Lagrangian manifolds with degenerate fold and applications to the theory of wave beams

29.10
13:40-14:00

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We discuss a geometric approach based on the Maslov canonical operator theory [MF01] to constructing the asymptotics of solutions of (pseudo)differential problems. The problem is associated with a geometric object – a Lagrangian manifold in the phase space. The type of singularity on this manifold determines the type of special function in terms of which the asymptotics can be expressed. In particular, singularities of the type of degenerate fold correspond to an asymptotic expression in terms of the Bessel function J_0.

This kind of singularities arises in the theory of wave beams, in particular, for Bessel and Laguerre-Gauss beams. Laguerre-Gauss beams are solutions of the three-dimensional Helmholtz equation in the paraxial approximation (which can be considered as the Shrödinger equation). The considered beams are the product of the Gaussian exponent and the Laguerre polynomials. The discussed approach to constructing asymptotics is based on studying the dynamics of the initial Lagrangian manifold along a Hamiltonian vector field with a Hamiltonian corresponding to the Schrödinger equation. It allows us to obtain an effective asymptotics of such beams in terms of Airy and Bessel functions of compound argument [DNT23].

One of the advantages of the discussed approach is that it is quiet universal. In particular, it allows to abandon the paraxial approximation and consider the original Helmholtz equation. In the talk the global asymptotics in terms of special functions of the solution of the Helmholtz equation with "initial" conditions generated by Laguerre-Gauss beams will be also given.

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[DNT23] S. Dobrokhotov, V. Nazaikinskii, and A. Tsvetkova, *Asymptotics of the localized bessel beams and Lagrangian manifolds*, Journal of Communications Technology and Electronics **68**:6 (2023), pp. 625–638.

[MF01] V. Maslov and M. Fedoruik, *Semi-classical approximation in quantum mechanics*, D. Reidel Publishing Company, 2001.