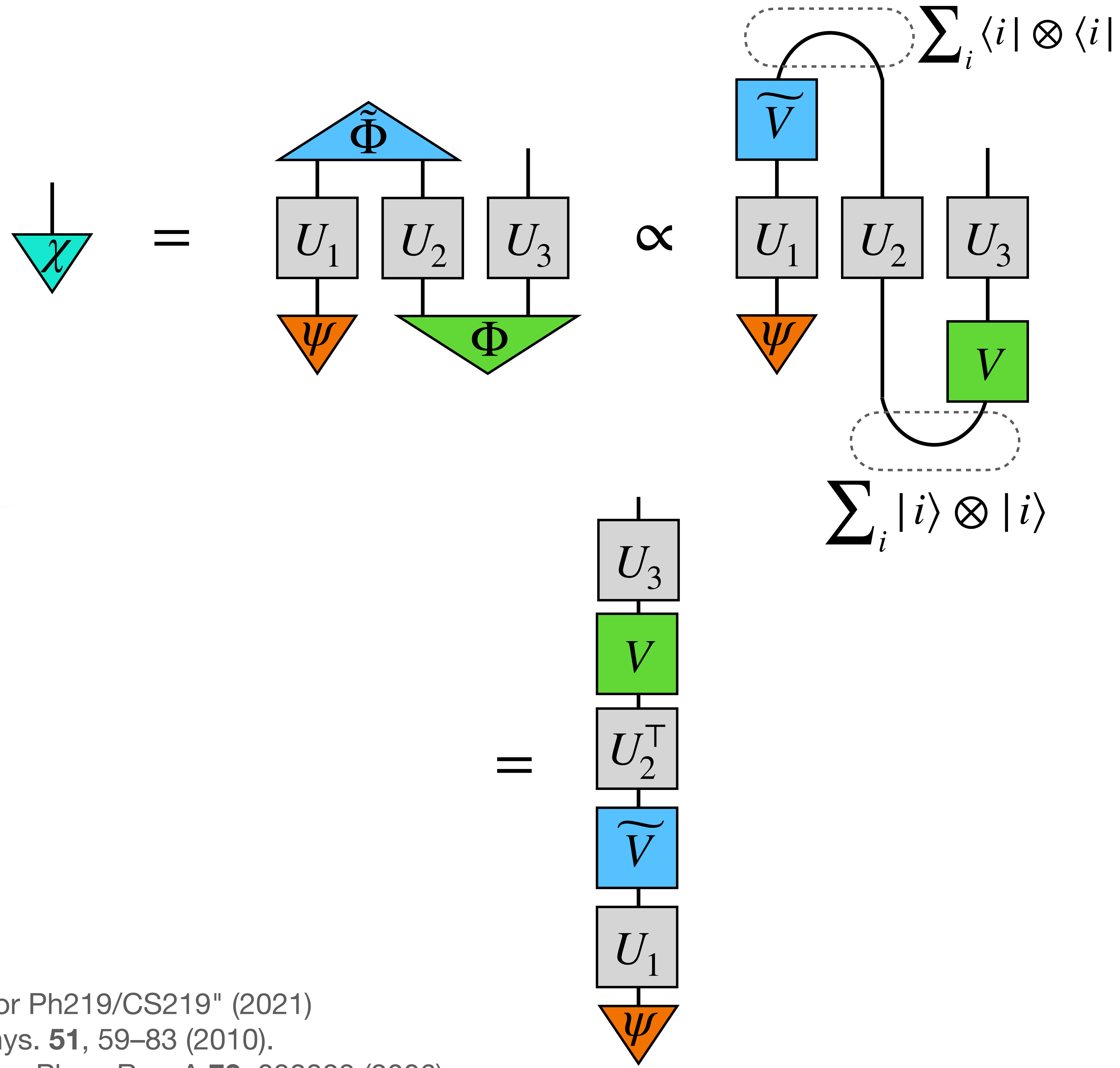


Calculations with tensor networks



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