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Admissibility of discontinuities in the solutions of a hyperbolic 2×2 system of conservation laws

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The work is devoted to the study of the problem of admissibility of discontinuities in the solutions of a hyperbolic system of two conservation laws describing quasitransverse waves in nonlinearly elastic weakly anisotropic media obtained in [Kul86]. The standard viscous regularization method is applied to the defining system of equations. Regularization leads to the situation where two different viscosity profiles may correspond to the discontinuity [KS95, CP23].

The solutions of the system of equations depend on the sign of the parameter multiplying the fourth-order nonlinear term of the flux function. In this work the study of the stability of viscosity profiles has been carried out for positive and negative values of the nonlinearity parameter $\varkappa$. A single stable structure is found for each of these cases. If $\varkappa > 0$, then the "upper" structure is stable, while, if $\varkappa < 0$, then the "lower" structure is stable. The analysis of the linear (spectral) stability of these two profiles has shown that one of them is stable while the other is unstable.

We have numerically solved the Riemann problem in the case when the initial discontinuity corresponds to two different viscosity profiles. The results of calculations have shown that the asymptotics of a nonstationary solution of the Riemann problem represents a linearly stable viscosity profile. A linearly unstable viscosity profile is not a solution to the Riemann problem.

This conclusion demonstrates that the definition of admissibility of a discontinuity should include the requirement of stability of the viscosity profile.

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[Kul86] A. Kulikovskii, *Equations describing the propagation of non-linear quasitransverse waves in a weakly non-isotropic elastic body*, Journal of Applied Mathematics and Mechanics **50** (1986), pp. 455–461.