Adiabatic approximation in quantum and classical mechanics, semi-rigid wall billiards and long nonlinear coastal waves

29.10
12:30-13:00

Sergey Dobrokhotov

Ishlinsky Institute for Problems in Mechanics of the RAS, Moscow

In the paper [DNT23], time-periodic solutions of a nonlinear system of shallow water equations in basins with shallow gentle shores localized in the vicinity of the coastline were constructed. In this work, the construction of such solutions is associated with special trajectories of a two-dimensional Hamiltonian system with a Hamiltonian $H = D(x_1, x_2)(p_1^2 + p_2^2)$, where the function D is the depth of the basin. We denote the coastline $\Gamma = \{D = 0\}$ and assume that $\nabla D|_\Gamma \neq 0$. As D turns to zero as $x \to \Gamma$ thus suitable solutions of the Hamiltonian system organize the so-called billiards with semi-rigid walls, woven from trajectories located between the standard caustics and the "non-standard" ones $Gamma$. (see also [BT24]). These billiards implies the asymptotic eigenfunctions of the linear operator $-\nabla(D(x)\nabla)$ for large eigenvalues and turn mentioned coastal long nonlinear waves. The existence of these billiards with semi-rigid walls is possible in the case of integrable Hamiltonian systems with Hamiltonian H, which practically does not happen in real situations. In this talk, we consider degenerate situations where "standard" caustics are very close to the coastline ("non-standard" caustics). Then "fast and slow" variables appear in the problem and we can apply the classical and quantum versions of the adiabatic approximation, the requirement of integrability disappears and it is always possible to construct effective asymptotic wave solutions having a small number of oscillations normal to the shore [DMV24] (which are analogs of Stokes and Ursell waves). The corresponding trajectories are strongly localized in the narrow vicinity of the coast, while they always enter the coastline and reflect from it at an angle of 90 degrees. Thus, we have asymptotic solutions similar to the "whispering gallery" type solutions known in acoustics, but at the same time for their existence due to a "degenerate" wall (coastline) the convexity of the two-dimensional region $(x : D(x) > 0)$ in which the pool is located is not required.

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[BT24] S. Bolotin and D. Treschev, *Another billiard problem*, Russ. J. Math. Phys. **31**:1 (2024), pp. 50–59.

[DMV24] S. Dobrokhotov, D. Minenkov, and M. Votiakova, *Asymptotics of long nonlinear coastal waves in basins with gentle shores*, Russ. J. Math. Phys. **31**:1 (2024), pp. 79–93.

[DNT23] S. Dobrokhotov, V. Nazaikinskii, and A. Tsvetkova, *Nonlinear effects and run-up of coastal waves generated by billiards with semi-rigid walls in the framework of shallow water theory*, Proc. Steklov Inst. Math. **322** (2023), pp. 105–117.