Minimum number of non-wandering points in addition to expanding attractors

28.10
15:30-15:50

Marina Barinova

HSE University

The structure of the non-wandering set of an Ω-stable diffeomorphism closely related to the topology of the ambient manifold and dynamical properties of the system. Moreover, meaningful conclusions can be drawn, even if only part of the non-wandering set is known.

Let $f : M^3 \to M^3$ be an Ω-stable diffeomorphism given on a closed connected 3-manifold M^3. Non-wandering set $NW(f)$ of the diffeomorphism f is a disjoint union of basic sets, each of which is compact, invariant and topologically transitive. If a basic set is a periodic orbit, then it is called trivial, otherwise — non-trivial.

A. Brown proved [A B10], that a non-trivial hyperbolic attractor $\mathcal{A}$ of a 3-diffeomorphism can be only several types:

1. 1- or 2-dimensional expanding attractor, i.e. $\dim \mathcal{A} = \dim W^u_x$, $x \in \mathcal{A}$. A local structure of such attractors is a direct product of a Cantor set and a disk. They can be orientable or non-orientable (in Grines sence).

2. 2- or 3-dimensional Anosov torus. It means that the restriction of the diffeomorphism on the attractor is conjugated to Anosov diffeomorphism given on 2- or 3-torus. In the last case $\mathcal{A}$ coincides with M^3. Anosov torus is always orientable.

We study a class of diffeomorphisms for which all non-trivial basic sets are attractors. Results from paper [BPY24] show that a non-trivial attractor can be either 1-dimensional non-orientable expanding or 2-dimensional expanding (orientable or not) in this case.

Suppose, that each non-trivial attractor is 2-dimensional. We obtained a lower estimates of a number of periodic points outside of non-trivial part of non-wandering set. The estimates are based on a number of connected components of the set $W^2_\Lambda \setminus \Lambda$, where Λ is the union of all non-trivial attractors of the diffeomorphism f. More precisely, the following theorem holds [Bar24].

Theorem 1. *Let $f : M^3 \to M^3$ be an Ω-stable diffeomorphism, given on a closed 3-manifold, Λ be a non-empty set of non-trivial basic sets of f. If Λ consists of 2-dimensional expanding attractors having a* *total of k_1 components of the set $W^s_\Lambda \setminus \Lambda$, each of which is homeomorphic to $\mathbb{R}P^2 \times \mathbb{R}$, and k_2 other components, then the number of points in the set $NW(f) \setminus \Lambda$ no less then $\frac{3}{2}k_1 + k_2$ and this estimate is exact.*

The article was prepared within the framework of the project ”International academic cooperation HSE University”.

[A B10] A. Brown, *Nonexpanding attractors: conjugacy to algebraic models and classification in 3-manifolds*, Journal of Modern Dynamics **4** (2010), pp. 517–548.

[Bar24] M. Barinova, *On isolated periodic points of diffeomorphisms with expanding attractors of codimension 1*, Arxiv. org, 2024.

[BPY24] M. Barinova, O. Pochinka, and E. Yakovlev, *On a structure of non-wandering set of an â¦-stable 3-diffeomorphism possessing a hyperbolic attractor*, Discrete and Continuous Dynamical Systems **44**:1 (2024), pp. 1–17.