Neutrino Oscillations and Superluminocity

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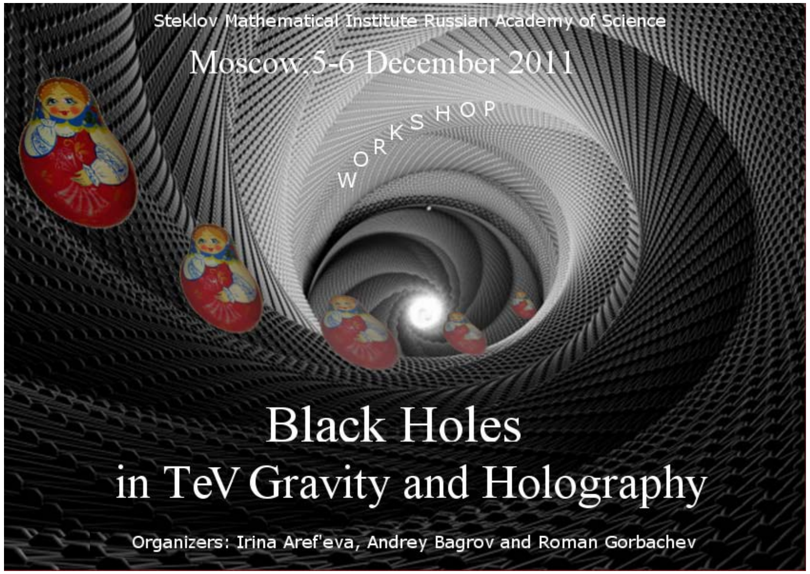


Moscow, December 5-6, 2011

Outlook

I.A., I.Volovich, arXiv:1110.0456.

- “OPERA”
- QFT models with superluminal velocity
- Cherenkov radiation and Cohen – Glashow problem
- New (dark) neutrino as a possible solution to the Cohen – Glashow problem
- Oscillations for supeluminal neutrino
- New neutrino in the D-brane framework

OPERA
(Oscillation Project with Emulsion-tRacking Apparatus)

- The special theory of relativity (a cornerstone of modern fundamental physics) has the upper limit of velocities which is the velocity of light in vacuum.

- The OPERA collaboration has recently announced the results about possible evidence for superluminal propagation of neutrinos.

- Obviously such an astonishing claim requires extraordinary standards of proof including confirmation by independent experiments.

Data

- **OPERA (Oscillation Project with Emulsion-tRacking Apparatus), 2011**

$$(v-c)/c=[2.48\pm 0.28(stat)\pm 0.30(sys)]\times 10^{-5}$$

$$<E_\nu>=17GeV$$

$$(v-c)/c=[2.37\pm 0.32(stat)^{+0.34}_{-0.24}(sys)]\times 10^{-5}$$

- **MINOS (Main Injector Neutrino Oscillation Search), 2007**

$$(v-c)/c=[5.1\pm 2.9]\times 10^{-5}$$

$$E_\nu \approx 3GeV$$

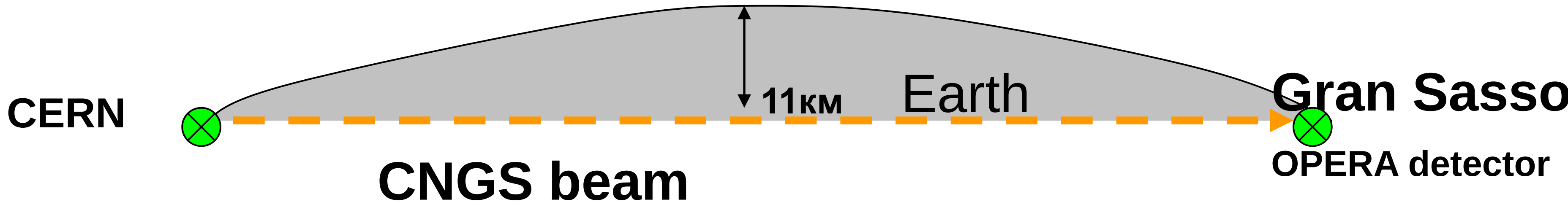
- **SN 1987A**

$$|v-c|/c<2\times 10^{-9}$$

$$E_\nu \approx 10\,MeV$$

Since c=299 792 458 м/c

OPERA, arXiv: 1109.4897



velocity = the precisely measured distance from CERN to OPERA/ time of neutrinos travelling

CNGS beam of muon neutrinos with mean energy of 17 GeV produced at CERN, travels about 730 km to the OPERA detector in the Gran Sasso Laboratory.

An early arrival time of the muon neutrinos with respect to the one computed assuming the speed of light in vacuum of 60 ns is reported.

