29.10
09:30-10:10

# Mathematical aspects of brain growth modeling

Pavel I. Plotnikov

*Lavryentyev Institute of Hydrodynamics, Novosibirsk*

The work is devoted to the analysis of the mathematical model of the volumetric growth of incompressible neo-Hookean material. Models of this kind are used in order to describe the evolution of the human brain under the action of an external load. The model consists of a nonlinear elastostatic equation and an evolution equation for a deformation vector field $\mathbf{u}$, pressure function $p$, and growth factor $w$. We study the structure of the governing equations and show that they formally can be reduced to a gradient flow of the marginal energy for the growth factor. We study in many details a variety of the homeostatic states. In particular, we show that the space of homeostatic deformation fields $\mathbf{u}$ coincides with the Möbius group of conformal transforms in $\mathbb{R}^3$. Moreover, the corresponding homeostatic growth factors equal conformal factors $e^f$ of the homeostatic deformation fields. Note that the homeostatic conformal factors forms 4-dimensional manifold in the space of smooth functions. We modify the Stopelli-Valent method in order to prove the local well-posedness of the initial-boundary value traction problem in a neighborhood of the homeostatic manifold. The main conclusion is the multiplicity of gradient flows for the growth factor.

In the work, we also study the long time behavior of solutions to the mathematical model of the volumetric growth of incompressible neo-Hookean material. We take an interest in the following question, which is of practical significance. Assume that a material is under the the temporary load vanishing after some moment (hydrocephalus natural or induced). This means that the external forces in the elasticity equations vanish for all sufficiently large values of the temporal variable. The question arises as to whether the growth process is reversible. In other words, does the mechanical system return to the initial configuration. More general question is the converges of the growth process to some final homeostatic state. We prove that for every infinitely smooth solutions $\mathbf{u}(t)$, $p(t)$, $w(t)$ to the governing equations, there is a conformal mapping $\mathbf{u}_\infty$ with the conformal factor $e^f$ such that the orbit $w(t) \to e^f$ as $t \to \infty$ in every Sobolev space. The proof is based on the extension the Lojasiewicz-Simon theory to the case of gradient flows with multiple potentials, which Hessians are not Fredholm operators of index zero.