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Brane world scenario:
exact solutions with an event horizon

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Introduction

- What is the problem?
  - Black hole creation may be a consequence of strong gravity at short distances attainable in high energy experiments if our space is realized on a hypersurface – three-brane in a multidimensional space-time.
  - Correct (or better exact) description of black hole geometry when the matter universe is strictly situated on the three-dimensional brane but gravity propagates into extra space dimensions is needed.
  - Appearance of delta-like singularities in matter distribution hidden under horizon for static locally stable black holes is a problem: in fact the matter (quarks and gluons) must be smoothly distributed.
  - Therefore one expects that rather black stars are created with matter both inside and outside an event horizon in a finite brane-surface volume.

Techniques

- Stress-energy tensor structure.
  - The Einstein equations in the bulk read,

$$ {}^{(5)}G_{AB} = \kappa_5 T_{AB}, \quad T_{AB} = \delta^{\mu}_{A}\delta^{\nu}_{B}\tau_{\mu\nu}\delta(z) $$

    with $\kappa_5 = 1/M_*^3$ and M_* is a Planck scale in five dimensions.
  - In order to define $\tau_{\mu\nu}$ let us introduce extrinsic curvature tensor $K_{\mu\nu}$.

$$ K_{\mu\nu} = -\frac{1}{2}\frac{\partial g_{\mu\nu}}{\partial z} \textit{ valid in the Gaussian normal coordinates} (i.e. g_{zz} = -1, g_{\mu z} = 0) \textit{ only!} $$

  - $\tau_{\mu\nu}$ is defined by the Israel-Lanczos junction conditions,

$$ [g_{\mu\nu}K - K_{\mu\nu}]^{+0}_{-0} = \kappa_5 \tau_{\mu\nu}. $$

  - $K^{+0}_{\mu\nu}$, and $K_{\mu\nu -0}$ are the extrinsic curvature tensors of hypersurfaces $z = +0$ and $z = -0$ correspondingly.

Techniques

- General construction.
  - To build a brane we search for a metric $g_{AB}(x, y)$ which is a bulk vacuum solution of the Einstein equations with event horizon.
  - Suppose that:
    - a) the induced metric $g_{\mu\nu}(x, y)$ is asymptotically flat for any hypersurface $y = const$ and inherits the horizon;
    - b) in the chosen coordinate systems $g_{5B}(x, y) = 0$ and the remaining metric components provide orbifold geometry $g_{AB}(x, y) = g_{AB}(x, -y)$;
    - c) Coordinate y is spacelike i.e. $g_{yy} \equiv g_{55} < 0$.
  - In order to generate a brane filled by matter we proceed the following transformation,

$$g_{AB}(x, y) \Longrightarrow g_{AB}(x, |z| + a).$$

  - Brane: $z = 0$.

Construction of the solution.

- Preparing of the suitable coordinate system.
  - We start from the metric describing a five-dimensional static neutral black hole in Schwarzschild coordinates $\{t, r, \theta_1, \theta_2, \theta\}$,

$$g_{AB} = \mathrm{diag}\left[U(r),\, -\frac{1}{U(r)},\, -r^2\cos^2\theta,\, -r^2\cos^2\theta\cos^2\theta_1,\, -r^2\right],$$

    where $U(r) = 1 - \frac{M}{r^2}$, M is related to the Schwarzschild-Tangherlini radius $M \equiv r^2_{Sch-T}$.
  - Let's define the Gaussian normal coordinates in respect to hypersurface with space-like normal vector $\theta = 0$.
  - The vector orthonormal to this hypersurface $n^A = [0, 0, 0, 0, 1/r]$.
  - The required change of coordinates acts on two variables $r = r(\rho, y)$, $\theta = \theta(\rho, y)$.

Construction of the solution.