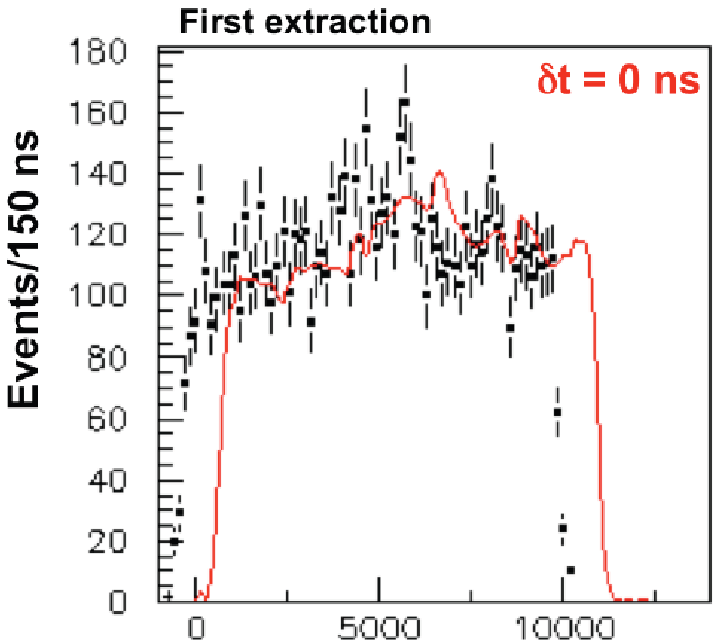
This anomaly corresponds to a relative difference of the muon neutrino velocity with respect to the speed of light

The neutrino time of flight measurement

The time of flight of CNGS neutrinos cannot be precisely measured at the single interaction level since any proton in the extraction time may produce the neutrino detected by OPERA

2 ”long” 10500 ns LHC extractions separated by 50ms (16 000) with mean neutrino energy of 13, 9GeV and with 42, 9GeV

From: arXiv:1109.4897



Comparison of the measured neutrino interaction time distributions (data points) and the proton PDF (red line) for the SPS extraction

Without correction	**1048,5 ns**
Corrections to….	**988 ns**
Net difference	**60** ns

”short” 3ns LHC extractions (20) (2-nd version of arXiv:1109.4897)

Recently the OPERA collaboration has reported 20 neutrino events obtained for very short proton extraction pulse, with a PDF gaussian mean width of only 3ms, in four bunches separated 524ns.

This 20 neutrinos also appears to precede light for 62.1 ± 3.7ns with a RMS (Root Mean Square)=16.4ns

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Possible explanations

- **Excluding of experimental errors**

  Systematic errors,…

- **Breaking of holy principles of the theoretical physics in 4-dim space-time**
  - **Lorentz invariance**
  - **Causality**

- **New physics in D-dimensional space-time**

→ **New neutrino in D-dim space-time**

There are various investigations of constraints on neutrino velocities and possible mechanisms for breaking the standard Lorentz invariance motivated by the OPERA claim

Breaking of Lorentz invariance

- **Explicit**
  - **Parameterization of Lorentz invariance breaking**
  - **A proper maximal velocity for each field**
  - **Models with higher space derivatives**

- **Non-explicit**
  - **Tachyon**

    Scalar tachyon

    Neutrino as a tachyon

Explicit Breaking of Lorentz invariance

Collady, Kostelecky, Phys.Rev.D58 (1998)

- " A proper maximal velocity for each field" **(PMV)**

S.Coleman, S.Glashow, Phys.Rev, D59 (1999)

From: invariance under space rotations and shift;
renormalizability

$$a,b = 1,\ldots n, \quad n \le 2$$

For real scalar fields

$$\Delta L = \frac{1}{2}\sum_{a,b}\partial_i\phi^a\epsilon_{ab}\partial^i\phi^b$$

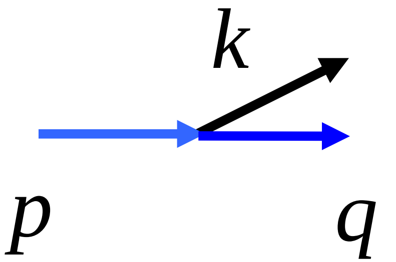
ϵ_{ab} - diagonal matrix

Energy-momentum relation

$$E_a{}^2 = c_a{}^2\vec{p}^2 + c_a{}^4 m_a{}^2$$

Instability (Cherenkov radiation) in PMV models

$$k_0{}^2 = \beta^2 \vec{p}_0{}^2$$



$$p_0{}^2 = \alpha^2 \vec{p}^2 + m^2$$

Radiation condition in PMV :

$$\frac{\beta}{\alpha}\sqrt{1+\frac{m^2}{\alpha^2 \vec{p}^2}} + \frac{1}{2}\left(1-\frac{\beta^2}{\alpha^2}\right)\frac{|\vec{p}\,|}{|\vec{k}\,|} < 1$$

PMV(particle
maximum velocity)

Cherenkov radiation

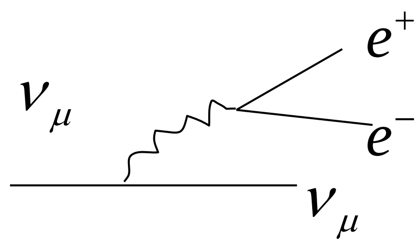
A.Cohen, S.Glashow, arXiv:1109.6562
New Constraints on Neutrino Velocities

Suppose that : ***mu neutrino*** **with energy ~ 10 GeV has superluminal velocity**

Cherenkov radiation

$$\nu_\mu \longrightarrow \begin{cases} \nu_\mu + \gamma \\ \nu_\mu + \nu_e + \overline{\nu}_e \\ \nu_\mu + e^+ + e^- \end{cases}$$

