Nonlinear long standing waves with support bounded by caustics or localized in the vicinity of a two-link trajectory

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Long, nonbreaking waves in bounded water basins with a shore are described by the system (see, e.g., [Mei89])

$$\eta_t + \langle \nabla, (D(\mathbf{x}) + \eta)\mathbf{u} \rangle = 0, \quad \mathbf{u}_t + \langle \mathbf{u}, \nabla \rangle \mathbf{u} + g\nabla\eta = 0, \quad \mathbf{x} \in \mathbb{R}^2. \quad (1)$$

The free surface elevation $\eta(\mathbf{x}, t)$ and the averaged over the depth horizontal fluid velocity $\mathbf{u}(\mathbf{x}, t) = (\mathbf{u}_1(\mathbf{x}, t), \mathbf{u}_2(\mathbf{x}, t))$ are unknown functions, $D(\mathbf{x})$ is the function of basin depth. The solution is assumed to be defined in the time-dependent domain occupied by water $\{\eta(\mathbf{x}, t) + D(\mathbf{x}) \geq 0\}$. After the transformation defined by the parametric formulas (see [DMN22]) $\mathbf{x} = \mathbf{y} - N(\mathbf{y}, t)\rho(\mathbf{y})|\nabla_{\mathbf{y}} D(\mathbf{y})|^{-2}\nabla_{\mathbf{y}} D(\mathbf{y})$, $\eta = N(\mathbf{y}, t)$, $\mathbf{u} = \mathbf{U}(\mathbf{y}, t)$, where $\rho(y)$ is a cutoff function, the system (1) can be considered as the perturbation of the linear system

$$N_t + \langle \nabla_{\mathbf{y}}, D(\mathbf{y})\mathbf{U} \rangle = 0, \quad \mathbf{U}_t + g\nabla_{\mathbf{y}} N = 0.$$

The solution $(N(\mathbf{y}, t), \mathbf{U}(\mathbf{y}, t))$ is now assumed to be defined in the fixed domain $\{D(\mathbf{y}) \geq 0\}$. After eliminating of the $\mathbf{U}$-component and considering a time-harmonic solution of the form $N(\mathbf{y}, t) = e^{i\omega t}\psi(\mathbf{y})$ we obtain the eigenfunction problem $-\langle \nabla, gD(\mathbf{y})\nabla\psi \rangle = \omega^2\psi$. In the present paper ([KT23]) asymptotic (with respect to large ω) eigenfunctions $\psi(\mathbf{y})$ are constructed in the following two situations: 1) the domain is of a form of an elliptic annulus bounded by two confocal ellipses (a basin with an island); 2) there is a periodic trajectory with two reflections from the boundary of the domain.

This is a joint work with Anna Tsvetkova.

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[DMN22] S. Yu. Dobrokhotov, D. S. Minenkov, and V. E. Nazaikinskii, *Asymptotic solutions of the cauchy problem for the nonlinear shallow water equations in a basin with a gently sloping beach*, Russ. J. Math. Phys. **29** (2022), pp. 28–36.

[KT23] A. I. Klevin and A. V. Tsvetkova, *Nonlinear long standing waves with support bounded by caustics or localized in the vicinity of a two-link trajectory*, Russ. J. Math. Phys. **30** (2023), pp. 543–551.

[Mei89] C. C. Mei, *The applied dynamics of ocean surface waves*, World Scientific, 1989.