- Our coordinate transformation has the following form.
  - $$|y| = \int_\rho^r \frac{\mathrm{sign}((r-\rho))\; x^2}{\sqrt{(x^2-M)\,(x^2-\rho^2)}}dx, \quad \theta = \int_\rho^r \frac{\mathrm{sign}((r-\rho)y)}{\sqrt{(x^2-M)\,(x^2-\rho^2)}}dx.$$
  - We have: inside the horizon $r < \rho < \sqrt{M}$ and outside the horizon $\sqrt{M} < \rho < r$.
  - The metric in new coordinates $\{t, \rho, \theta_1, \theta_2, y\}$, reads,

    $$g_{AB}(x,y) = \mathrm{diag}\left[U(r),\, -\frac{r^2 {r_\rho}^2}{\rho^2 U(r)},\, -r^2\cos^2\theta,\, -r^2\cos^2\theta\cos^2\theta_1,\, -1\right],$$

    where $r = r(\rho, y)$, $\theta = \theta(\rho, y)$.
  - The final answer for black star metric and $\tau_{\mu\nu}$ has the following form:

$$g_{AB}^{final}(x,z) = g_{AB}(x,y)|_{y=|z|+a},\; \kappa_5\tau_{\mu\nu}(x,a) = \left(\frac{\partial g_{\mu\nu}}{\partial y} - g_{\mu\nu}\frac{g^{\lambda\delta}\partial g_{\lambda\delta}}{\partial y}\right)\Big|_{y=a}$$

Construction of the solution.

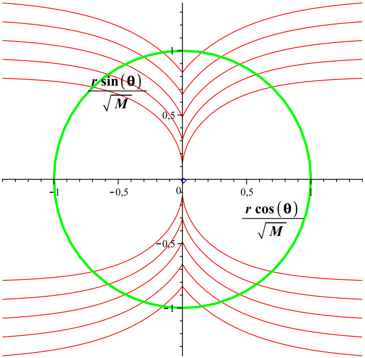

Figure 1: Pairs of hypersurfaces symmetric in respect to the horizontal axis to be glued into a brane are shown by red curves. The circle of horizon in dim = 5 is depicted by green line. $g_{AB}(x,y) \Longrightarrow g_{AB}(x,|z|+a)$, $a = 0.69868\sqrt{M} \div \pi\sqrt{M}/2$

Construction of the solution.

- Some technical remarks.
  - Note, that situation on the horizon is O.K. All quantities that must be continues are continues. for example for scalar curvature on the brane ${}^{(4)}R$ we have the following limit:

$$ {}^{(4)}R(a) = -2\,\frac{B+1-\cos^2 a - 4\,|\sin a|\sqrt{1+B}\sqrt{B}\cos a}{(1+B)\cos^2 a}, $$

$$ B(a) \equiv \lim_{\rho\to\sqrt{M}} \frac{r(\rho,a)-\rho}{\rho-\sqrt{M}} = \frac{1}{2}\left(\cosh\left(\frac{2a}{\sqrt{M}}\right)-1\right). $$

  - In this construction space-time is asymptotically flat and the following asymptotic takes place:

$$ {}^{(4)}R = \frac{4M^2a^2}{\rho^8}\left(1+O\left(\frac{a^2}{\rho^2}\right)\right). $$

Matter distribution.

- Here and below we use new radial coordinate $R(\rho, a) \equiv r(\rho, a) \cos\theta(\rho, a)$ on the brane.
- The total mass in 4+1 dimension is given by

$$\mathcal{M} = \frac{3}{16\pi\kappa_5} \int\limits_{t=const} d^{(4)}V\, {}^{(5)}R_{AB}\xi^A m^B \equiv \int\limits_0^\infty dR\, f_5(R).$$

- $\mathcal{M}$ does not depends on the value of parameter a!
- The exact calculations show that the 3-dim Komar integral, ${}^{(4)}\mathcal{M}_{eff} = 0$.
- compare with $g_{00} - 1 = O(1/R^2)$ but not $O(1/R)$ <= infinite size of an extra dimension.