Kinematically allowed $v_\nu > 1$ **Suppressed!**



Threshold energy $E_0 = 2m_e / \sqrt{\delta}, \quad \delta = v_\nu^2 - v_e^2$

v_e **Max velocity**

$v_e = 1$ **up to** 10^{-15}

$$E_0 = 2m_e/\sqrt{\delta}|_{OPERA} \approx 140 \text{ MeV}$$

Cherenkov radiation

Rate of radiation

$$\Gamma = \frac{G_F^2}{14 \cdot 192\pi^3} E^5 \delta^3, \quad E = 17.5\,GeV,$$

$$\tau = \frac{1}{\Gamma} = 1126\,km/c, \quad \frac{N}{N_0} = e^{-730/1126} \cong 0.5$$

ν_μ with energy E_0 after covering L will have energy E

$$E^{-5} - E_0^{-5} = 5k\delta^3 \frac{G_F^2}{192\pi^3} L$$

The original beam would be strongly depleted

Conclusion: observation of neutrino with E ~ 12.5 GeV and superluminal velocity is impossible

Instability of superluminal neutrino

Rate of radiation

$$\Gamma = \frac{G_F^2}{14 \cdot 192\pi^3} E^5 \delta^3, \quad E = 17.5 GeV,$$

$$\tau = \frac{1}{\Gamma} = 1126 km / c, \quad \frac{N}{N_0} = e^{-730/1126} \simeq 0.5$$

ICARUS collaboration. ”A search for analogue to Cherenkov radiation by high energy neutrinos at superluminal speeds in Icarus”, arXiv1110.3763v2.

The original beam would be strongly depleted

Mohanty, Rao, 11111.2723

Conclusion: observation of neutrino with E ~ 12.5 GeV and superluminal velocity is impossible

$$E^4(p) = p^2 + p^4 / M^2$$

Proposal

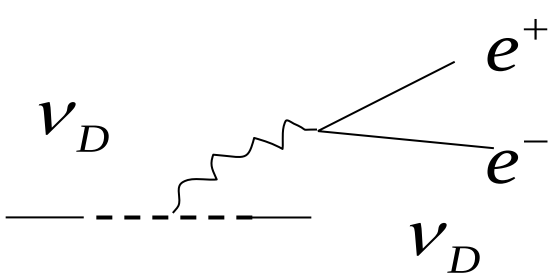
New dark superluminal neutrino

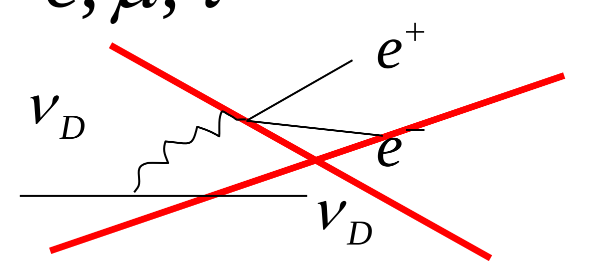
Interaction only via oscillations **I.A., I.Volovich, arXiv:1110.0456.**

$$\mathcal{L} = \mathcal{L}_{\mathcal{EW}} + \Delta\mathcal{L}$$

$$\Delta\mathcal{L} = i\,\overline{\nu}_{DL}(\gamma^{\mu}\partial_{\mu} + \delta\,\gamma^{i}\partial_{i})\nu_{DL} + \frac{1}{2}m_D\overline{\nu}^{c}_{DL}\nu_{DL} + \sum_{l} m_{Dl}\overline{\nu}^{c}_{DL}\,\nu_{lL} + h.c.$$

$$\bar{\psi} = \psi^{+}\gamma^{0}, \quad \psi^{c} = C\psi, \qquad l = e, \mu, \tau$$





Oscillations for Lorentz invariant models

$$i\gamma^{\mu}\partial_{\mu}\psi_a - m_{ab}\psi_b = 0, \qquad a,b = 1,2$$

Pontekorvo, 1957
Gribov, Pontekorvo 1969

$$m = \begin{pmatrix} m_a & m_{ab} \\ m_{ab} & m_b \end{pmatrix}$$

Initial data: $\psi_1(x,0) = 0, \quad \psi_2(x,0) = e^{ikx}$

$|k| \to \infty :$

$$P_{a\to b}(t) = |\psi_b(x,t)|^2 =$$

$$= \sin^2(2\theta)\,[\sin^2(\frac{\Delta m^2}{4E}t) - \frac{\Delta m^2}{4E^2}\sin(\frac{\Delta m^2}{4E}t)\sin(2Et)] + O(\frac{m_{1,2}^2}{E^4})$$

$$\Delta m^2 = m_1^2 - m_2^2, \quad E = |k|$$

$$U^{+} m\ U = \begin{pmatrix} m_1 & 0 \\ 0 & m_2 \end{pmatrix}$$

$$U = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}$$

$$m_{1,2}^2 = \frac{m_a^2 + m_b^2}{2} \pm \frac{1}{2}\sqrt{(m_a^2 - m_b^2)^2 + 4m_{ab}^4}$$

Oscillations for PMV

Mass states

$$H|\nu_i> \;=\; E_i|\nu_i>$$

$$P|\nu_i> \;=\; p_i|\nu_i>$$

Assume dispersion relations

$$E_i = \sqrt{(1+\delta_i)p^2 + m_i^2 + \frac{p^4}{M_i^2}}$$

Amplitude

$$\mathcal{A}_{BA}(x,t) \;=\; <\nu_B|\nu_A(t,x)>$$

$$= \; e^{i(p_1 x - E_t)}\left[\delta_{BA} + \sum\left(U_{Bi}U^*_{Ai}e^{i(p_i-p_1)x - i(E_i-E)t} - 1\right)\right]$$

$$p_i \;=\; E - \frac{m_i^2}{2E} - \frac{E}{2}\delta_i + \frac{m_i^2}{4E}\delta_i$$

$$e^{i(-\frac{\Delta m^2_{i1}}{2E} - \frac{E}{2}\delta_i + \frac{m_i^2}{4E}\delta_i)x}$$

