31.10
10:40-11:10

The Gaussian multiplicative chaos for the sine-process

Alexander I. Bufetov

Steklov Mathematical Institute of the RAS, Moscow

Grigori Olshanski [Ols11] posed the problem of deciding when two determinantal measures are mutually absolutely continuous and of finding the corresponding Radon—Nikodym derivative.

Olshanski proved that the determinantal point process on $\mathbb{Z}$ governed by the Gamma-kernel is quasi-invariant under the group of finite permutations of $\mathbb{Z}$ and computed the Radon—Nikodym derivative, a multiplicative functional given by a generalized Euler product.

In development of Olshanski's programme it has been shown ([Buf18], Sept. 2014) that determinantal point processes governed by integrable kernels satisfying a regularity condition are quasi-invariant under the group of finite permutations in the discrete case and under the group of diffeomorphisms with compact support in the continuous case.

The Radon—Nikodym derivative is found explicitly as a regularized multiplicative functional over pairs of particles of our configuration. The key point is the equivalence of reduced Palm measures of the same order for our processes (that are rigid in the sense of Ghosh and Peres [GP17]): in this case the Radon—Nikodym derivative is given by a regularized multiplicative functional over particles of our process, in other words, a random Euler product. In the particular case of the sine-process the square root of the Radon—Nikodym derivative coincides, up to a linear multiple, with the "stochastic zeta-function" studied by Chhaibi, Najnudel, Nikeghbali [CNN17].

Recall now that the Gaussian multplicative chaos is a random measure introduced by Mandelbrot, Peyrière and Kahane in development of the 1941 theory of the local structure of turbulence by Andrei Nikolaevich Kolmogorov [Kol41]. Since the work of Yan Fyodorov and his collaborators [FHK12, FK14, FKS16, FS16] the convergence to the Gaussian multiplicative chaos for characteristic polynomials of random matrices has been a subject of intense research (see, e.g., [Ber17, BWW, LN, Web15], and references therein).

The random Euler product obtained as the square root of the Radon—Nikodym derivative of Palm measures of a determinantal point process naturally arises in the problem of minimality of realizations of our process in the Hilbert space that governs it.

In joint work with Yanqi Qiu and Alexander Shamov [BQS21] it is proved that almost every realization of a determinantal point process governed by an orthogonal projection is a complete set for the underlying reproducing kernel Hilbert space. The result had been conjectured by Lyons and Peres; in the case of the discrete space, it had been proved by Lyons; in the rigid case and, in particular, for the sine process, by Ghosh [Gho15].

Our complete set is however not minimal: indeed, almost every realization of the sine process has excess 1 for the Paley—Wiener space, that is, the configuration stays complete and becomes minimal after one particle is removed [Buf]. The reason for the excess one is that the suitably rescaled random Euler product converges in distribution to the Gaussian multiplicative chaos.

[Ber17] N. Berestycki, *An elementary approach to Gaussian multiplicative chaos*, Electron. Commun. Probab. **22** (2017), Paper No. 27.

[BQS21] Alexander I. Bufetov, Yanqi Qiu, and Alexander Shamov, *Kernels of conditional determinantal measures and the Lyons–Peres completeness conjecture*, J. Eur. Math. Soc. **23**:5 (2021), pp. 1477–1519.

[Buf] Alexander I. Bufetov, *The sine-process has excess one*, arXiv:1912.13454, 57 pp.

[Buf18] Alexander I. Bufetov, *Quasi-symmetries of determinantal point processes*, Ann. Probab. **46**:2 (2018), pp. 956–1003.

[BWW] Nathanaël Berestycki, Christian Webb, and Mo Dick Wong, *Random Hermitian matrices and Gaussian multiplicative chaos*, arXiv:1701.03289, 61 pp.

[CNN17] Reda Chhaibi, Joseph Najnudel, and Ashkan Nikeghbali, *The Circular Unitary Ensemble and the Riemann zeta function: the microscopic landscape and a new approach to ratios*, Invent. math. **207** (2017), pp. 23–113.

[FHK12] Y. V. Fyodorov, G. A. Hiary, and J. P. Keating, *Freezing transition, characteristic polynomials of random matrices, and the Riemann zeta function*, Phys. Rev. Lett. **108** (2012), 170601, 5 pp.

[FK14] Y.V. Fyodorov and J. Keating, *Freezing transitions and extreme values: random matrix theory, and disordered landscapes*, Philos. Trans. R. Soc. Lond. Ser. A Math. Phys. Eng. Sci. **372** (2014), no. 2007, 20120503.

[FKS16] Y.V. Fyodorov, B. A. Khoruzhenko, and N. Simm, *Fractional Brownian motion with Hurst index $H = 0$ and the Gaussian Unitary Ensemble*, Ann. Probab. **44**:4 (2016), pp. 2980–3031.

[FS16] Y.V. Fyodorov and N. Simm, *On the distribution of maximum value of the characteristic polynomial of GUE random matrices*, Nonlinearity **29** (2016), pp. 2837–2855.

[Gho15] Subhroshekhar Ghosh, *Determinantal processes and completeness of random exponentials: the critical case*, Probab. Th. Relat. Fields **163** (2015), pp. 643–665.

[GP17] Subhroshekhar Ghosh and Yuval Peres, *Rigidity and tolerance in point processes: Gaussian zeros and ginibre eigenvalues*, Duke Math. J. **166**:10 (2017), pp. 1789–1858.

[Kol41] A.N. Kolmogorov, *The local structure of turbulence in incompressible viscous fluid for very large Reynolds numbers*, Dokl. Akad. Nauk SSSR **30**:4 (1941).

[LN] Gaultier Lambert and Joseph Najnudel, *Subcritical multiplicative chaos and the characteristic polynomial of the $C\beta E$*, arXiv:2407.19817, 35 pp.

[Ols11] Grigori Olshanski, *The quasi-invariance property for the Gamma kernel determinantal measure*, Adv. Math. **226**:3 (2011), pp. 2305–2350.

[Web15] C. Webb, *The characteristic polynomial of a random unitary matrix and gaussian multiplicative chaos – the L^2-phase*, Electron. J. Probab. **20**:104 (2015).