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Existence of localized motions near an unstable equilibrium position

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We discuss a new type of motion—motion localized in the vicinity of an unstable equilibrium position. Let us consider a dynamical system whose equilibrium position is non-degenerate and unstable in Lyapunov sense, and its degree of instability is greater than zero and less than the number of degrees of freedom. The energy at the equilibrium position is assumed to be zero. It is shown that for any sufficiently small positive value of the total energy of the system, there is a motion of the system with a given energy value that begins at the boundary of the region where motion is possible and does not leave a small neighborhood of the equilibrium position. We call such motions as localized motions.

An essential condition for the presence of such movements is the limitation of system movements in "unstable directions." For natural systems with gyroscopic and dissipative forces, this is ensured by the conservation or non-increase of the total mechanical energy. The use of topological methods [Bor67] in the analysis of such motions makes possible to abandon the condition of analyticity of the first integrals and the condition of non-resonance of purely imaginary roots of the characteristic equation. The presence of time-dependent gyroscopic and dissipative forces, as well as forces with incomplete dissipation, does not interfere with the proof of the existence localized solutions.

As an example, we consider the restricted circular three-body problem. Two massive bodies, due to mutual gravitational attraction, move in the same plane in circular orbits with a constant speed around their center of mass. A third body of sufficiently small mass does not affect the motion of massive bodies, it is under the influence of attractive forces to two massive bodies On a rotating orbital plane with a beginning at this center of mass and one of the axes passing through the centers of massive bodies, taking into account inertial forces, there are five positions of relative equilibrium of a small body - libration points. Three collinear libration points have a degree of instability equal to unity, therefore, according to the Kelvin-Chetaev theorem, they cannot be stabilized by adding dissipative and gyroscopic forces. Nevertheless, in accordance with the above, localized trajectories should exist near these unstable collinear libration points. Numerical simulations for the parameters of the Earth-Moon system convincingly illustrate our theoretical study.

This is a joint work with Eugene Kugushev.

[Bor67] K. Borsuk, *Theory of retracts*, Polish Scientific Publishers, 1967.