Oscillations for supeluminal neutrino

2-neutrino system

$$E_i = \sqrt{(1+\delta_i)p^2 + m_i^2}$$

$$P_{AA}(x,t) \;=\; 1 - 2\sin^4\theta \sin^2\left(\frac{\Delta m_{12}^2}{4E}x + (\frac{E}{2} - \frac{m_1^2}{4E})\delta_2\, x\right)$$

$$\frac{1}{L_{0,\delta}} = \left(\frac{\Delta m_{i1}^2}{(eV^2)}\frac{(Gev)}{E} + \underbrace{\frac{E\delta_i}{(GeV)}\cdot 10^{18}}_{first\;correction} - \underbrace{\frac{m_i^2}{(eV^2)}\frac{(GeV)}{2E}\delta_i}_{2-nd\;correction}\right)\frac{1}{(2.5km)}$$

If active neutrino is superluminal then

ν_μ, ν_τ mixing (MINOS, $E = 3\text{GeV}$)

$L_{0,\delta}$ ~ $0.4 \cdot 10^{-7} mm$

Group velocities

$$K = \begin{pmatrix} \sqrt{(1+2\delta)p^2 + m_1^2} & M \\ M & \sqrt{p^2 + m_2^2} \end{pmatrix} \approx \begin{pmatrix} p(1+\delta+\frac{1}{2}\frac{m_1^2}{p^2}) & M \\ M & p(1+\frac{1}{2}\frac{m_2^2}{p^2}) \end{pmatrix}$$

In ultrarelativistic approximation

$$K = \begin{pmatrix} p & M \\ M & (1+\delta)p \end{pmatrix}$$

$$\lambda_{1,2} = \begin{cases} p(1+\frac{\delta}{2}) \pm \frac{\delta}{2}p, & M = 0 \\ p(1+\frac{\delta}{2}) \pm M, & \frac{\delta^2 p^2}{4M^2} << 1 \end{cases}$$

Usual+Superluminal

Superluminal+Superluminal

- **The present of the superluminal propagation for small p**
  Disappearance of the second superluminal propagation for large p

Mass corrections

$$p(1+\frac{1}{2}\delta) + \frac{1}{4}\frac{m_1^2 + m_2^2}{p} \pm \frac{1}{2}\sqrt{\delta^2 p^2 + \delta(m_1^2 - m_2^2) + 4M^2}$$

Group velocities for momentum dependent mixing

$$K = \begin{pmatrix} p & M \\ M & (1+\delta)p \end{pmatrix} \qquad V_{1,2} = \frac{\partial \lambda_{1,2}}{\partial p}$$

$$V_{1,2} = 1 + \frac{\delta}{2}\left(1 \pm \epsilon(p)\right)$$

$$\epsilon(p) \equiv \frac{1}{\sqrt{1 + \frac{4M^2}{\delta^2 p^2}}} \cdot \left[\, 1 + \frac{4M}{\delta^2 p} \frac{\partial M}{\partial p} \right]$$

To have both group velocities superluminal we have to assume $|\epsilon| < 1$

Group velocities for momentum dependent mixing

$$V_{1,2} = 1 + \frac{\delta}{2}(1 \pm \epsilon(p))$$

We want ϵ interpolate between

- Near $p = 0$: $\epsilon \sim 1$
- Near $p = \infty$: $\epsilon \sim 0$, for example for $p > p_{Opera/Minus}$

Find M from

$$\bar{M} = \frac{M}{\delta}$$

$$\frac{\partial \bar{M}}{\partial p} = \frac{p}{4\bar{M}} \left[\epsilon(p) \sqrt{1 + \frac{4\bar{M}^2}{p^2}} - 1 \right]$$

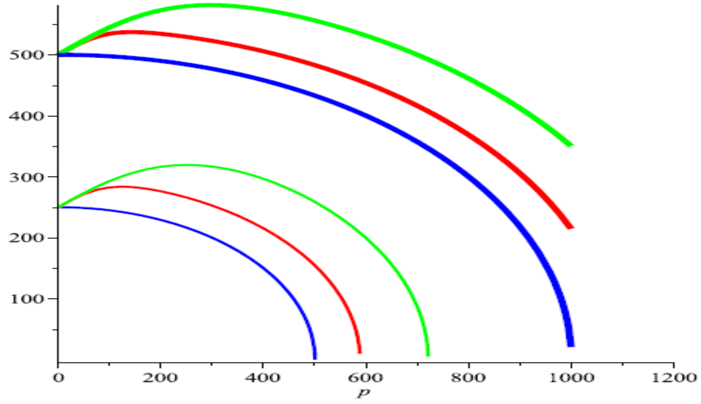
Group velocities for momentum dependent mixing

$$V_{1,2} = 1 + \frac{\delta}{2}(1 \pm \epsilon(p))$$

$$\frac{\partial \bar{M}}{\partial p} = \frac{p}{4\bar{M}}\left[\epsilon(p)\sqrt{1+\frac{4\bar{M}^2}{p^2}} - 1\right]$$

