Subharmonic functions

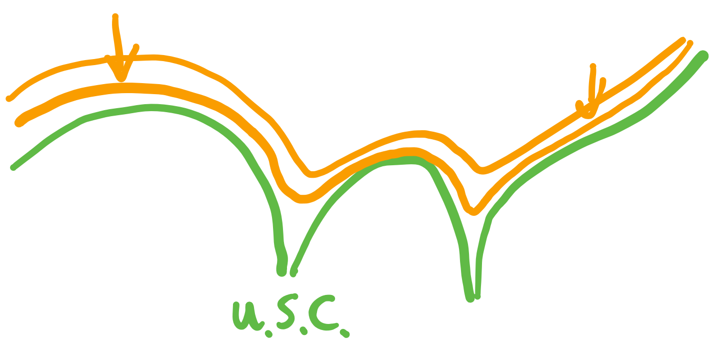
$u : \Omega \longrightarrow [-\infty, +\infty)$

$\subset \mathbb{C}$

1. upper semicontinuous

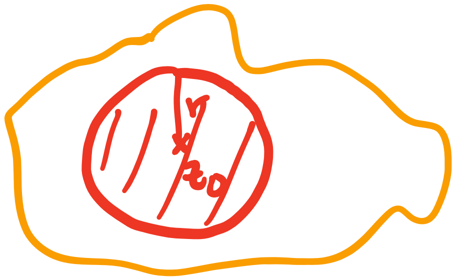
~~lower~~

the monotonic limits from above of continuous functions



2. sub mean value property :

$z_0 \in \Omega$, $r > 0$ $\quad$ $|z - z_0| \leq r$ contained inside Ω



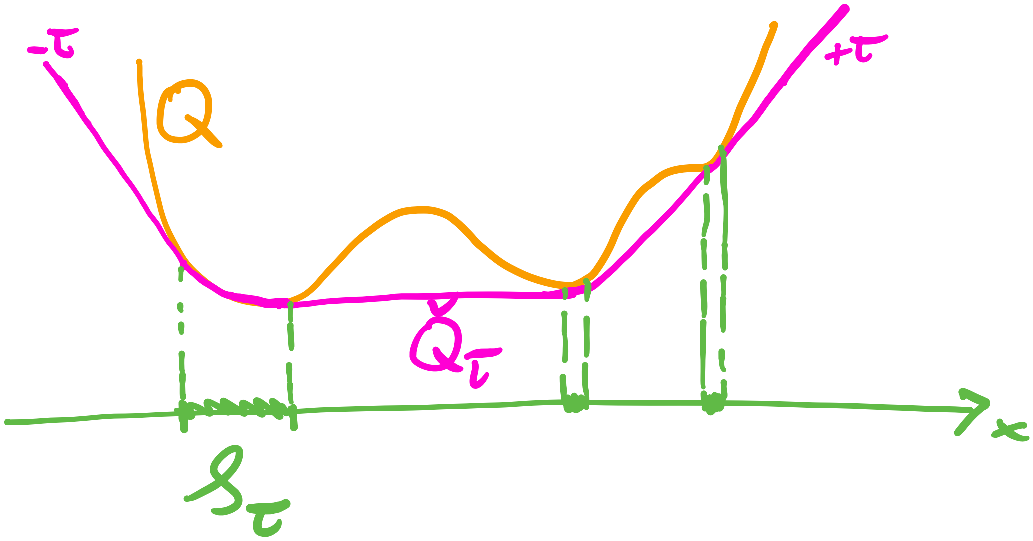
$$u(z_0) \le \fint_{|z-z_0|=r} u$$

sub-mean value property.

for all such disks given by z_0, r.

Obstacle problem in $\mathbb{R}^1$ with convexity in place of subharmonicity.

$\mathrm{Conv}_\tau(\mathbb{R})$: convex functions on $\mathbb{R}$, and $|f(x)| \le \tau|x| + O(1)$ as $|x| \to \infty$.



$$\partial_z,\ \bar\partial_z,\ \Delta_z = \partial\bar\partial_z \qquad \boxed{z = x + iy}$$

$$\begin{cases} \partial_z = \frac{1}{2}(\partial_x - i\,\partial_y) \\ \bar\partial_z = \frac{1}{2}(\partial_x + i\,\partial_y) \end{cases} \qquad \begin{cases} \partial_x = \frac{\partial}{\partial x} \\ \partial_y = \frac{\partial}{\partial y} \end{cases}$$

$$\Delta_z = \partial_z \bar\partial_z = \frac{1}{4}\underbrace{\left(\partial_x^2 + \partial_y^2\right)}_{\text{usual Laplacian}}$$

$$h = z^m \bar z^n$$

$$\partial_z h = m z^{m-1} \bar z^n$$

$$\partial_{\bar{z}} h = n z^m \bar{z}^{n-1} \qquad \text{exercise!}$$