NMS-flows on 4-manifolds

28.10
15:00-15:30

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In this talk, we consider the so-called *NMS-flows* f^t, that is, *non-singular* (without fixed points) Morse-Smale flows given on closed orientable n-manifolds M^n, $n \geq 2$. The non-wandering set of such a flow consists of a finite number of periodic hyperbolic orbits.

In the case of a small number of orbits, the known invariants can be significantly simplified and, most importantly, the classification task can be brought to realization by describing the admissibility of the obtained invariants. In [PS22], an exhaustive classification of flows with two orbits on arbitrary closed n-manifolds was obtained. In the work [Uma90], the problem of classification for three-dimensional Morse-Smale flows with a finite number of special trajectories is solved. The topological equivalence of non-singular flows, under assumptions of different generality, on the 3-sphere, is obtained, for example, in [Fra85], [Yu16].

In [GPS23], it was found that the only orientable 4-manifold admitting NMS-flows with exactly one saddle periodic orbit, assuming that it is *twisted* (its invariant manifolds are non-orientable), is $\mathbb{S}^3 \times \mathbb{S}^1$. It is also proved there that such flows are divided into eight equivalence classes. We should immediately note that in the case of a non-twisted orbit, the number of equivalence classes of such flows is infinite, as follows from the work [PS20], and among them there are flows with wildly embedded invariant manifolds of the saddle orbit.

This paper is devoted to the topological equivalence of four-dimensional NMS-flows with exactly one saddle periodic orbit, assuming that it is untwisted.

[Fra85] J. Franks, *Nonsingular Smale flows on S^3*, Topology **24**:3 (1985), pp. 265–282.

[GPS23] V. Galkin, O. Pochinka, and D. Shubin, *Classification of NMS-flows with unique twisted saddle orbit on orientable 4-manifolds*, Arxiv (2023).

[PS20] O. Pochinka and D. Shubin, *On 4-dimensional flows with wildly embedded invariant manifolds of a periodic orbit*, Applied Mathematics and Nonlinear Sciences **5**:2 (2020), pp. 261–266.

[PS22] O.V. Pochinka and D.D. Shubin, *Non-singular Morse-Smale flows on n-manifolds with attractor repeller dynamics*, Nonlinearity **35**:3 (2022), p. 1485.

[Uma90] Ya. L. Umanskii, *Necessary and sufficient conditions for topological equivalence of three-dimensional Morse-Smale dynamical systems with a finite number of singular trajectories*, Math. Sb. **181**:2 (1990), pp. 212–239.

[Yu16] B. Yu, *Behavior of nonsingular Morse-Smale flows on* S^3, Discrete and Continuous Dynamical Systems **36**:1 (2016), p. 509.