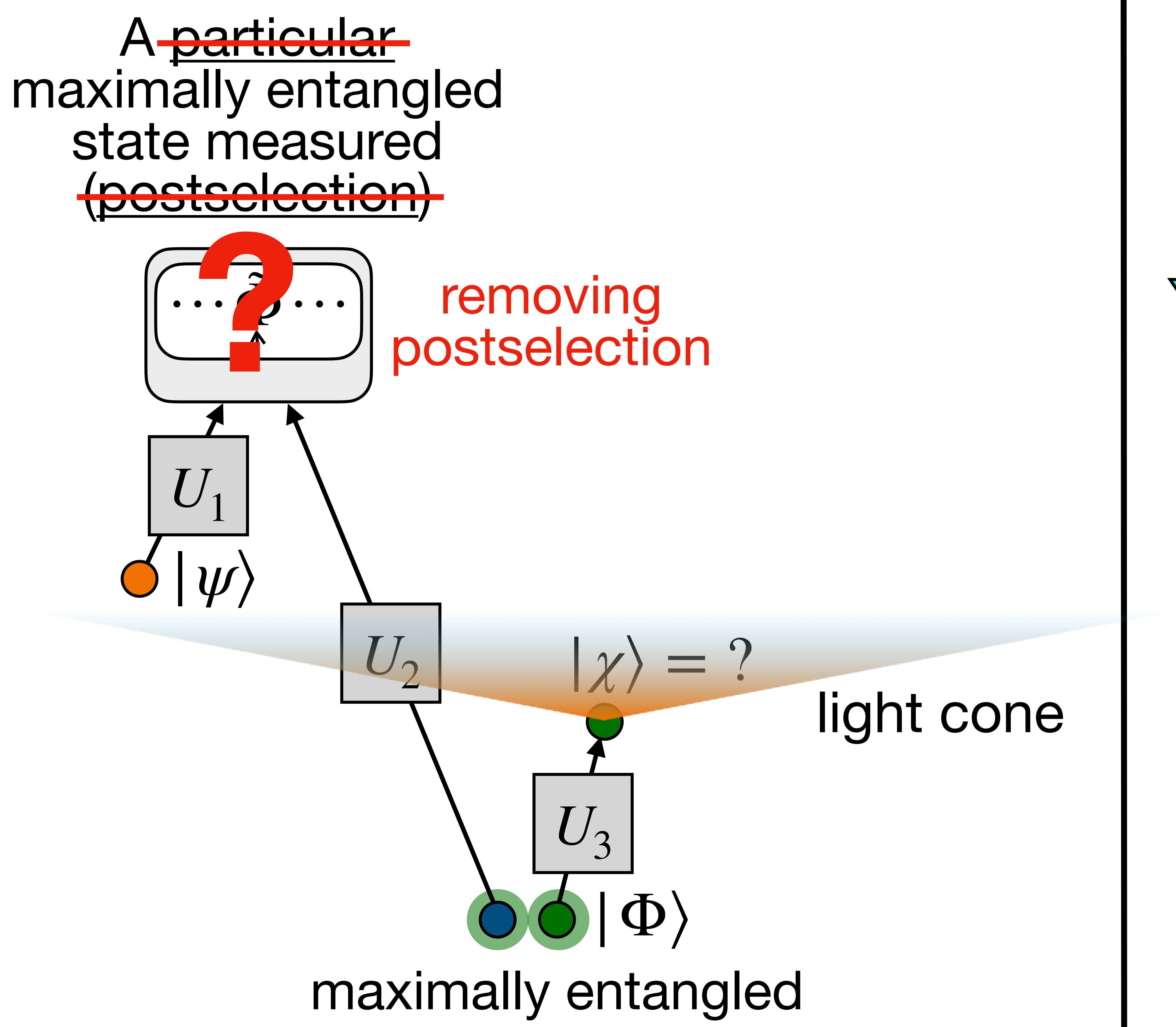

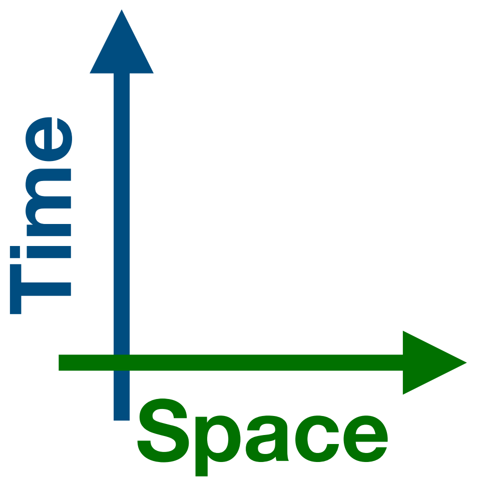

