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Morse-Smale systems on simply connected manifolds

Ilya Saraev

HSE University, Nizhny Novgorod

A Morse-Smale system dynamical system given on a closed manifold M^n is a structurally stable system whose non-wandering set consists of finite number of orbits. In 1961, Smale obtained inequalities connecting the number of fixed point and periodic orbits of such systems and Betty numbers of the ambient manifold similar to Morse inequalities for Morse function. In particular, it was proven, that if a Morse-Smale diffeomorphism $f : M^n \to M^n$ does not have periodic points with one-dimensional unstable invariant manifolds, then the homology group $H_1(M^n)$ is trivial. However, for $n \geq 3$ it does not mean that the fundamental group $\pi_1(M^n)$ of M^n is trivial. The following theorem enhances this result.

Theorem 1. *Let M^n be a closed manifold, $n \geq 3$, and $f : M^n \to M^n$ is a Morse-Smale diffeomorphisms such that $(n-1)$-dimensional invariant manifold of arbitrary saddle periodic point does not contain heteroclinic submanifolds. If $k_1 = 0$ then M^n is simply connected.*

For $n = 3$, the unique simply connected manifold is the sphere S^3 and Theorem 1 follows from [Bon+02] and [OE]. For $n \geq 4$, there are a numerous of simply connected manifolds not homeomorphic to sphere (for instance, $S^k \times S^l$, $k, l \geq 2, k + l = n$), but a complete classification of smooth simply connected manifolds is known only for $n = 4$ due to non-trivial results of Rochlin, Freedman, Donaldson and Furuta (see [A05] for references).

To proof Theorem 1, we obtain the following topological version of well known smooth result that can be of independent interest:

Proposition 1. *Let Q^{n-1}, M^n be closed topological manifolds, Q^{n-1} is simply connected and locally flat in M^n. Then there exists an embedding $e : Q^{n-1} \times [-1, 1] \to M^n$ such that $e(Q^{n-1} \times \{0\}) = Q^{n-1}$.*

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[Bon+02] C. Bonatti, V. Grines, V. Medvedev, and E. Pecou, *Three-dimensional manifolds admitting Morse-Smale diffeomorphisms without heteroclinic curves*, Topology and Applications **117** (2002), pp. 335–344.

[OE] Pochinka O.V. and Osenkov E.M., *The unique decomposition theorem for 3-manifolds, admitting Morse-Smale diffeomorphisms without heteroclinic curves*, to appear.