Simple example $\epsilon(p) = e^{-p^2/a^2}$ $\bar{M} = \frac{M}{\delta}$

$$\bar{M}(p) = \frac{1}{2}\sqrt{\left(\frac{1}{2}\sqrt{\pi}a\,\mathrm{erf}(\frac{p}{a}) + C_1\right)^2 - p^2}$$

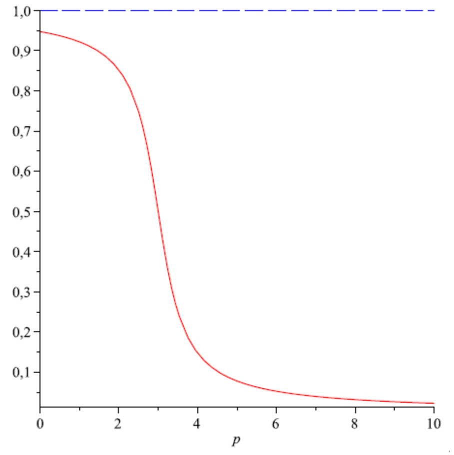


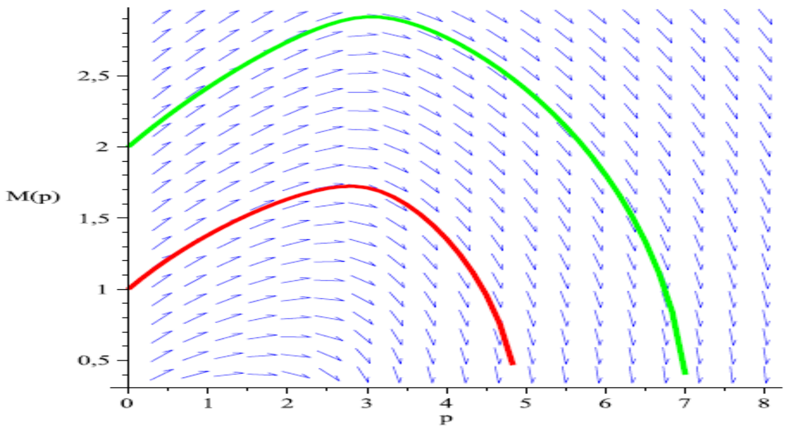
a=1 blue lines
a=100 red lines
a=250 green lines
C_1=1000 thin lines
C_1= 500 thick lines

Group velocities for momentum dependent mixing

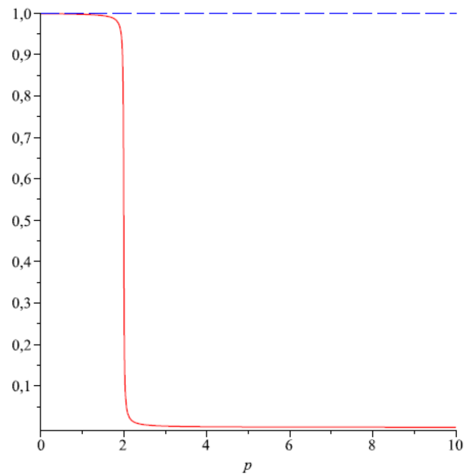
Sharp change of velocity

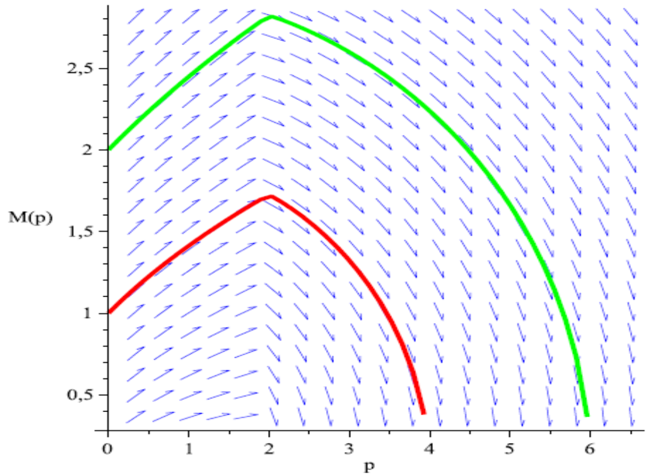
$$\epsilon = \frac{1}{2} - \arctan\left(\frac{p-b}{a}\right)\pi^{-1}$$





Problem with the cut





Wigner distribution

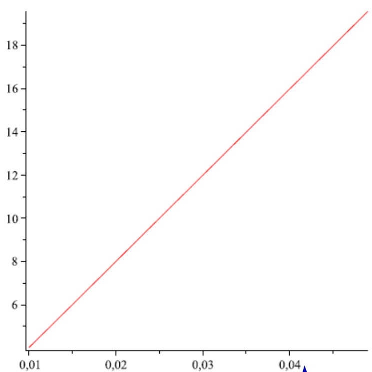
Group velocities for momentum dependent mixing

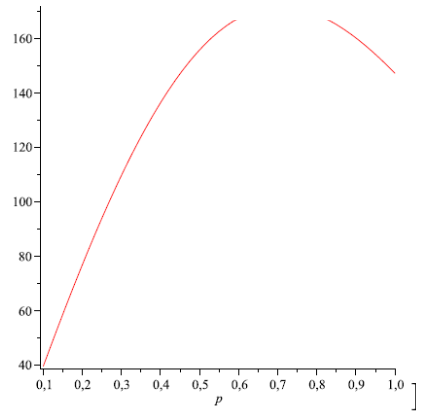
M(p) from a "fundamental theory" (for example SFT)