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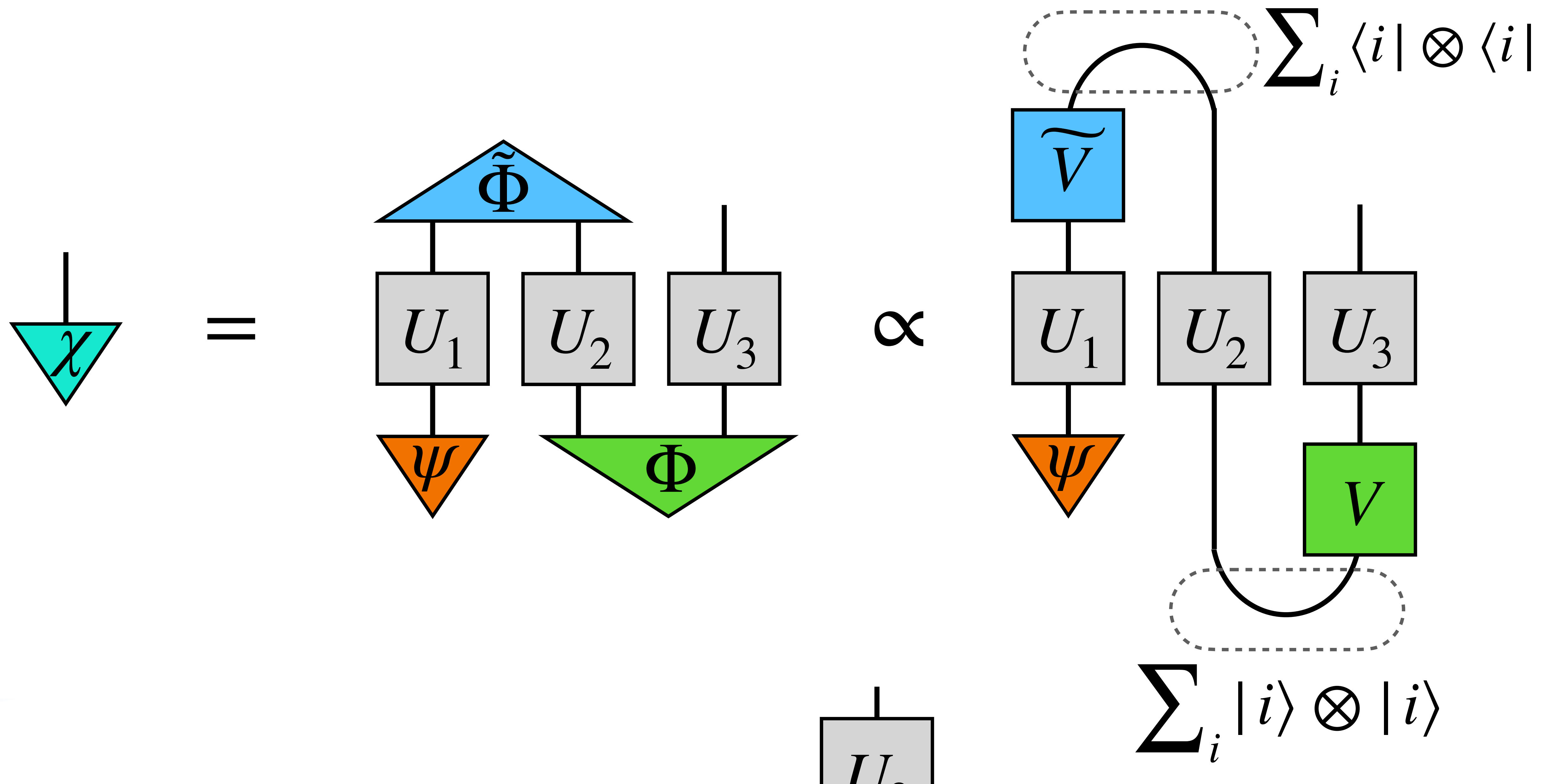

