Abstracts

28.10
13:30-14:00

The two Levi-Civita regularizations

Alain Albouy

Observatoire de Paris, CNRS, France

Levi-Civita published in 1916, and republished in 1920 and 1924, a proof that the flow of the 3-dimensional 3-body problem may be regularized, that is, replaced by a smooth flow which passes the binary collisions without experiencing any kind of singularity. This proof is reproduced by Siegel in his book (1956). But from 1965, all the authors on the regularization of the binary collisions considered that Levi-Civita only did the 2-dimensional case, and that the 3-dimensional case was first proposed in 1964 by Kustaanheimo, who extended in a sophisticated way the 2-dimensional regularization that Levi-Civita used in 1906. This is extremely surprising. What is then the status of Levi-Civita's result in 1916? He claimed to regularize the flow, without any hypothesis on the value of the energy. If we replace his second regularization by the Kustaanheimo-Stiefel regularization, we cannot regularize the binary collisions with zero energy. If we replace it by Moser's regularization (1970), which indeed Moser relates to Levi-Civita's second regularization (see [J M70] p. 615), we have the same problem. I will propose a very simple improvement of Levi-Civita's second regularization which confirms his argument. The improved regularization is still simpler than the KS regularization.

I wish to thank G.F. Gronchi for informing me of the 3-dimensionality of [T L20].

[J M70] J. Moser, *Regularization of kepler's problem and the averaging method on a manifold*, Communications on Pure and Applied Math. (1970), pp. 609–636.

[T L20] T. Levi-Civita, *Sur la régularisation du problème des trois corps*, Acta Mathematica **42** (1920), pp. 99–144.

Dissipative effects in Hamiltonian mean-field model

29.10
11:10-11:30

Artem Alexandrov

Institute for Information Transmission Problems & Moscow Institute of Physics and Technology & HSE University, Moscow

Hamiltonian mean-field model is the simplest possible realization of many-particle system with long-range interactions. This model has the following Hamiltonian,

$$H_{\mathrm{HMF}} = \sum_{i=1}^{N} \frac{p_i^2}{2m} + \frac{2K}{N} \sum_{i<j}^{N} \cos\left(\phi_j - \phi_i\right), \tag{1}$$

where $\forall i \ \phi_i \in [-\pi, +\pi)$ are phase variables, K is the coupling constant, and N^{-1} factor in the interaction term is needed for the correctly defined $N \to \infty$ limit (so-called *Kac prescription*). It is well-known fact that due to the weak convergence between empirical (discrete) & continuous measures, the HMF model has a well-defined hydrodynamic limit, which is described by the Vlasov equation. In physics HMF model is known as *kinetic* XY-model because of the kinetic energy term in the Hamiltonian. From the physical point of view, XY-model with quenched disorder is of a great interest. This quenched disorder can be modelled by adding into the equations of motion for ϕ_i random quantities ω_i with known distribution function $g(\omega)$.

In addition to quenched disorder, the dissipative processes are also common for physics. The equations of motion for model with disorder and dissipation are given by

$$m\ddot{\phi}_i + \dot{\phi}_i = \omega_i + \frac{2K}{N} \sum_{j=1}^{N} \sin\left(\phi_j - \phi_i\right). \tag{2}$$

In theory of dynamical systems, this model is known as *Kuramoto model with inertia*. All the statement about the weak convergence of measures hold for this model, too. My talk is devoted to generalization