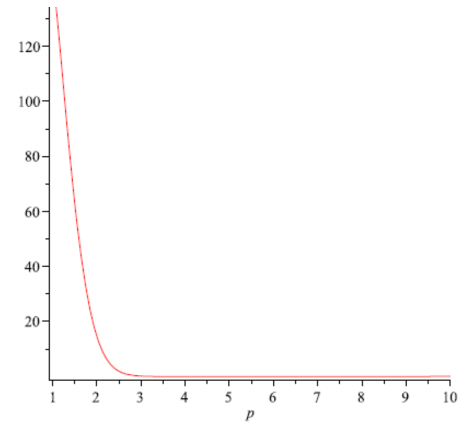
Kostelecky, Samuel, Phys.Rev.D39 (1998)

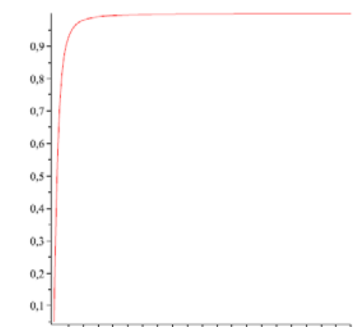
$$M(p) = (1 - e^{-p^2/M_1^2})M_0$$

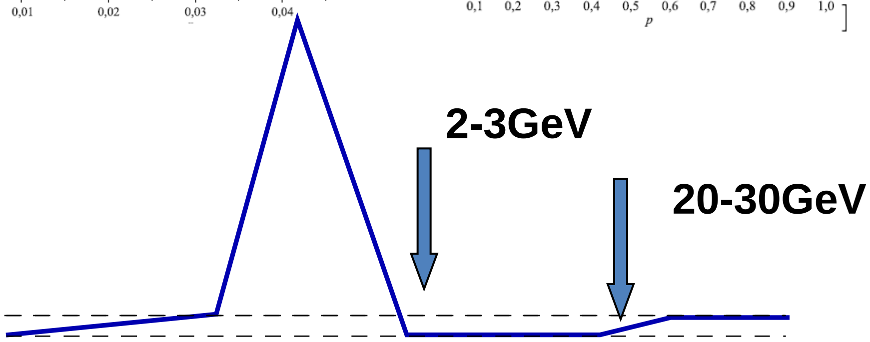
$$M_0 = 1MeV; M_1 = 1GeV; \delta = 10^{-5}$$











Problem with the pick

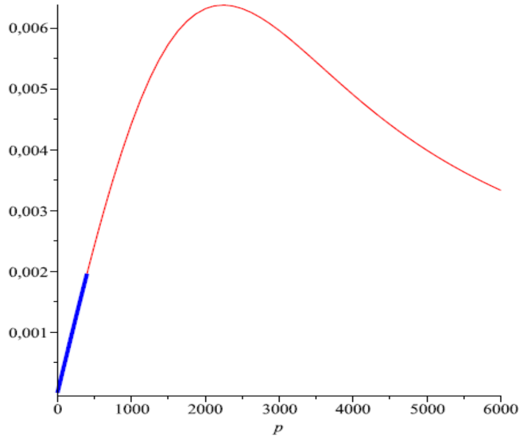
Mixing angle for momentum dependent mixing

M(p) from a “fundamental theory” (for example SFT)

