[T L20] T. Levi-Civita, *Sur la régularisation du problème des trois corps*, Acta Mathematica **42** (1920), pp. 99–144.

Dissipative effects in Hamiltonian mean-field model

29.10
11:10-11:30

Artem Alexandrov

Institute for Information Transmission Problems & Moscow Institute of Physics and Technology & HSE University, Moscow

Hamiltonian mean-field model is the simplest possible realization of many-particle system with long-range interactions. This model has the following Hamiltonian,

$$H_{\text{HMF}} = \sum_{i=1}^{N} \frac{p_i^2}{2m} + \frac{2K}{N} \sum_{i<j}^{N} \cos\left(\phi_j - \phi_i\right), \tag{1}$$

where $\forall i$ $\phi_i \in [-\pi, +\pi)$ are phase variables, K is the coupling constant, and N^{-1} factor in the interaction term is needed for the correctly defined $N \to \infty$ limit (so-called *Kac prescription*). It is well-known fact that due to the weak convergence between empirical (discrete) & continuous measures, the HMF model has a well-defined hydrodynamic limit, which is described by the Vlasov equation. In physics HMF model is known as *kinetic* XY-model because of the kinetic energy term in the Hamiltonian. From the physical point of view, XY-model with quenched disorder is of a great interest. This quenched disorder can be modelled by adding into the equations of motion for ϕ_i random quantities ω_i with known distribution function $g(\omega)$.

In addition to quenched disorder, the dissipative processes are also common for physics. The equations of motion for model with disorder and dissipation are given by

$$m\ddot{\phi}_i + \dot{\phi}_i = \omega_i + \frac{2K}{N} \sum_{j=1}^{N} \sin\left(\phi_j - \phi_i\right). \tag{2}$$

In theory of dynamical systems, this model is known as *Kuramoto model with inertia*. All the statement about the weak convergence of measures hold for this model, too. My talk is devoted to generalization of Kuramoto model with inertia for the case with additional stochasticity (dynamical disorder) and for the more complicated structure of couplings, where the interaction is defined by the weighted non-directed graph. The outline of talk consists of

1. Brief review of HMF model
2. Usage of graphons to construct $N \to \infty$ limit
3. Application of Penrose method to analyze stability of incoherent state (uniformly distributed phases)

The concrete set of statements will be adjusted in accordance with timing. The talk is based on papers [CM19a, CM19b, CMM23] and the author's paper [AG24].

[AG24] A. Alexandrov and A. Gorsky, *Penrose method for kuramoto model with inertia and noise*, Chaos, Solitons & Fractals **183** (2024).

[CM19a] H. Chiba and G. S. Medvedev, *The mean field analysis of the Kuramoto model on graphs I. The mean field equation and transition point formulas*, Discrete and Continuous Dynamical Systems - Series A **39**:1 (2019), pp. 131–155.

[CM19b] H. Chiba and G. S. Medvedev, *The mean field analysis of the Kuramoto model on graphs II. Asymptotic stability of the incoherent state, center manifold reduction, and bifurcations*, Discrete and Continuous Dynamical Systems **39**:7 (2019), pp. 3897–3921.

[CMM23] H. Chiba, G. S. Medvedev, and M. S. Mizuhara, *Bifurcations and patterns in the Kuramoto model with inertia*, Journal of Nonlinear Science **33**:5 (2023).