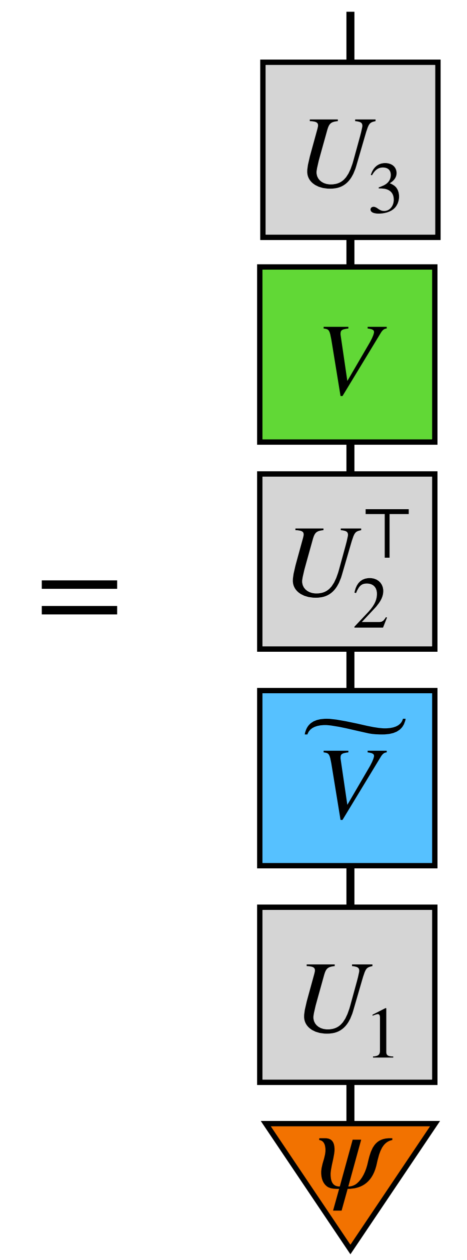

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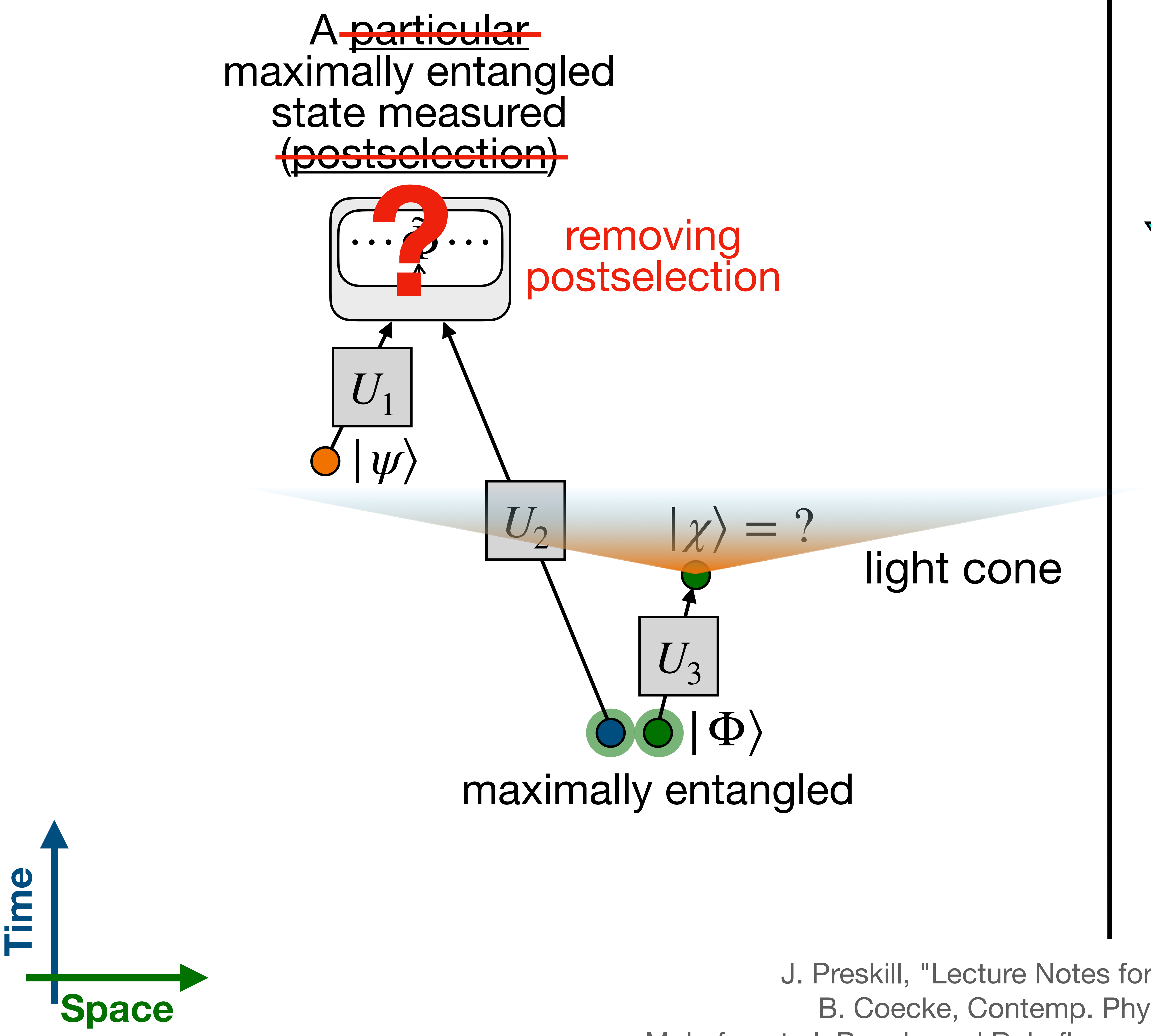
