

Another billiard problem

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Let (M, g) be a Riemannian manifold, $\Omega \subset M$ a domain with smooth boundary Γ , and ϕ be a smooth function such that $\phi|_{\Omega} > 0$, $\phi|_{\Gamma} = 0$, and $d\phi|_{\Gamma} \neq 0$. We study the geodesic flow of the metric $G = g/\phi$ in Ω . The G -distance from any point of Ω to Γ is finite, so the geodesic flow is incomplete. Regularization of the flow in a neighborhood of Γ establishes a natural reflection law from Γ . This leads to a certain billiard-like problem in Ω . We obtain a normal form for the regularized flow near Γ and for the corresponding billiard map of $T^*\Gamma$. This leads to a version of Lazutkin's theorem [V L72] on the existence of caustics for convex billiards. Our work was motivated by the results of Dobrohotov and Nazaikinskii, see e.g. [SVB13], on the quasi-classical approximation for the wave equation $u_{tt} = \nabla \cdot (\phi \nabla u)$ in Ω degenerating on Γ . The talk is based on the paper [SD24].

This is a joint work with Dmitry Treschev.

- [SD24] S. Bolotin and D. Treschev, *Another billiard problem*, Russ. J. Math. Phys. **31** (2024), pp. 50–59.
- [SVB13] S. Dobrokhoto, V. Nazaikinskii, and B. Tirozzi, *Two-dimensional wave equation with degeneration on the curvilinear boundary of the domain and asymptotic solutions with localized initial data*, Russ. J. Math. Phys. **20** (2013), pp. 389–401.
- [V L72] V. Lazutkin, *Existence of a continuum of closed invariant curves for a convex billiard*, Uspehi Mat. Nauk **27** (1972), pp. 201–202.