28.10 13:00-13:30

Six problems in classical KAM Theory

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In a series of recent papers ([LL20], [LL23], [LL24], [LL]) we developed a "singular KAM Theory" which allows to investigate primary and secondary KAM tori exponentially close to the singularities (separatrices) arising near simple resonances in nearly-integrable generic Hamiltonian systems. In particular, such a theory allows to get upper bound on the measure of the "non-torus set", which, according to conjectures made by Arnold, Kozlov and Neishdadt in [AKN88] and [AKN06], are sharp up to a logarithmic correction, at least in the case of natural (or mechanical) Hamiltonian systems.

After briefly reviewing the main ideas and techniques on which Singular KAM Theory is based, I will discuss six open problems which naturally arise in this context.

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