Methods for description of open quantum system dynamics based on the thermodynamical approach

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Kawasaki-Ganton Projection Operator

The projection operator acts on an arbitrary matrix A according to the formula

$$\mathcal{P}(t)A = \rho_{rel}(t)\,\mathrm{Tr}\,A + \sum_m \{\mathrm{Tr}(AP_m) - (\mathrm{Tr}\,A)\langle P_m\rangle^t\}\frac{\partial \rho_{rel}(t)}{\partial \langle P_m\rangle^t}, \tag{1}$$

where $\rho_{rel}(t)$ is the quasi-equilibrium density matrix with which we approximate the exact one.

Projected evolution equation

The Kawasaki-Gunton projection operator has the following properties

$$\mathcal{P}(t)\mathcal{P}(t') = \mathcal{P}(t),$$
$$\mathcal{P}(t)\frac{\partial \rho(t)}{\partial t} = \frac{\partial \rho_{rel}(t)}{\partial t}.$$

Using these properties we apply this projector to the Liouville-von Neumann equation

$$\frac{\partial}{\partial t}\rho(t) = \mathcal{L}(t)\rho(t), \tag{2}$$

where $\mathcal{L}(t)$ is an arbitrary generator.
The resulting dynamics equation has the following form

$$\frac{\partial}{\partial t}\mathcal{P}(t)\rho(t) = \mathcal{P}(t)\mathcal{L}(t)\mathcal{P}(t)\rho(t) + \mathcal{P}(t)\mathcal{L}(t)U(t,t_0)\mathcal{Q}(t_0)\rho(t_0) + \mathcal{P}(t)\mathcal{L}(t)\int_{t_0}^{t} U(t,s)\mathcal{Q}(s)\mathcal{L}(s)\mathcal{P}(s)\rho(s)ds, \tag{3}$$

Two-level system in an external field

The evolution equation for a two-level system in an external field has the following form

$$\frac{\partial}{\partial t}\rho(t) = \frac{i\Omega}{2}[\sigma_+ + \sigma_-, \rho(t)] + \gamma_0(N+1)(\sigma_-\rho(t)\sigma_+ - \frac{1}{2}\{\sigma_+\sigma_-, \rho(t)\}) + \gamma_0 N(\sigma_+\rho(t)\sigma_- - \frac{1}{2}\{\sigma_-\sigma_+, \rho(t)\}). \tag{4}$$

The act of the generator $\mathcal{L}(t)$ can be represented in the following form

$$\mathcal{L}: \begin{pmatrix}\mathrm{Tr}\,A\\ \vec{v}\end{pmatrix} \to \begin{pmatrix}0 & \vec{0}^{\,T}\\ \vec{b} & G\end{pmatrix}\begin{pmatrix}\mathrm{Tr}\,A\\ \vec{v}\end{pmatrix}, \tag{5}$$

where $\vec{v}$ are the coordinates of the density matrix in the basis of the Pauli matrices, $G = \begin{pmatrix}-\frac{\gamma}{2} & 0 & 0\\ 0 & -\frac{\gamma}{2} & \Omega\\ 0 & -\Omega & -\gamma\end{pmatrix}$, $\vec{b} = \begin{pmatrix}0\\0\\-\frac{\gamma_0}{2}\end{pmatrix}$.

Jaynes principle and the form of ρ_{rel}

In order to get the ρ_{rel} form, the Jaynes principle is used. The considered entropies are the Gibbs entropy $S = -\mathrm{Tr}\,\rho(t)\ln(\rho(t))$

$$\rho^G_{rel}(t) = \frac{1}{Z_G}\exp\left(-\sum_i F_i(t)P_i\right), \tag{6}$$

where $Z_G = \mathrm{Tr}\exp(-\sum_i F_i(t)P_i)$.

and the Renyi entropy $S^R_q = \frac{1}{1-q}\ln(\mathrm{Tr}\,\rho(t)^q)$.

$$\rho^R_{rel} = \frac{1}{Z_R}\left(1 + \frac{q-1}{q}\sum_m F_m\left(\langle P_m\rangle^t - P_m\right)\right)^{\frac{1}{q-1}}, \tag{7}$$

where $Z_R = \mathrm{Tr}\left(1 + \frac{q-1}{q}\sum_m F_m\left(\langle P_m\rangle^t - P_m\right)\right)^{1/(q-1)}$.

The evolution equation

If we take as the parameter set $\{P_m\}$ the set $\{\sigma_1, \sigma_3\}$, then the evolution equation (3) will take the form

$$\frac{\partial}{\partial t}\begin{pmatrix}\mathrm{Tr}(\rho_{rel})\\ \langle\sigma_1(t)\rangle\\ 0\\ \langle\sigma_3(t)\rangle\end{pmatrix} = \begin{pmatrix}0\\ -\langle\sigma_1(t)\rangle\frac{\gamma}{2}\\ 0\\ -\langle\sigma_3(t)\rangle\gamma - \frac{\gamma_0}{2}\mathrm{Tr}(\rho_{rel})\end{pmatrix} + \int_{t_0}^{t} ds \begin{pmatrix}0\\0\\0\\ -\Omega^2 e^{\frac{1}{2}(s-t)\gamma}\langle\sigma_3(s)\rangle\end{pmatrix}. \tag{8}$$

If we separate the small parameter λ: $\Omega \to \lambda\Omega$ and substitute $\langle\sigma_3(t)\rangle = e^{-\gamma t}\langle\sigma_3'(t)\rangle - \frac{\gamma_0}{2\gamma}$ we get the equation

$$\frac{\partial}{\partial t}\langle\sigma_3'(t)\rangle = -\lambda^2\int_{t_0}^{t}\Omega^2\exp\left[\frac{\gamma}{2}(t-s)\right]\langle\sigma_3'(s)\rangle\,ds + \lambda^2\int_{t_0}^{t}\Omega^2\exp\left[\frac{\gamma}{2}(t+s)\right]\frac{\gamma_0}{2\gamma}ds.$$

Dissipative Wick rotation

As can be seen in the exponent is a positive value. Suppose that $\gamma < 0$, then the kernel of the integral converges to the delta function in the Bogolubov-van Hove scaling $t \to \lambda^{-2}t$. The solution is the function

$$\langle\sigma_3(t)\rangle = e^{(-\frac{\gamma}{\lambda^2} + \frac{\Omega^2}{\gamma})t}\left(\langle\sigma_3(0)\rangle + \frac{\gamma_0}{2\gamma}\right) - \frac{\gamma_0}{2\gamma}$$

In the result, we get the evolution of the averages

$$\begin{cases}\mathrm{Tr}(\rho_{rel})(t) = 1,\\ \langle\sigma_1(t)\rangle = \langle\sigma_1(0)\rangle\, e^{-\frac{\gamma}{2}t},\\ \langle\sigma_3(t)\rangle = e^{(-\frac{\gamma}{\lambda^2} + \frac{2\Omega^2}{\gamma})t}\langle\sigma_3(0)\rangle - \frac{\gamma_0}{2\gamma}\left(1 - e^{(-\frac{\gamma}{\lambda^2} + \frac{2\Omega^2}{\gamma})t}\right).\end{cases} \tag{9}$$

Conclusion

In this work, quasi-equilibrium density matrices have been obtained (6)-(7) based on the principle of maximum entropy. A two-level system in an external field was considered with the usage of projective operators.