$$M(p) = (1 - e^{-p^2/M_1^2})M_0$$

$$M_0 = 1MeV; M_1 = 1GeV; \delta = 10^{-5}$$

$$\tan 2\theta = \frac{2M}{(1+\delta)p}$$

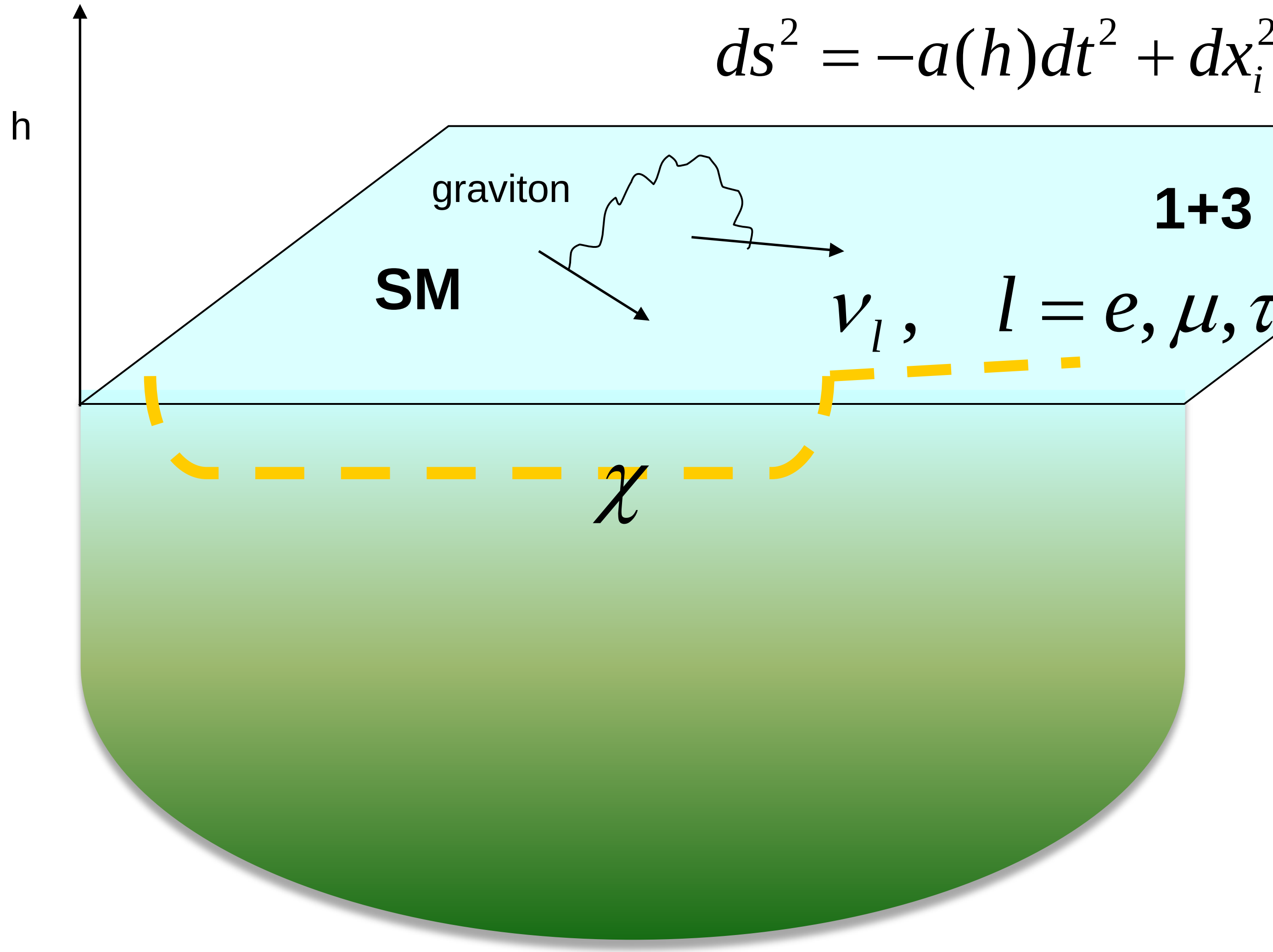


$$\tan 2\theta \approx \frac{2M_0}{p}(1 - e^{-p^2/M_1^2})(1-\delta) \propto \begin{cases} \frac{2M_0 p}{M_1^2}, & p \to 0 \\ \frac{2M_0}{p}, & p \to \infty \end{cases}$$

