4. Quantum physics, quantum information and quantum dynamical semigroups

The low-density limit for a system with continues spectrum

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An important problem in the theory of open quantum systems is the description of the master equations for the reduced dynamics of an open quantum system in various approximations on the basis of its exact microscopic dynamics at interaction with the environment. A well-known approximation is the weak coupling limit, at which the intensity of interaction of the system with the environment is a small quantity [1], and the closely related stochastic limit [2]. Another approximation is the low-density limit, where the density of surrounding particles is a small quantity while an interaction between a system and a reservoir is not assumed to be small. Such a limit describes collisional decoherence [3]. The study of the low-density limit in the theory of open quantum systems goes back to [4] and was developed in other papers, see for example [5, 6]. In the low-density limit for open quantum systems, the quantum Langevin equation [7] and the quantum stochastic differential equation with a quantum Poisson process [8] were derived. The connection between this approximation and collisional decoherence models is established in [9]. In the present paper, we consider the low-density limit for a quantum system interacting with an ideal Fermi gas in thermal equilibrium at non-zero temperature. The low-density limit is defined as follows

$$n_\varepsilon = \varepsilon n_0,\ t_\varepsilon = \varepsilon^{-1} t_0,\ \varepsilon \downarrow 0, \tag{1}$$

where n, t is the density of gas and time respectively. Let H^S be a self-adjoin operator with dense domain of definition in Hilbert space $\mathscr{H}^S$ accosted with Hamiltonian of system. By H^R denote the Hamiltonian of ideal Fermi gas be the self-adjoin operator in Fermi-Fock space $\mathscr{F}$. Then the general Hamiltonian is

$$H^G = H^S \otimes I + I \otimes H^R + V, \tag{2}$$

where V is a operator of interaction. We will consider the operator V of the scattering type, i.e. the gas particles cannot be absorbed by the system and their number is constant. The general form of such operator is $V = \sum_{j=1}^{\nu} Q_j \otimes a^\dagger(f_j)\, a(f_j)$ [4, 10], where $a^\dagger, a$ are creation and annihilation operators. Let $\mathscr{U}^S, \mathscr{U}^R$ be a some sub-algebra of algebra of system observables and CAR-algebra for reservoir respectively. The evolution $Y \to Y(t) = \mathcal{T}(t)\, Y, Y \in \mathscr{U}^S \otimes \mathscr{U}^R$ of the whole system with reservoir generated by the Hamiltonian (2), is given as a solution of the von Neumann equation

$$\frac{d}{dt} Y = i\left[H^G, Y\right]. \tag{3}$$

The initial state of the system is given as a functional $\omega^S : \mathscr{U}^S \to \mathbb{C}$, $\omega^S(X) = \mathrm{Tr}(X\rho)$ generated by a density matrix ρ which is trace-class operator $\rho \in J_1\left(\mathscr{H}^S\right)$. We will consider the reservoir state as a thermal equilibrium associated with the Kubo-Martin-Schwinger functional $\omega_{\mu_\varepsilon} : \mathscr{U}^R \to \mathbb{C}$ related with chemical potential μ_ε denoted by n_ε in limit (1) with fixed temperature.

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Case $\dim\left(\mathscr{H}^S\right) < \infty$ is well considered [4], but the obtained results do not apply to the case of an infinite-dimensional system when the Hamiltonian H^S has a continuous spectrum and the operators Q_j are unbounded. The purpose of the paper is to describe the reduced dynamic in this case. Reduced dynamic $\rho \to \widetilde{\mathcal{T}}_\varepsilon(t)\rho$ implicitly determined by relation

$$\operatorname{Tr}\left(X\widetilde{\mathcal{T}}_\varepsilon(t)\rho\right) = \left[\omega^S \otimes \omega_{\mu_\varepsilon}\right]\left(\mathcal{T}\left(\varepsilon^{-1}t\right)X \otimes I\right),\ \varepsilon \downarrow 0. \tag{4}$$

We study equation (4) with using of reduced n-particle evolution operators $R_n^\varepsilon(t)$ which are denoted like

$$(g_0 \otimes g_1 \otimes \cdots \otimes g_n, R_n^\varepsilon(t) f_0 \otimes f_1 \otimes \cdots \otimes f_n) = \\ = \left[\omega^S \otimes \omega_{\mu_\varepsilon}\right]\left(\mathcal{T}\left(\varepsilon^{-1}t\right)\left(|f_0\rangle\langle g_0| \otimes a^\dagger(f_n)\dots a^\dagger(f_1)\, a(g_1)\, a(g_n)\right)\right).$$

$R_n^\varepsilon(t)$ can be considered like quantum analog of BBGKY hierarchy. With evaluating this method we derive the reduced dynamics in the limit (4) in some special cases. The obtained equations are considered with the application of scattering theory and Kato-Birman theory [11]. This paper deals with the existence and completeness of wave operators and their use to derive the reduced dynamics of the system.

References

[1] H. Spohn, J.L. Lebowitz. *Irreversible thermodynamics for quantum systems weakly coupled to thermal reservoirs.* // Adv. Chem. Phys. 2007. Vol. 38. P. 109–142.

[2] L. Accardi, Y.G. Lu, I. Volovich *Quantum theory and its stochastic limit.* Springer Science & Business Media, 2013.

[3] S. Campbell, B. Vacchini. *Collision models in open system dynamics: A versatile tool for deeper insights?.* // Europhys. Lett. 2021. Vol. 133. No. 6. 60001.

[4] R. Dumcke. *The low density limit for an N-level system interacting with a free Bose or Fermi gas.* // Commun. Math. Phys. 1985. Vol. 97. P. 331–359.

[5] L. Accardi, Y.G. Lu. *The low-density limit of quantum systems.* // J. Phys. A: Math. Gen. 1991. Vol. 24 No. 15. 3483.

[6] S. Rudnicki, R. Alicki, R. Sadowski. *The low-density limit in terms of collective squeezed vectors.* // J. Math. Phys. 1992. Vol. 33. No. 7. P. 2607–2617.

[7] L. Accardi, A.N. Pechen, I.V. Volovich. *A stochastic golden rule and quantum Langevin equation for the low density limit.* // Infin. Dimension. Anal. Quantum Probab. Relat. Top. 2003. Vol. 6. No. 03. P. 431–453.

[8] A.N. Pechen. *Quantum stochastic equation for a test particle interacting with a dilute Bose gas.* // J. Math. Phys. 2004. Vol. 45. No. 1. P. 400–417.

[9] S.N. Filippov, G.N. Semin, A.N. Pechen. *Quantum master equations for a system interacting with a quantum gas in the low-density limit and for the semiclassical collision model.* // Physical Review A. 2020. Vol. 101. No. 1. 012114.

[10] E.B. Davies. *Markovian master equations.* // Commun. Math. Phys. 1974. V. 39. P. 91–110.

[11] M. Reed, B. Simon *Methods of modern mathematical physics. III. Scattering theory.* Academic Press, Inc, 1979