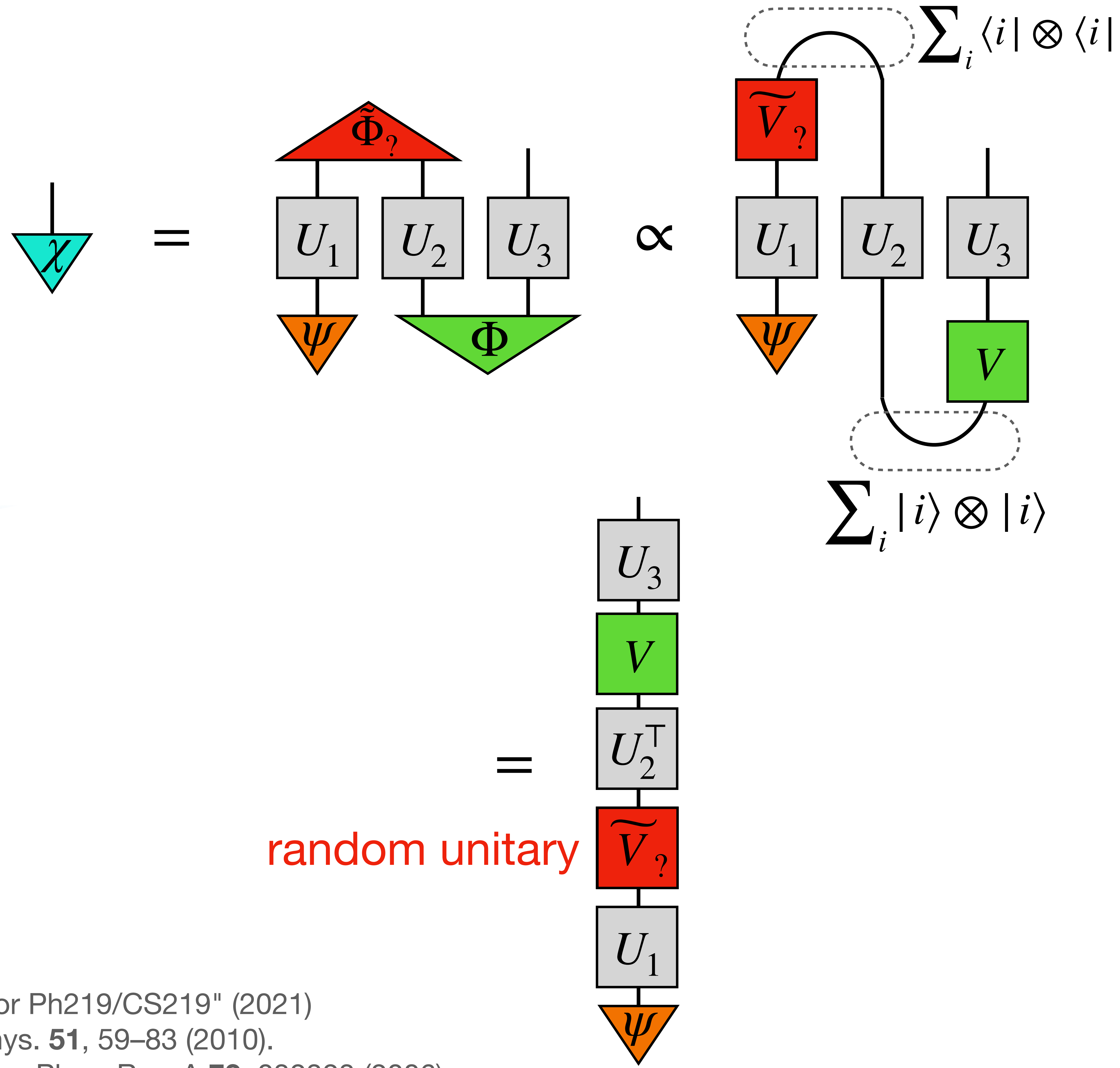


Calculations with tensor networks



