The Property of Unique Continuation in Certain Spaces Spanned by Rational Functions on Compact Nowhere Dense Sets

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Abstract. It has been known for over a century that certain large classes of functions defined on a compact nowhere dense subset X of the complex plane, and obtained as limits of analytic functions in various metrics, can sometimes inherit the property of *unique continuation* characteristic of the approximating family. The first example of the transfer of the uniqueness property in this way to $R(X)$, the space of functions that can be uniformly approximated on X by a sequence of rational functions whose poles lie outside of X, was obtained by M. V. Keldysh around 1940, but apparently never published. Later in 1975 A. A. Gonchar exhibited a qualitatively definitive improvement of Keldysh's example, and my goal here is to extend that result to $R^p(X, dA)$, $p \geq 2$, the evidently larger space obtained as the closure of the rational functions in $L^p(X, dA)$, where dA denotes 2-dimensional Lebesgue, or area, measure. Among other things, this will require obtaining a suitable notion for the existence of boundary values along the outer boundary of a Swiss cheese for functions in $R^p(X)$.