D-dimensional Dark Neutrino

We are living on D-brane **Rubakov,Shaposhnikov,1983**

$$ds^2 = -a(h)dt^2 + dx_i^2 + dh^2, \quad a > 0$$

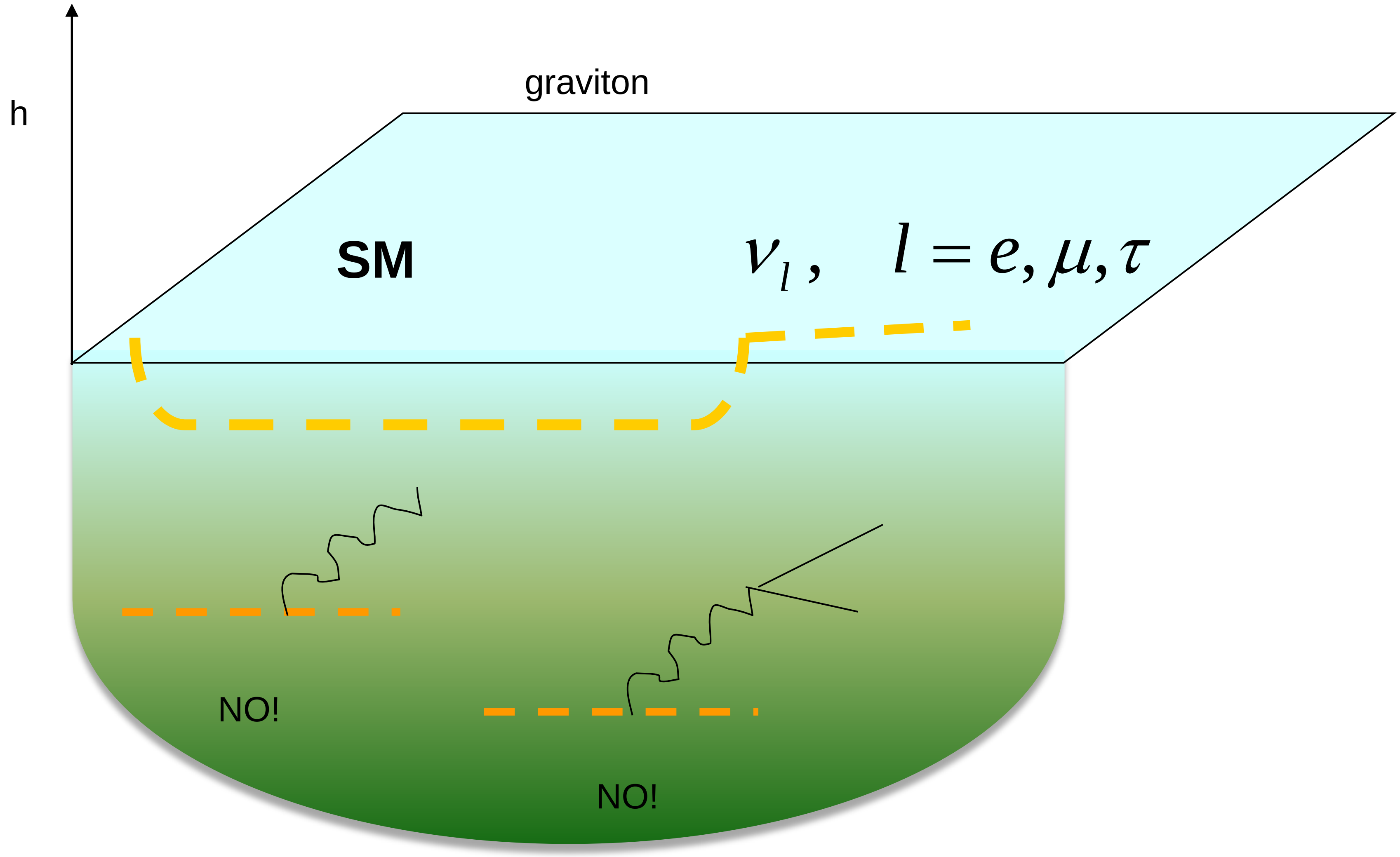


Kiritsis, 1999

Gubser, 2011

D-dimensional Dark Neutrino

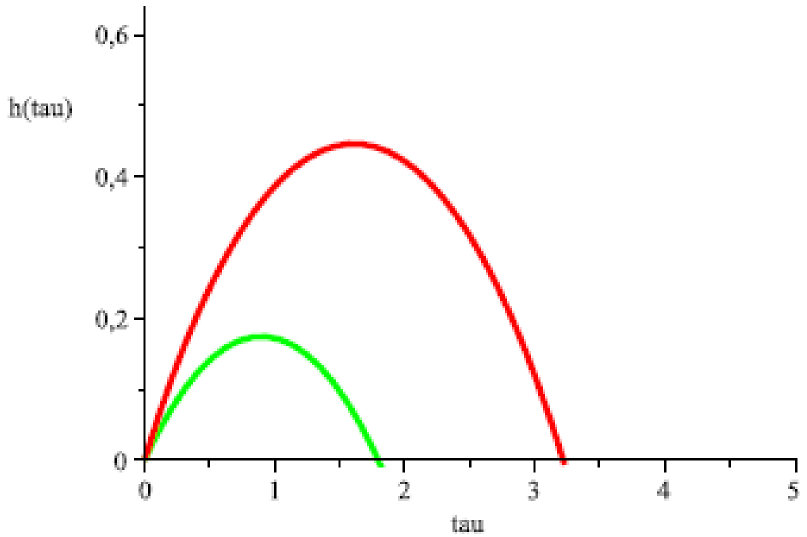
$$ds^2 = -a(h)dt^2 + dx_i^2 + dh^2, \quad a > 0$$

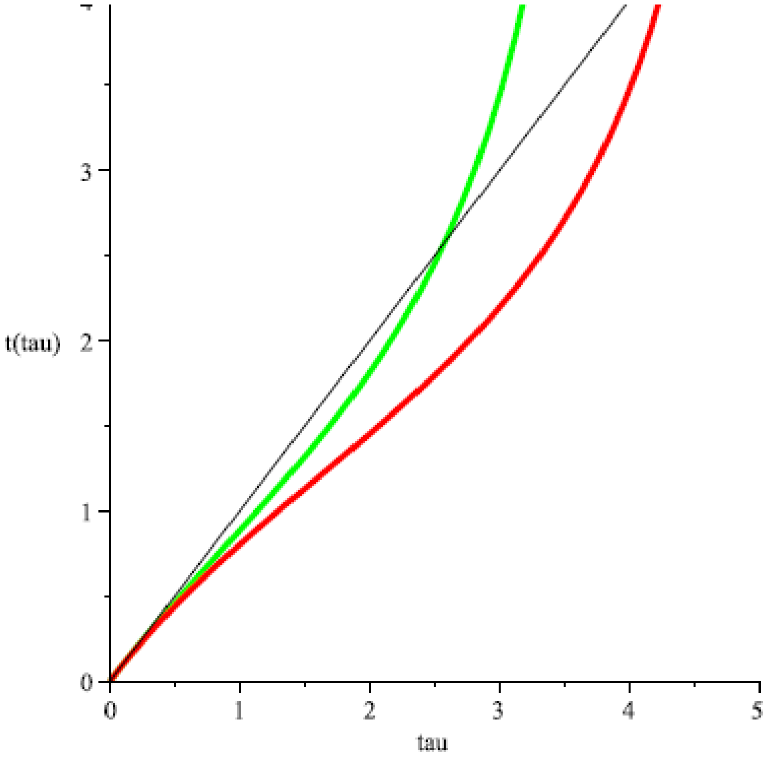


In the bulk

Geodesics

$$\ddot{h} + \frac{1}{2}k^2 e^{k^2 h}(\dot{t})^2 = 0$$
$$\ddot{t} + k^2 \dot{t}\dot{h} = 0$$
$$\ddot{x} = 0$$





Conclusion

- Dark neutrino solves the Cohen-Glashow problem with instability of neutrino due to Cherenkov radiation

- New formula to neutrino oscillation

- New experiments to check the neutrino velocity