Matter distribution.

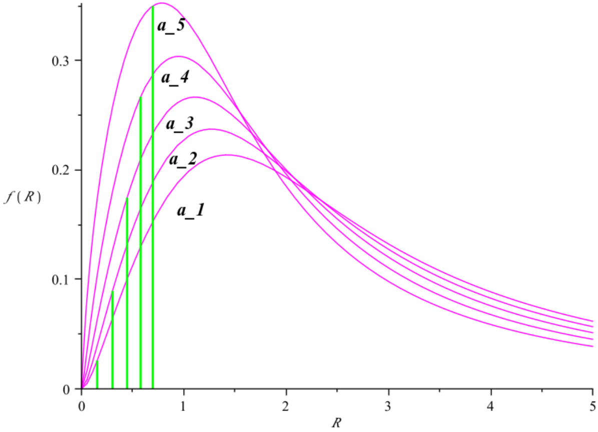

Figure 2: The matter-density radial distributions $f_5(R, a)$ on the brane with $M = 1$ are presented by a magenta colored line. The corresponding horizons are indicated by green lines. $a_1 > a_2 > a_3 > a_4 > a_5$

Matter distribution.

- Effective 4-D stress-energy tensor $S_{\mu\nu}$.
  - projection of Einstine equations onto the brane: SMS equations

$$^{(4)}G_{\mu\nu} \equiv G_{\mu\nu} = \kappa_5^2 \Sigma_{\mu\nu} - E_{\mu\nu} \equiv \kappa_4 S_{\mu\nu}, \quad \kappa_4 \equiv \frac{1}{M_{Pl}^2},$$

  - where

$$\Sigma_{\mu\nu} = \frac{1}{24}\Big(-2\tau\tau_{\mu\nu} + 6\tau_\mu^\sigma \tau_{\sigma\nu} + g_{\mu\nu}(-3\tau^{\sigma\rho}\tau_{\sigma\rho} + \tau^2)\Big),$$

  - and

$$E_{\mu\nu} =^{(5)} C^A_{BCD} n_A n^C q^B_\mu q^D_\nu .$$

  - Compare with 5-D Einstein equations:

$$^{(5)}G_{AB} = \kappa_5 \delta^\mu_A \delta^\nu_B \tau_{\mu\nu} \delta(z).$$

Matter distribution.

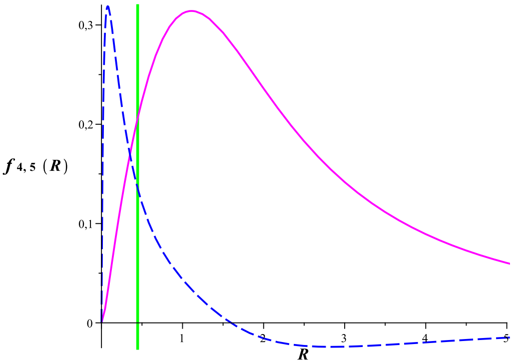

Figure 3: The matter-density radial distribution $f_5(R)$ on the brane with $a = 1.1, M = 1$ is presented for $\kappa_5 = 1$ by a magenta colored line. The effective matter-density $f_4(R)$ is shown by blue line for the value $\kappa_4 = 50$ to compare with $f_5(R)$. The horizon is indicated by green line.

Conclusions

- Results.
  - We have shown that by cut-and-paste method in special Gaussian normal coordinates one can build the exact geometry of multidimensional black star with *horizon*, generated by a *smooth* matter distribution in our universe.
  - In our approach, for a given total mass, the profiles of available configurations for matter distribution are governed by the parameter a which is presumably related to the collision kinematics when a black object ("black hole") is created by partons on colliders.
- Generalizations.
  - charged and rotated black stars as well as black rings.
    compact extra dimensions and warped geometries.

Thanks for attention!!!!!