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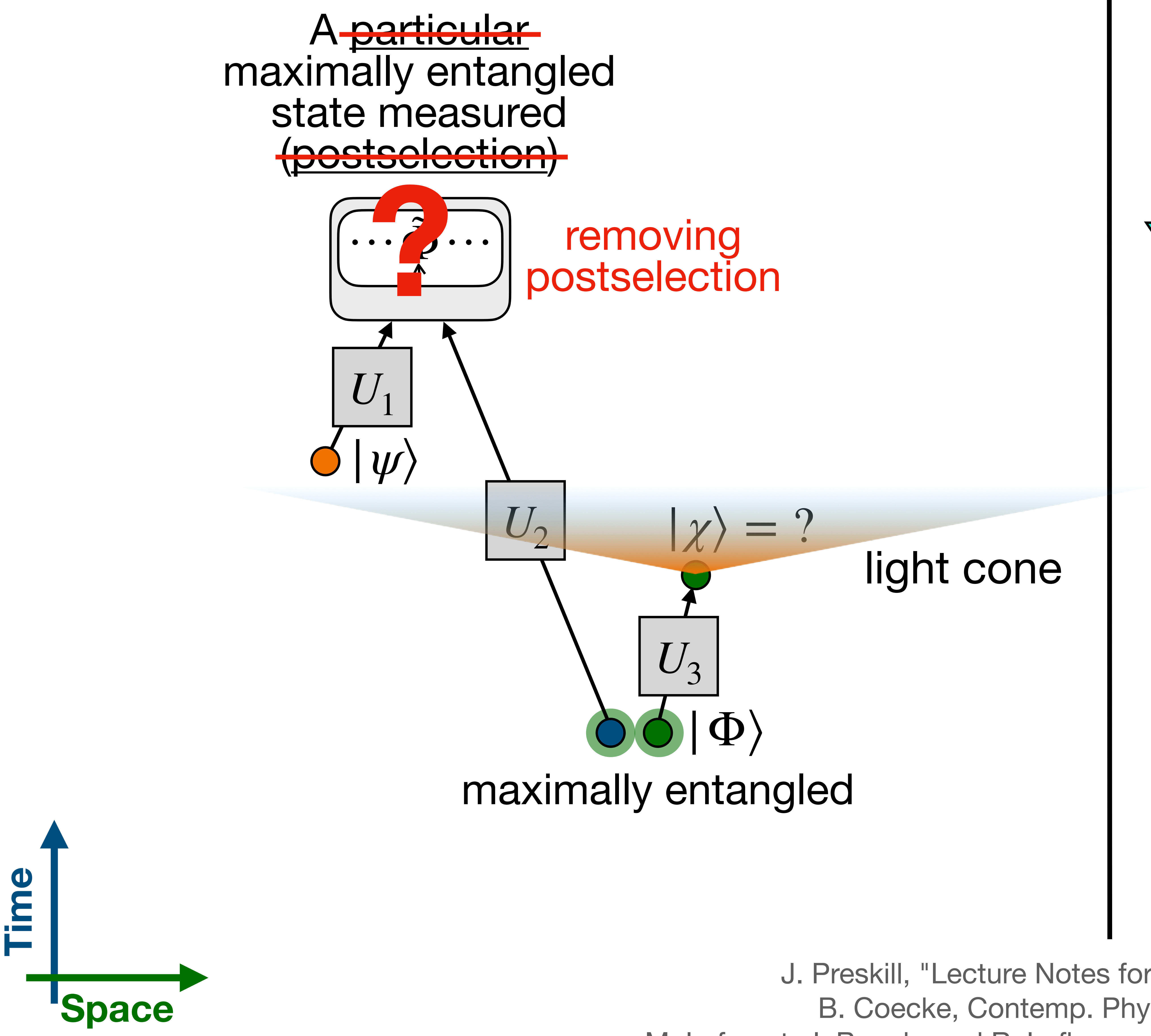

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