On relation between exact and Markovian correlation functions for unitary dynamics with random Hamiltonian

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Introduction. In the theory of open quantum systems the Markovian approximation is very widespread. Usually it assumes the Gorini-Kossakowski-Sudarshan-Lindblad equation for density matrix dynamics and some specific formulae for correlation functions in terms of dynamical map for this equation [1]. These formulae are usually called regression formulae. We will call such a correlation functions the Markovian correlation functions, and we give more explicit definition of them below. Here we construct an explicit and simple example of a model, for which dynamics of the density matrix is defined by a dynamical semigroup with the Gorini-Kossakowski-Sudarshan-Lindblad generator, but exact correlation functions do not coincide with the Markovian correlation functions.

Model and definitions. We consider a unitary evolution with a random time-dependent Hamiltonian

$$\tilde{H}(t) = \xi \frac{1}{2\sqrt{t}} H,$$

where H is a fixed n-dimensional Hermitian matrix, ξ is a real random variable with the standard normal distribution.

Let $U(t)$ be a solution of the equation

$$\frac{d}{dt} U(t) = -i\tilde{H}(t) U(t)$$

for $t > 0$ such that $U(+0) = I$.

Definition 1. Suppose that A and B are $n \times n$ matrices and $t \geqslant s \geqslant 0$. Then the exact correlation function $\langle A(t)B(s)\rangle$ is defined by the formula

$$\langle A(t)B(s)\rangle \equiv \mathbb{E}\,\mathrm{Tr}\, A\mathcal{U}_t(\mathcal{U}_s)^{-1}(B(\mathcal{U}_s\rho)),$$

where $\mathcal{U}_t$ is a superoperator denfined as $\mathcal{U}_t(X) = U(t)X(U(t))^\dagger$ for an arbitrary $n \times n$ matrix X.

Definition 2. Let us define

$$\Phi_t \equiv \mathbb{E}\mathcal{U}_t.$$

Suppose that A, B_1, B_2 are $n \times n$ matrices and $t \geqslant s \geqslant 0$. Then Markovian correlation function $\langle B_2(s)A(t)B_1(s)\rangle$ is defined by the formula

$$\langle B_2(s)A(t)B_1(s)\rangle_M \equiv \mathrm{Tr}\, A\Phi_t(\Phi_s)^{-1}(B_1(\Phi_s\rho)B_2),$$

in particular for $B_2 = I$, $B_2 = B$

$$\langle A(t)B(s)\rangle_M \equiv \mathrm{Tr}\, A\Phi_t(\Phi_s)^{-1}(B(\Phi_s\rho)).$$

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Theorem 1. The operator-valued function Φ_t has the explicit form

$$\Phi_t = e^{\mathcal{L}t},$$

where $\mathcal{L}$ has the Gorini-Kossakowski-Sudarshan-Lindblad form, namely,

$$\mathcal{L}(\rho) = H\rho H - \frac{1}{2}H^2\rho - \frac{1}{2}\rho H^2.$$

The semigroup Φ_t defines evolution of the density matrix as $\rho(t) = \Phi_t(\rho(0))$.

Main result. The main result of the talk consists in the explicit formula, which expands the exact correlation functions in terms of Markovian correlation functions.

Theorem 2. Let A, B be $n \times n$ matrices, $t \geqslant s \geqslant 0$, then

$$\langle A(t)B(s)\rangle = \langle A(t)B(s)\rangle_M + \langle(\Phi_{2(\sqrt{ts}-s)}(A) - A)(t)B(s)\rangle_M$$

$$\sum_{k=1}^{\infty} \frac{(\sqrt{ts}-t))^k}{k!} \sum_{m_1,m_2=0}^{k} \binom{k}{m_1}\binom{k}{m_2} \langle (H^{k-m_1})(s)(H^{k-m_2}\Phi_{2(\sqrt{ts}-s)}(A)H^{m_2})(t)(H^{m_1}B)(s)\rangle_M.$$

Thus, we have obtained non-trivial corrections to Markovian correlation functions for this simple model. These corrections are also repesented as sums of Markovian correlation functions, hence the exact correlation functions are still defined by Markovian correlation functions (but not only of the same operators), but there relation is much more complex than for the Markovian case.

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References

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