On some geometric ideas in the method of averaging

29.10
12:00-12:30

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We present a topological-analytical method for proving some results of the N.N. Bogolyubov method of averaging [Bog45, BM63] for the case in which the time interval is infinite. The essence of the method is to combine topological methods of proving the existence of a periodic solution or a solution which never leaves some subset of the phase space, with the theorem of N.N. Bogolyubov on the averaging on a finite time interval. In particular, the considered approach allows us to abandon the non-degeneracy condition on the Jacobi matrix from the classical theorems of the averaging method.

We will consider the averaging procedure in the periodic and non-periodic cases; we will discuss and explain the geometry that underlies the classical results on averaging in these two cases. We will also briefly discuss the method of averaging for a part of variables and some results concerning the averaging in a neighborhood of an elliptic equilibrium in the non-periodic case. In particular, we will introduce a procedure of hyperbolization of an elliptic equilibrium, with the help of which new results on averaging over an infinite time interval can be obtained.

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[BM63] N.N. Bogolyubov and Y.A. Mitropolskij, *Asymptotic methods in the theory of non-linear oscillations*, Moscow: Nauka, 1963.

[Bog45] N.N. Bogolyubov, *On some statistical methods in mathematical physics*, Kiev: Izdat. Akad. Nauk Ukr. SSR, 1945.