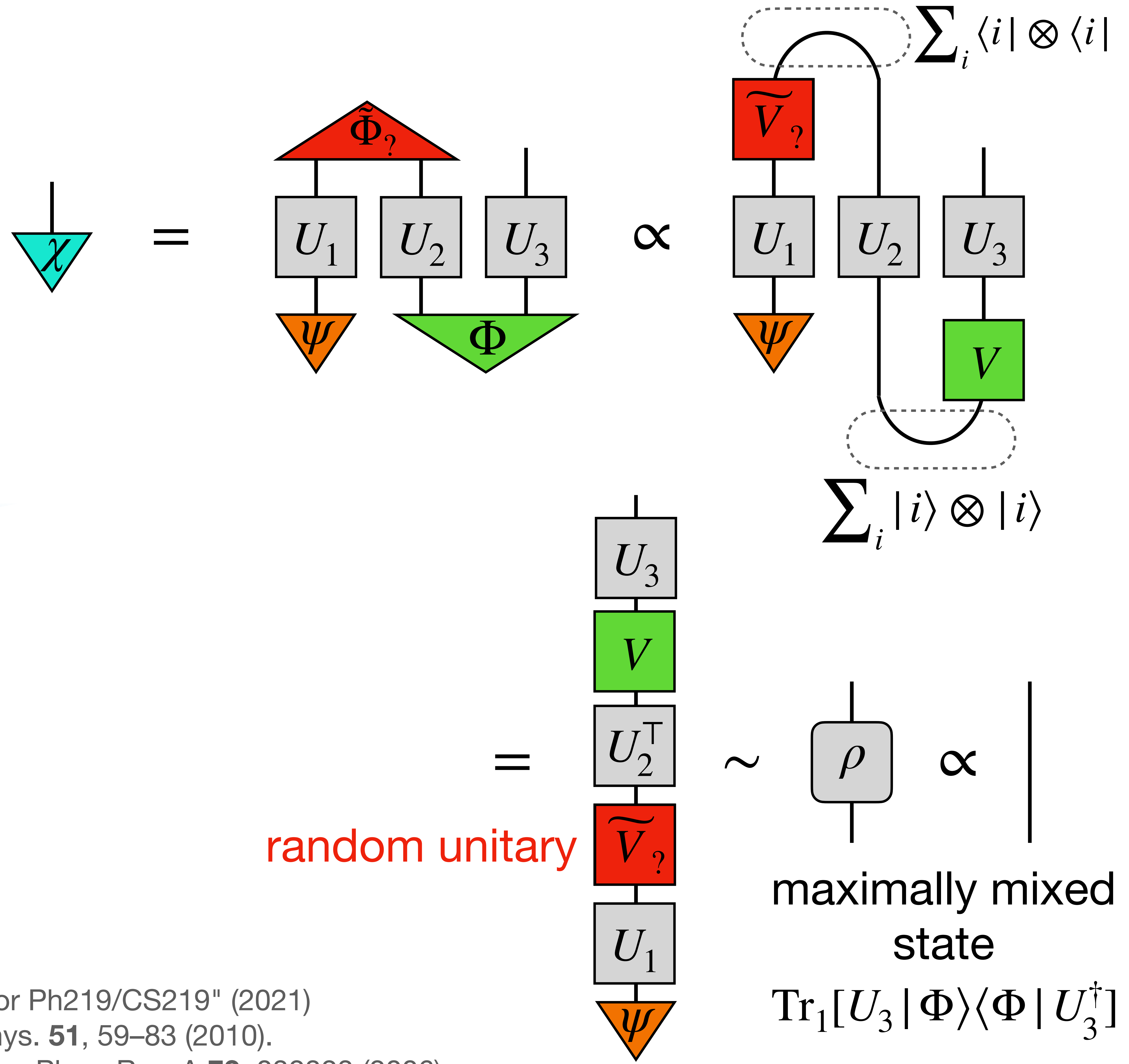

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